Functions

What is a function? f "of" x / f "at" x

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$.

How do we identify the domain and range of a function?

- First must know what these terms mean...define each.
- Must know how to indicate domain and range using correct notation (Set and Bracket)

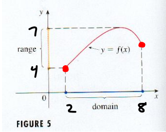


FIGURE 5

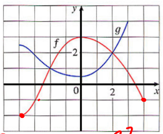
Domain → all possible x values

$\{x \mid 2 \le x \le 8, x \in R\}$

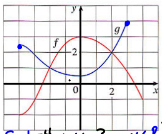
Range → all possible y values

$\{y \mid 4 \le y \le 7, y \in R\}$

Examples:



$\{x \mid -4 \le x \le 4, x \in R\}$
$\{y \mid -2 \le y \le 3, y \in R\}$



$\{x \mid -4 \le x \le 3, x \in R\}$
$\{y \mid 0.5 \le y \le 4, y \in R\}$

Set Notation $-2 < x \le 5$ new way → Bracket $(-2, 5]$

-3 -2 -1 0 1 2 3 4 5 6 7

$x \ge 3$ OR $3 \le x < \infty$ $[3, \infty)$

-1 0 1 2 3 4 5

$-2 \le x \le 5$ $[-2, 5]$

-3 -2 -1 0 1 2 3 4 5

$1 \le x < 4$ $[1, 4)$

-1 0 1 2 3 4 5

$x \ge -2$ OR $-2 \le x < \infty$ $[-2, \infty)$

-3 -2 -1 0 1 2 3 4

$-3 < x \le 5$ $(-3, 5]$

-4 -3 -2 -1 0 1 2 3 4 5 6

$x < 4$ $-\infty < x < 4$ $(-\infty, 4)$

-3 -2 -1 0 1 2 3 4 5 6

Notation	Set Description	Picture
(a, b)	$\{x \mid a < x < b\}$	a b
$[a, b]$	$\{x \mid a \le x \le b\}$	a b
$[a, b)$	$\{x \mid a \le x < b\}$	a b
$(a, b]$	$\{x \mid a < x \le b\}$	a b
(a, ∞)	$\{x \mid x > a\}$	a
$[a, \infty)$	$\{x \mid x \ge a\}$	a
$(-\infty, b)$	$\{x \mid x < b\}$	b
$(-\infty, b]$	$\{x \mid x \le b\}$	b
$(-\infty, \infty)$	R (set of all real numbers)	

$x \in R$

1. Express the interval in terms of inequalities and graph the interval.
   (a) $(-2, 6)$ (b) $[-3, -2]$
   (c) $(1, 4]$ (d) $[-2, 1.5]$
   (e) $[3, \infty)$ (f) $(-\infty, 2)$
   (g) $(-\infty, 1]$ (h) $(-\frac{3}{2}, \infty)$
2. Express the inequality in interval notation and graph the corresponding interval.
   (a) $x < 2$ (b) $0 < x < 3$
   (c) $-1 \le x < 2$ (d) $x > 1$
   (e) $-1 \le x \le 3$ (f) $x \le -1$

1c) $1 < x \le 4$

-1 0 1 2 3 4 5 6

g) $-\infty < x \le 1$ OR $x \le 1$

-2 -1 0 1 2 3

2c) $[-1, 2)$

-3 -2 -1 0 1 2 3 4 5

1c) $(1, \infty)$

-1 0 1 2 3 4 5 6 7

Function Notation

- Must understand the notation associated with determing the values of functions

I. From a graph $f(x)$ (x, y) $(x, f(x))$

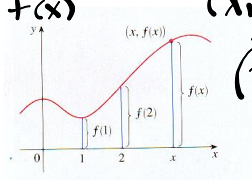


FIGURE 4

II. From a table of values

x	$f(x)$

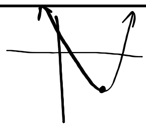
III. From an explicit formula (Equation)

$f(x) = -2x^2 + 5x - 3$ ← Explicit formula !

$f(-3) = ?$ $f(\$) = ?$

$f(2 - h) = ?$

Domain & Range



$y=3(x-5)^2-4$

$\{x \mid x \in R\}$

$\{y \mid y \geq -4, y \in R\}$

a) $y=-2(x+7)^2-3$ b) $y=3x^2+12x-5$

Feb 3-10:29 AM

5b) $f(x)=x^2-2$

$f(-x)=(-x)^2-2$

x^2-2

$f(-x)-2$

x^2-2-2

x^2-4

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6c) $f(x)=\dfrac{x-2}{x}$

$$f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}-2}{\frac{1}{x}}$$

$$\frac{1-2x}{\cancel{x}}\cdot\frac{\cancel{x}}{1}$$

$$\boxed{1-2x}$$

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$f(x)=\dfrac{x-2}{x}$ $f(-x)=\dfrac{(-x)-2}{(-x)}$

$=\dfrac{-x-2}{-x}$

$=\dfrac{x+2}{x}$

$f(f(-x))$

$$f\left(\frac{x+2}{x}\right)=\frac{\left(\frac{x+2}{x}\right)-2}{\left(\frac{x+2}{x}\right)}$$

$$=\frac{\frac{x+2-2x}{x}}{\frac{x+2}{x}}$$

$$=\frac{-x+2}{\cancel{x}}\cdot\frac{\cancel{x}}{x+2}$$

$$=\frac{-x+2}{x+2}$$

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Bracket Notation

Notation used to describe "SETS" of numbers...

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Check-Up

1. If $\sin\theta = -\frac{1}{\sqrt{10}}$ and $\cos\theta < 0$ find $\tan\theta$

2. Determine the domain and range of the quadratic $f(x) = -5x^2 + 10x - 3$.

Domain $x \in \mathbb{R}$

$f(-5) = -5(-5)^2 + 10(-5) - 3$
$= -5(25) - 50 - 3$
$= -125 - 50 - 3$
$= -178$

$f(x) = -5x^2 + 10x - 3$ y-int

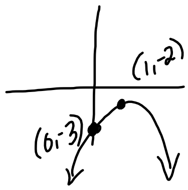
$= -5(x^2 - 2x + 1 - 1) - 3$

$= -5(x-1)^2 + 5 - 3$

$= -5(x-1)^2 - 2$

Max Value is -2 when $x = 1$

Range: $y \le -2$



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Domain → watch out for:

ex:

$f(x) = \frac{3}{x-2}$ → $x - 2 \neq 0$, $x \neq 2$

$x \in \mathbb{R}$, $x \neq 2$

$(-\infty, 2), (2, \infty)$

$f(x) = \sqrt{x+5}$ → $x + 5 \ge 0$, $x \ge -5$

Domain: $x \ge -5$

$[-5, \infty)$

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Check for Understanding...

Select the best response for each of the following:

$y = \frac{1}{x}$

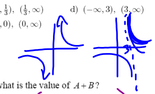
1. Find the domain of $f(x) = \sqrt{2x+3}$.

a) $[0, \infty)$ b) $(0, \infty)$ c) $[-\frac{3}{2}, \infty)$
d) $(-\frac{3}{2}, \infty)$ e) $[0, \frac{3}{2})$

$2x + 3 \ge 0$
$2x \ge -3$
$x \ge -3/2$

2. Find the range of the function $y = \frac{1}{x-3}$.

a) $(3, \infty)$ b) $(-\infty, 3)$
c) $(-\infty, \frac{1}{3}), (\frac{1}{3}, \infty)$ d) $(-\infty, 3), (3, \infty)$
e) $(-\infty, 0), (0, \infty)$



3. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6 (B) -3 (C) -1 (D) 2
(E) It cannot be determined from the information given.

(hint: system of equations)

$3 = 2(2)^3 + A(2)^2 + B(2) - 5$
$3 = 16 + 4A + 2B - 5$
$3 = 11 + 4A + 2B$
$-8 = 4A + 2B$

$-37 = 2(-2)^3 + A(-2)^2 + B(-2) - 5$
$-37 = -16 + 4A - 2B - 5$
$-16 = 4A - 2B$

① − ②

$-8 = 4A + 2B$ ①
$-16 = 4A - 2B$ ②
$8 = 4B$
$\boxed{2 = B}$

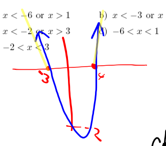
sub $B = 2$ into ①

$-8 = 4A + 2(2)$
$-8 = 4A + 4$
$-12 = 4A$
$-3 = A$

$2 + (-3)$
-1

4. Solve: $x^2 - x > 12$

a) $x < -6$ or $x > 1$ b) $x < -3$ or $x > 4$
c) $x < -2$ or $x > 3$ d) $-6 < x < 1$
e) $-2 < x < 3$



$x^2 - x > 12$
$x^2 - x - 12 > 0$
$(x-4)(x+3) = 0$
$x = 4$ $x = -3$

① find x-int

chart analysis

	$(x-4)$	$(x+3)$	$f(x)$
$x < -3$	−	−	+
$-3 < x < 4$	−	+	−
$x > 4$	+	+	+

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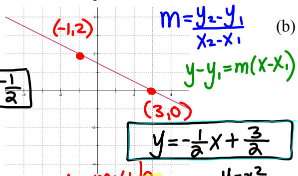
Functions continued...

1. Determine the equation that describes each of the following:

(a)



$m = -\frac{1}{2}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = m(x - x_1)$

$\boxed{y = -\frac{1}{2}x + \frac{3}{2}}$

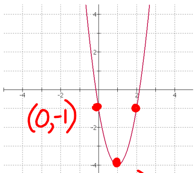
$y = mx + b$

$0 = -\frac{1}{2}(3) + b$

$0 = -\frac{3}{2} + b$

$1.5 = \frac{3}{2} = b$

(b)



$y = x^2$

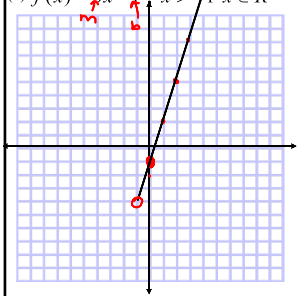
x	y
-3	9
0	0
1	1
2	4
3	9

$y = a(x-h)^2 + k$

$\boxed{y = 3(x-1)^2 - 4}$

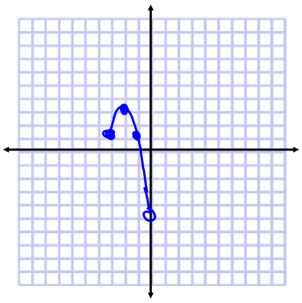
2. Sketch each of the following:

(a) $f(x) = 3x - 1, \quad x > -1 \quad x \in R$



(b) $f(x) = -2(x+2)^2 + 3, \; -3 \le x < 0, \; x \in R$

$h = -2 \quad k = 3$



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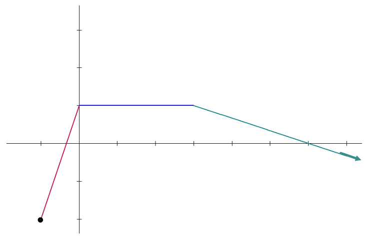
Quick Review of Functions

- What is a function?
- Domain and Range-----> Remember to look at restrictions on functions
- Function Notation
- How to check for a function (Table and Graph)

Let's head into a new direction...

What is the equation of the function that would describe the graph shown below???



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Piecewise Defined Functions

Definition:

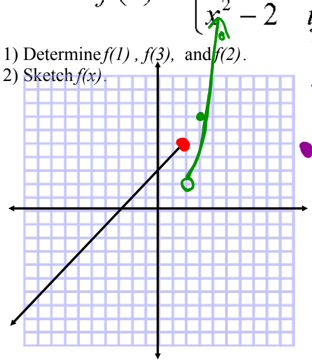
- **Functions defined by different formulas in different parts of their domains**

Example:

$$f(x) = \begin{cases} x+3 & if \; x \le 2 \\ x^2 - 2 & if \; x > 2 \end{cases}$$

1) Determine $f(1)$, $f(3)$, and $f(2)$.

2) Sketch $f(x)$.



$y = x+3$

x	y
0	3
1	4
2	5

$y = x^2 - 2$

x	y
0	2
3	7
4	14

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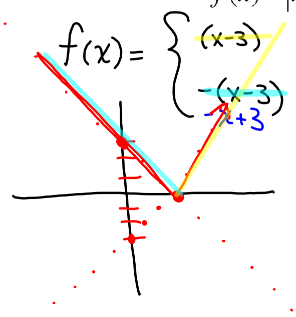
More Practice...

- Express the following absolute value function as a piecewise function
- Sketch the function

$$f(x) = |x - 3|$$

$$f(x) = \begin{cases} (x-3) & \text{if } f(x) \ge 0 \quad x-3 \ge 0, \; x \ge 3 \\ -(x-3) = -x+3 & \text{if } f(x) \le 0 \quad x < 3 \end{cases}$$



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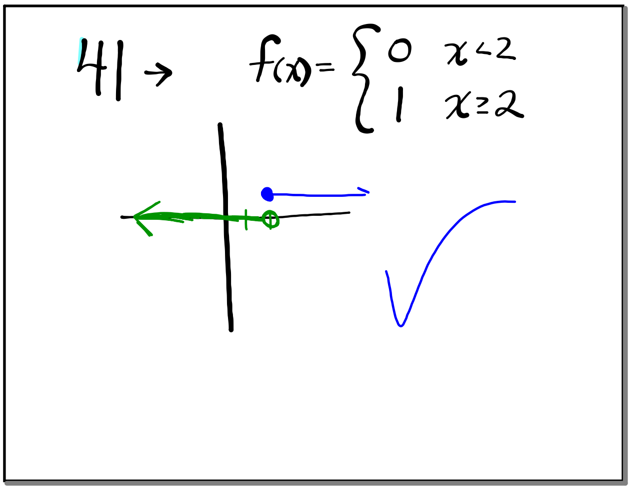

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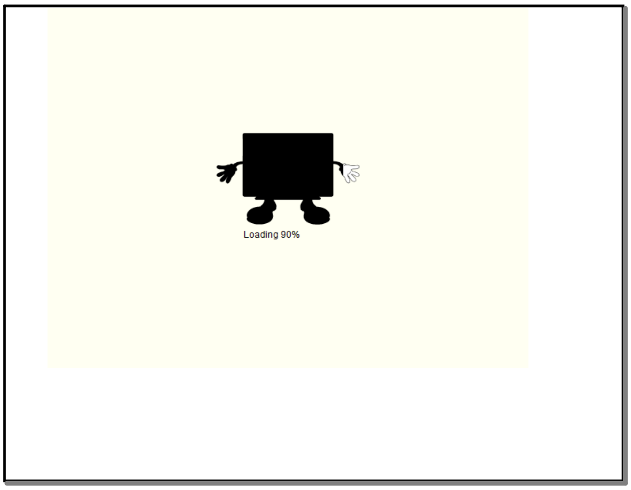

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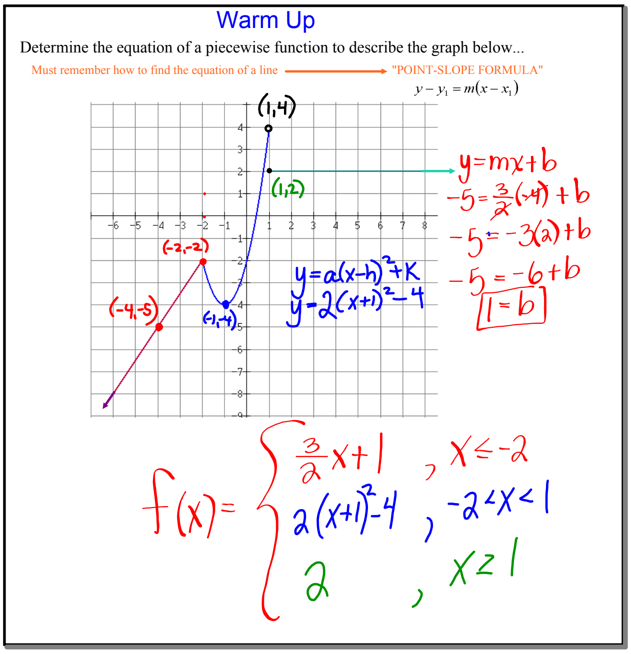

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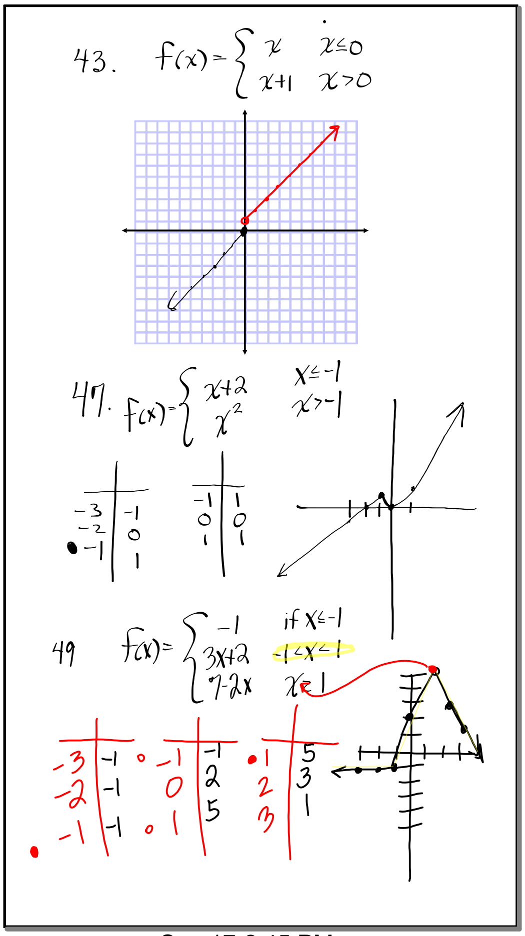

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Sketch the following piecewise function:

$$f(x)=\begin{cases}-2x+3, & \text{if } x\le -2\\ (x+1)^2-2, & \text{if } -2<x\le 1\\ 3x-1, & \text{if } 1<x<2\\ 5, & \text{if } x\ge 2\end{cases}$$

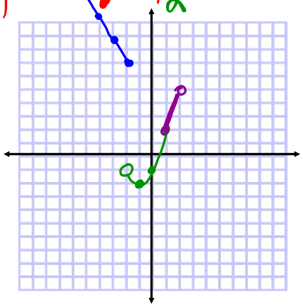
$y=-2x+3$

x	y
-4	11
-3	9
-2	7

x	y
-2	-1
-1	-2
0	-1
1	2

x	y
1	2
2	5

x	y
2	5
3	5
4	5



Given the function: $f(x)=-3|4-3x|+2$

(a) Evaluate *f*(2)

(b) Express *f*(x) as a piecewise function

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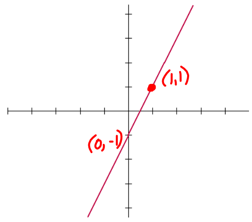
Catalog of Essential Functions (Relation)

1. Linear



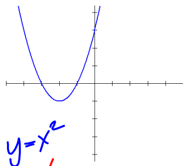
- Straight line
- Equation will be degree one
- Should be able to identify *slope intercepts*, and *equation* from the graph

$y=mx+b$

$y=\frac{2}{1}x-1$

$y=2x-1$

2. Quadratic



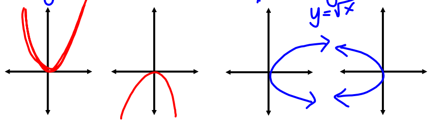
- Parabola (U-Shaped)
- Either *x* or *y* will be squared (Not both!)
- Should know the 4 basic quadratic functions
- Should be able to apply transformations to the basic quadratic functions

$y=a(x-h)^2+k$

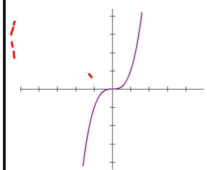
*new $x=y^2$

$y=\sqrt{x}$

$y=x^2$



3. Cubic



- S-Shaped
- Will work with functions having *x* raised to the third power
- Should be able to apply transformations to the basic quadratic functions

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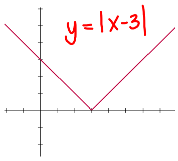
4. Absolute Value

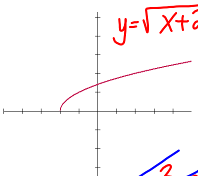
$y=|x-3|$



- V-Shaped
- Equation will have a variable within the absolute value bars
- Should be able to apply transformations to the basic absolute value functions

5. Square Root

$y=\sqrt{x+2}$



- Half parabola
- Equation will have a variable under the square root sign
- Should be able to apply transformations to the basic square root function

$\sqrt{x^2}=\sqrt{36}$

$x=\pm 6$

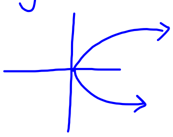
$\sqrt{36}$

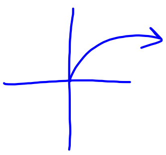
6

$y^2=x$

$y=\pm\sqrt{x}$

$y=\sqrt{x}$





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Warm-Up...

Given the function $f(x)=\begin{cases}2-x^2 & \text{if } x<1\\ 3 & \text{if } x=1\\ 2x-1 & \text{if } 1<x\le 3\\ (x-4)^2 & \text{if } x>3\end{cases}$

Evaluate the following: f(-1) f(1) f(3) f(2)

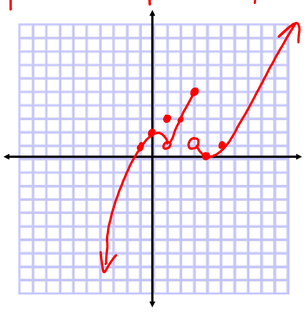
- Draw a sketch of this function

x	y
-1	1
0	2
1	1

x	y
1	3

x	y
1	1
2	3
3	5

x	y
3	1
4	0
5	1



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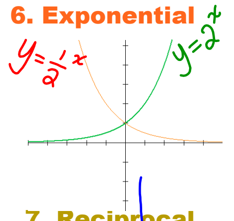
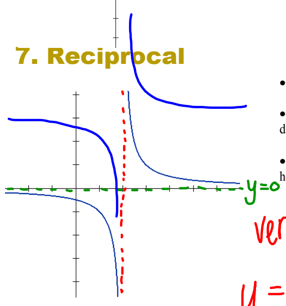
6. Exponential

$y = \left(\frac{1}{2}\right)^x$ $y = 2^x$



- Steadily increasing or decreasing
- Base will be a number and variable will appear in the exponent
- Should be able to identify the horizontal asymptote

7. Reciprocal



- Will have two branches
- Equation will have a variable within denominator of a rational expression
- Be able to identify the vertical and horizontal asymptotes

vertical asymptote $x=2$

$y = \frac{1}{x-2} + 3$

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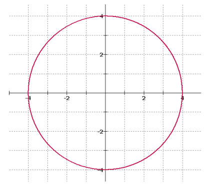
8. Circle

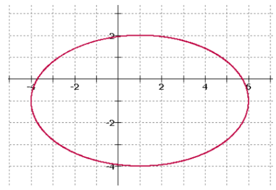


- General form: $(x-h)^2 + (y-k)^2 = r^2$

* center: (h, k)
* radius $= r$

- Be able to identify the function that would describe either just the top or bottom of the circle.

9. Ellipse



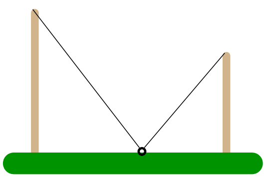
- General form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Where…
- Center: (h, k)
- a > b
- If a is the denominator of the "y" term the ellipse will have a vertical major axis.

Sep 9-10:19 PM

PRACTICE...

Two vertical poles of lengths 6 feet and 8 feet stand on level ground, with their bases 10 feet apart. There is an anchor located at some point along the ground between the two poles. A cable will run from the top of one pole to the anchor and then up to the top of the other pole. Determine a function, in terms of the distance from one of the poles to the anchor, that would represent the total length of this cable.



Sep 13-9:40 PM

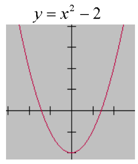
Symmetry

Even

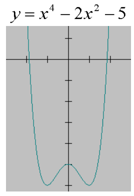
$f(-x) = f(x)$

Even functions are symmetric about the y-axis

$y = x^2 - 2$



$y = x^4 - 2x^2 - 5$

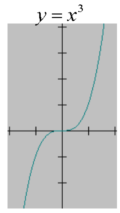


Odd

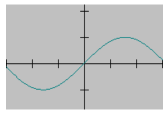
$f(-x) = -f(x)$

Odd functions are symmetric about the origin

$y = x^3$



$y = \sin(x)$



Sep 6-9:13 PM

New Functions from Old Functions...TRANSFORMATIONS

- Translations
- Stretches
- Reflections

Sep 11-9:40 PM

Translation

- To *translate* or *shift* a graph is to move it up, down, left, or right without changing its shape.
- Translation is summarized by the following table and illustration:

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward
$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward
$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right
$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

$y = f(x) + K$ $\uparrow$
$y = f(x) - K$ $\downarrow$
$y = f(x+h)$ $\leftarrow$
$y = f(x-h)$ $\rightarrow$

Feb 4-1:35 PM

Translations illustrated...

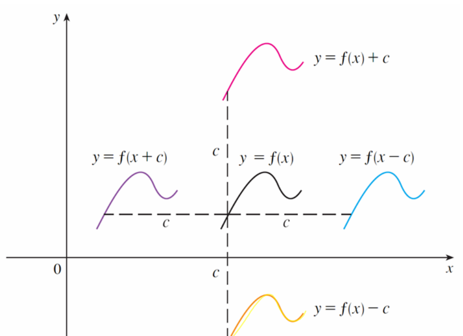
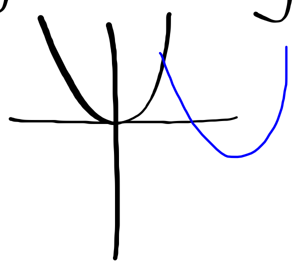


FIGURE 1
Translating the graph of f

Feb 4-1:34 PM

new

$y = x^2 \qquad y = (x-5)^2 - 2$



$h = 5 \quad K = -2$

$y = a(x-h)^2 + K$

mapping notation $(x, y) \rightarrow (x+h, ay+K)$

Transformational Form $y - K = a(x-h)^2$

Sep 18-2:39 PM

P12 → #1.

$h=0$

$k=5$

a) $y-5=f(x)$

$y=f(x)+5$

up 5 units!

1d) $y+3=f(x-7)$

$y=f(x-7)-3$

7 right, 3 down

$h=7$ $\quad k=-3$

3 a) $y=f(x+10)$

$(x,y)\to(x-10,y)$

d) $y-3=f(x-1)$

$y=f(x-1)+3$

$(x,y)\to(x+1,y+3)$

Sep 18-2:45 PM

Identify the translations for each of the following...

$f(x)=(x+7)^2$

$f(x)=|x|+3$

$f(x)=\sqrt{x-3}-2$

$f(x)=\dfrac{1}{x-5}+7$

Feb 4-1:17 PM

Using Mapping Notation to Describe Transformations:

*Think of this as a set of instructions to follow to TRANSFORM a graph

x	y = x²
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

(x,y)

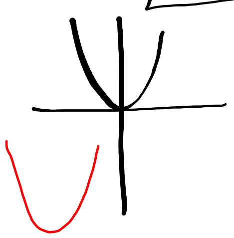
x	y = x² + 2
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

$(x,y)\to(x,\ y+2)$

x	y = (x − 5)²
2	9
3	4
4	1
5	0
6	1
7	4
8	9

$(x,y)\to(x+5,\ y)$

$$\boxed{y=1(x+3)^2-10}$$



$h=-3 \qquad k=-10$

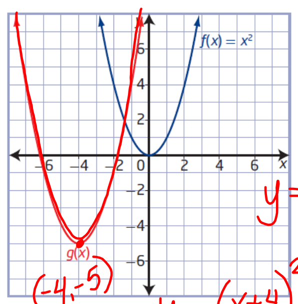
$(x,y)\to(x+h,y+k)$

$(x-3,y-10)$

Feb 4-2:36 PM

Determine the Equation of a Translated Function

a)

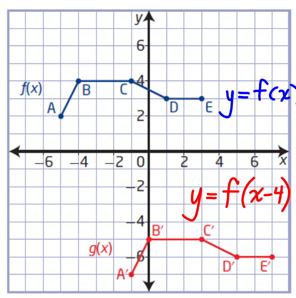


$y=f(x+4)-5$

$(-4,-5)$

$y=(x+4)^2-5$

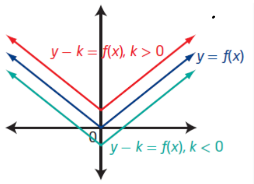
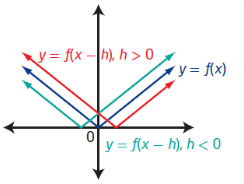
b)



$y=f(x)$

$y=f(x-4)-9$

Feb 4-2:44 PM

Transformation from $y = f(x)$	Mapping	Example
A vertical translation If $k > 0$, the translation is up. If $k < 0$, the translation is down.	$(x, y) \rightarrow (x, y + k)$	
A horizontal translation If $h > 0$, the translation is to the right. If $h < 0$, the translation is to the left.	$(x, y) \rightarrow (x + h, y)$	

Feb 4-2:47 PM

Practice Problems...

Page 13 - 15
#3, 5, 6, 7, 10, 11, 18

Feb 4-2:50 PM

Check-Up...

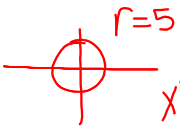
Copy and complete the table. #1, 3, 8

p13 #8

Translation	Transformed Function	Transformation of Points
vertical ↗	$y = f(x) + 5$	$(x, y) \rightarrow (x, y + 5)$
horizontal	$y = f(x + 7)$	$(x, y) \rightarrow (x - 7, y)$
h	$y = f(x - 3)$	$(x,y) \rightarrow (x+3, y)$
v	$y = f(x) - 6$	$(x,y) \rightarrow (x, y-6)$
horizontal and vertical	$y + 9 = f(x + 4)$ $y = f(x+4) - 9$	$(x,y) \rightarrow (x-4, y-9)$
horizontal and vertical	$y = f(x-4) - 6$ $y + 6 = f(x-4)$	$(x, y) \rightarrow (x + 4, y - 6)$
h & v	$y = f(x+2) + 3$	$(x, y) \rightarrow (x - 2, y + 3)$
horizontal and vertical	$y = af(x - h) + k$	$(x,y) \rightarrow (x+h, ay+k)$

p12 # 2a,c #4b

Feb 4-2:50 PM



$x^2+y^2=25$

Reflections and Stretches

Focus on . . .

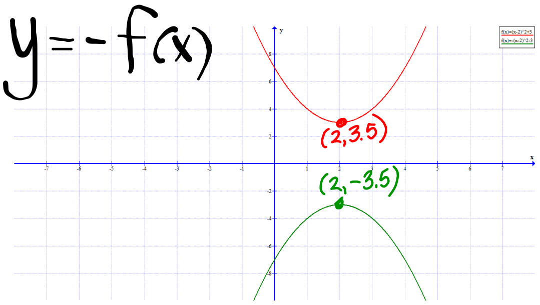
- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

Feb 5-2:43 PM

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

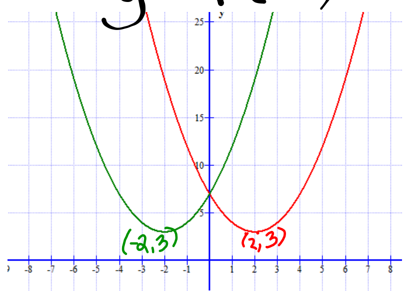
$y = -f(x)$



Feb 5-2:46 PM

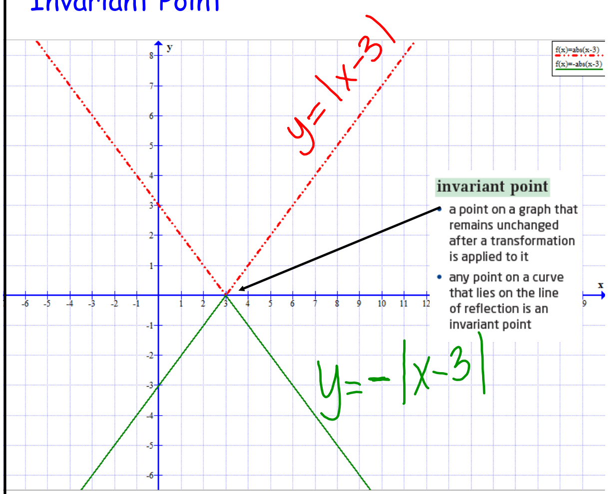
- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

$y = f(-x)$



Feb 5-3:07 PM

Invariant Point



invariant point

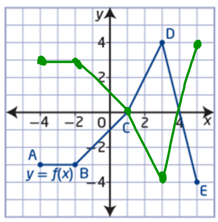
- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

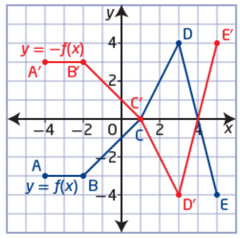
Feb 5-3:34 PM

Remember...

- When the output of a function $y = f(x)$ is multiplied by -1, the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below

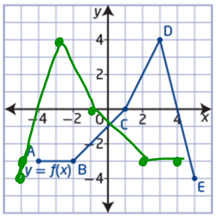


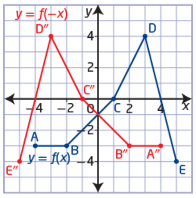


Feb 5-3:01 PM

Remember...

- When the input of a function $y = f(x)$ is multiplied by -1, the result, $y = f(-x)$, is a reflection of the graph in the y-axis.
- Sketch $y = f(-x)$ on the axis below





Feb 5-3:05 PM

Stretches and Compressions...

stretch

- a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Feb 5-3:13 PM

Vertical and Horizontal Stretches

vertical stretch $y = af(x)$ $\frac{1}{a}y = f(x)$

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $\boxed{y = af(x)}$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.

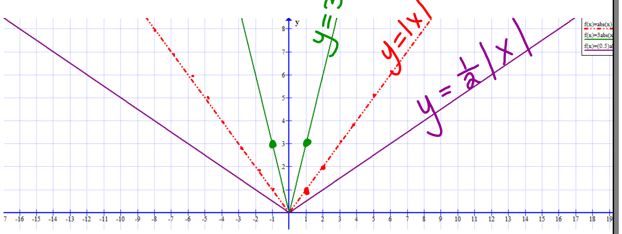
horizontal stretch

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $\boxed{y = f(bx)}$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Feb 5-3:14 PM

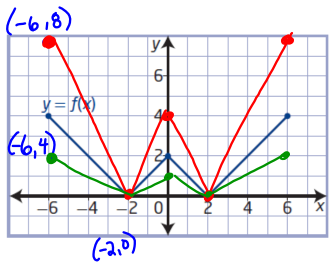
Vertical Stretch or Compression...

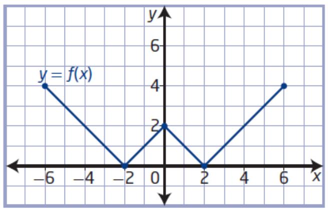
- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a, the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis.



$y = |x|$ $y = 3|x|$ $y = \frac{1}{2}|x|$

Feb 5-3:21 PM





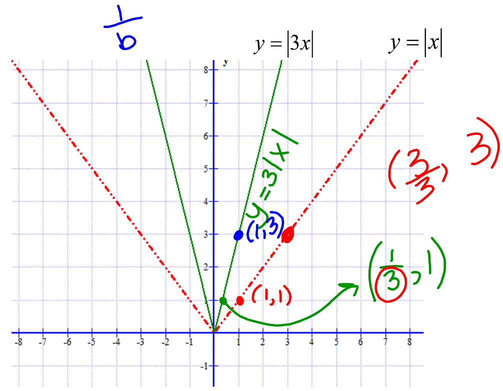
Sketch each of the following:

a) $g(x) = 2f(x)$

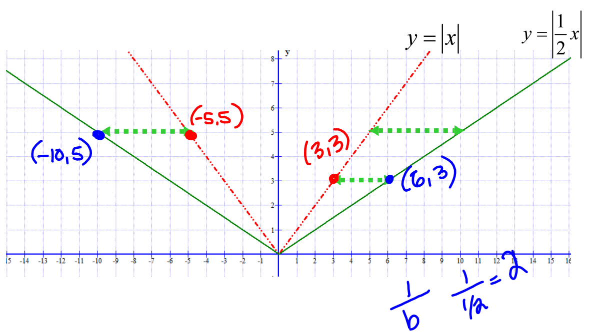
b) $g(x) = \frac{1}{2}f(x)$

Feb 6-3:06 PM

Horizontal Stretch or Compression...



Feb 5-3:31 PM



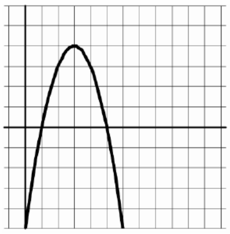
Feb 6-2:46 PM

Horizontal Stretch or Compression...

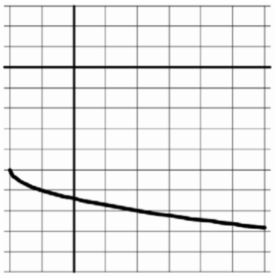
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b, the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

Feb 6-3:02 PM

Example 1: Apply $f\left(\frac{1}{2}x\right)$ to the graph.



Apply f(2x) to the given graph.



Feb 6-3:04 PM

Practice problems...

Page 28 - 31
#5, 6, 7, 8, 9, 14, C4

Feb 7-1:14 PM

Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

(2) $y = f\left(-\frac{1}{3}x\right)$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

(4) $y - 5 = -2f(-2x+6)$

Feb 7-1:21 PM

Summary of Transformations...

Transformations of the graphs of functions	
$f(x)+c$	shift $f(x)$ up c units K up
$f(x)-c$	shift $f(x)$ down c units K down
$f(x+c)$	shift $f(x)$ left c units h ←
$f(x-c)$	shift $f(x)$ right c units h →
$f(-x)$	reflect $f(x)$ about the y-axis x's are opposite
$-f(x)$	reflect $f(x)$ about the x-axis y's are opposite
$cf(x)$	When $0 < c < 1$ – vertical shrinking of $f(x)$ wider When $c > 1$ – vertical stretching of $f(x)$ narrow **Multiply the y values by c** a
$f(cx)$	When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by c** $\frac{1}{b}$

$y = f(x)$

$y = a(x-h)^2 + K$

$y = 4x^2$ $(x,y) \to (x, ay)$

$y =$

-3	36
-2	16
-1	4
0	0
1	4
2	16
3	36

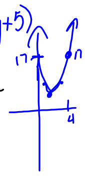
$y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4

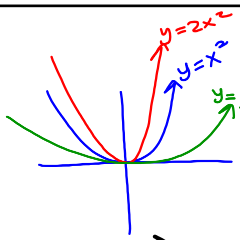
$y = 3(x-2)^2 + 5$

$(x,y) \to (x+2, 3y+5)$

x	y
-1	32
0	17
1	8
2	5
3	8
4	17



Feb 6-3:19 PM



horizontal stretch

vertical stretch

$$(x,y) \rightarrow \left(\frac{1}{b}x+h,\ ay+k\right)$$

$$y = af\left(b(x-h)\right)+k$$

$$y = 2\left(3(x+5)^2\right)-7$$

Sep 22-2:26 PM

Sept 21, 2015 Page 28

#1, 2 chart only

#3, 5, 6, 7, 9

1a)

x	$f(x)=2x+1$	$g(x)=-f(x)$	$h(x)=f(-x)$
-4	-7	$+7$	9

Sep 21-3:05 PM

$$y=f(x) \longrightarrow y=af\left(b(x-c)\right)+d$$

Mapping Rule: $(x,y)\rightarrow(bx+c,\ ay+d)$

Important note for sketching...

Transformations should be applied in following order:
1. Reflections
2. Stretches
3. Translations

Remember....RST

Feb 7-1:41 PM

The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

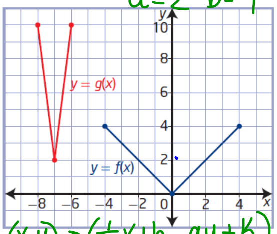
The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

Feb 7-2:29 PM

Write the Equation of a Transformed Function Graph

$k=2$ $h=-7$
$a=2$ $b=4$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



x	y
−8	10
−7	2
−6	10

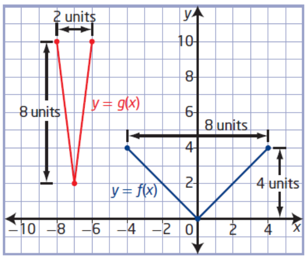
x	y
−4	4
0	0
4	4

$(x,y)\to\left(\frac{1}{b}x+h,\ ay+k\right)$

$(x,y)\to\left(\frac{1}{4}x-7,\ 2y+2\right)$

Solution

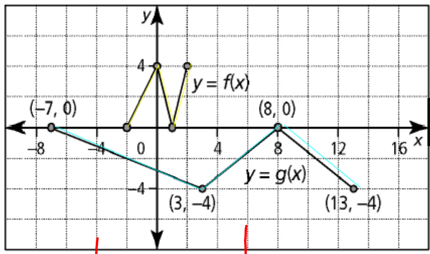
The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.



How could you use the mapping $(x, y) \to \left(\frac{1}{b}x + h,\ ay + k\right)$ to verify this equation?

Feb 7-2:54 PM

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



$\frac{1}{\frac{1}{5}} = 5$

x	y
−7	0
3	−4
8	0
13	−4

x	y
−2	0
0	4
1	0
2	4

$y = -f\left(\frac{1}{5}(x-3)\right)$

$(x,y)\to(5x+3,\ -y)$

Feb 7-2:39 PM

Example...

The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

Feb 7-2:38 PM

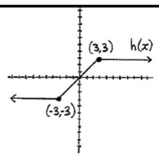
Practice Problems...

Pages 39 - 41

#3, 4, 6, 7, 8, 10, 13, 14

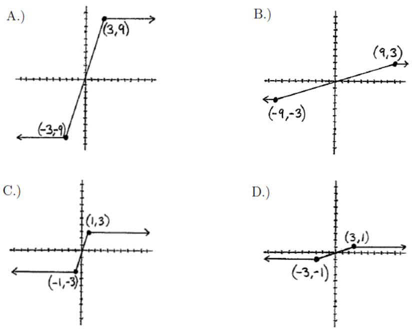
Feb 7-3:00 PM

Warm-Up...



Given the graph of $h(x)$ above, match the following four functions with their graphs.

29.) $3h(x)$ 30.) $\frac{1}{3}h(x)$ 31.) $h(3x)$ 32.) $h(\frac{x}{3})$



Feb 10-11:29 PM

Page 39. #3,4,7,9,10

$y = af(b(x-h)) + k$

3 a) $y-4 = f(x-5)$

$y = f(x-5)+4$

5 right, 4 up

b) $y+5 = 2f(3x)$

$y = 2f(3x) - 5$

$a=2$ $b=3$ $h=0$ $k=-5$

c) $y = \frac{1}{2}f(\frac{1}{2}(x-4))$

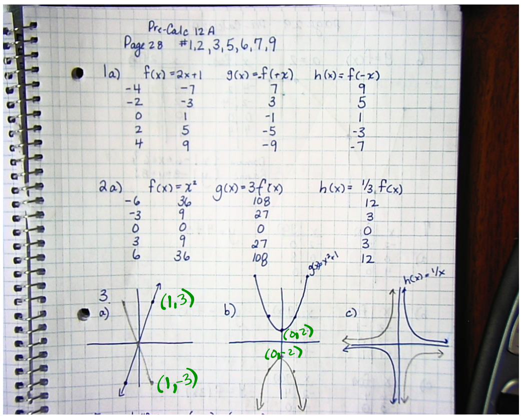
$a=\frac{1}{2}$ $b=\frac{1}{2}$ $h=4$ $K=0$

d) $y = -3f(2(x+2)) - 2$

x-axis reflection $a=3$ $b=2$ $h=-2$ $k=-2$

Sep 22-3:09 PM

page 28 solutions 1,2,3,5,6,7,9



Pre-Calc 12 A

Page 28 #1,2,3,5,6,7,9

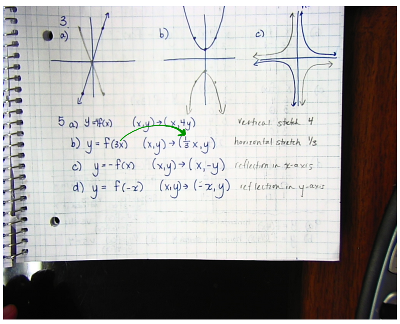
1a)

x	$f(x)=2x+1$	$g(x)=-f(x)$	$h(x)=f(-x)$
-4	-7	7	9
-2	-3	3	5
0	1	-1	1
2	5	-5	-3
4	9	-9	-7

2a)

x	$f(x)=x^2$	$g(x)=3f(x)$	$h(x)=\frac{1}{3}f(x)$
-6	36	108	12
-3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

3 a) (1,3) (1,-3) b) $g(x)=x^2+1$ (0,2) (0,-2) c) $h(x)=1/x$



3 a) b) c)

5 a) $y=4f(x)$ $(x,y)\to(x,4y)$ vertical stretch 4

b) $y=f(3x)$ $(x,y)\to(\frac{1}{3}x,y)$ horizontal stretch 1/3

c) $y=-f(x)$ $(x,y)\to(x,-y)$ reflection in x-axis

d) $y=f(-x)$ $(x,y)\to(-x,y)$ reflection in y-axis

Sep 23-2:15 PM

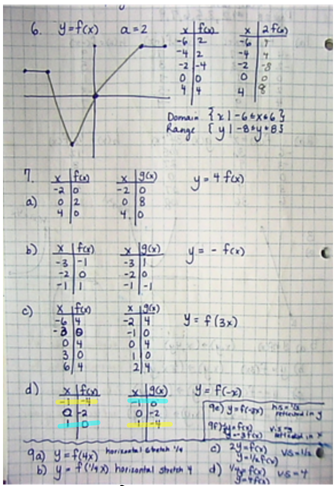
$y=f(x)$

$y=-f(x)$ $y=f(-x)$

reflection on x-axis | reflection on y axis

Sep 23-2:22 PM

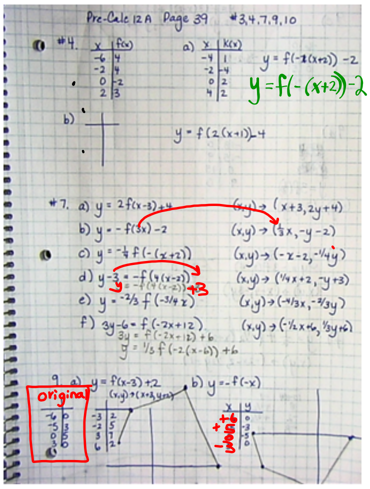
Page 29 #6



$2y=f(x)$
$y=\frac{1}{2}f(x)$

$\frac{1}{4}y=f(x)$
$y=4f(x)$

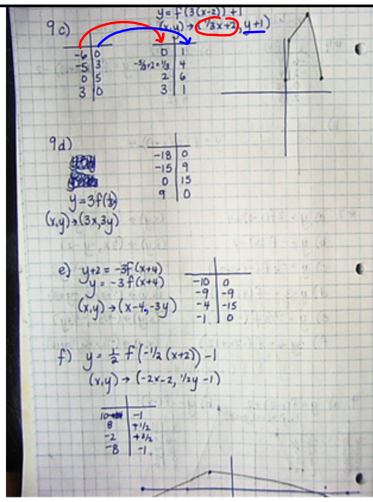
Sep 23-2:16 PM



Sep 23-2:16 PM

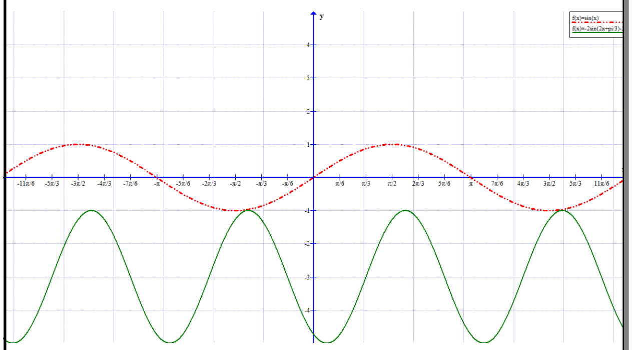
$3y-6=f(-2x+12)$
$3y=f(-2x+12)+6$
$y=\frac{1}{3}f(-2x+12)+2$
$y=\frac{1}{3}f(-2(x-6))+2$
$(x,y)\rightarrow(-\frac{1}{2}x+6,\frac{1}{3}y+2)$

Sep 23-2:42 PM



Sep 23-2:17 PM

How did the graph of y=f(x) become the green graph, y = g(x) ??



Feb 10-11:59 PM

#10 R, S, T

$y = af(b(x-h))+k$

$(x,y) \rightarrow (\frac{1}{b}x+h, ay+k)$

Sep 23-2:50 PM

Inverse of a Relation

An inverse function is a second function which undoes the work of the first one.

1. Introduction

Suppose we have a function f that takes x to y, so that

$$f(x) = y.$$

An inverse function, which we call f^{-1}, is another function that takes y back to x. So

$$f^{-1}(y) = x.$$

For f^{-1} to be an inverse of f, this needs to work for every x that f acts upon.

Did You Know?

The −1 in $f^{-1}(x)$ does not represent an exponent; that is $f^{-1}(x) \neq \frac{1}{f(x)}$.

Feb 11-12:01 AM

$y = f(1-x)$

$y = f(-x+1)$

$y = f(-(x-1))$

Sep 23-3:08 PM

4e) $y=-f(2-2x)-2$

$y=-f(-2x+2)-2$

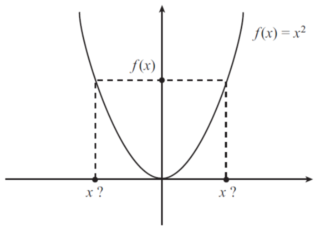
☺ $y=-f(-2(x-1))-2$

f) $y=-2f(-\frac{1}{2}x-1)+1$

$y=-2f(-\frac{1}{2}(x+2))+1$

Sep 23-3:09 PM

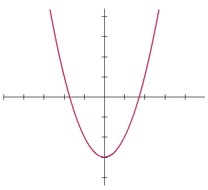
Not all functions have inverses. For example, let us see what happens if we try to find an inverse for $f(x)=x^2$.



Feb 11-12:18 AM

A function is said to be a one-to-one function if it never takes on the same value twice.

Look at this function...



Sep 18-11:55 PM

If a function is a one-to-one function then it will posses what is called an inverse function.

If f is a one-to-one function with domain A and range B. Then its **inverse function,** f^{-1} has domain B and range A and is defined by

$$f^{-1}(y)=x \Leftrightarrow f(x)=y$$

for any y in B.

→ domain of f^{-1} = range of f

range of f^{-1} = domain of f

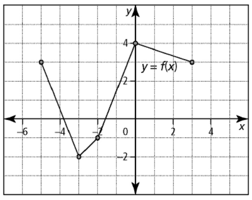
In plain english....the x and y coordinates will just switch places

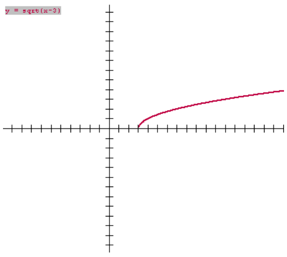
Feb 11-12:19 AM

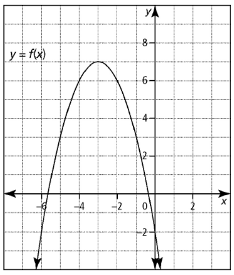
The inverse of a relation is found by interchanging the x-coordinates and y-coordinates of the ordered pairs of the relation. In other words, for every ordered pair (x, y) of a relation, there is an ordered pair (y, x) on the inverse of the relation. This means that the graphs of a relation and its inverse are reflections of each other in the line $y = x$.

$(x, y) \rightarrow (y, x)$

How does this play out graphically?







Sep 19-12:21 AM

What if given the function algebraically?

Determine algebraically the equation of the inverse of each function.

a) $f(x) = 3x - 6$ **b)** $f(x) = \frac{1}{2}x + 5$

c) $f(x) = \frac{1}{3}(x + 12)$ **d)** $f(x) = \frac{8x + 12}{4}$

Feb 11-12:19 AM

Determine the inverse for each of the following functions:

1. $f(x) = 2x - 5$

$y = 2x - 5$

$x = 2y - 5$

$x + 5 = 2y$

$\frac{1}{2}x + \frac{5}{2} = y$

or

$\frac{x+5}{2} = y$

$f^{-1}(x) = \frac{x+5}{2}$

$f(f^{-1}(x)) = \cancel{2}\left(\frac{x+5}{\cancel{2}}\right) - 5$

$x + 5 - 5$

x

$f^{-1}(f(x)) = \frac{(2x-5)+5}{2}$

$= \frac{2x}{2}$

$= x$

2. $f(y) = \sqrt{x - 3} + 4$

$x = \sqrt{y - 3} + 4$

$(x - 4)^2 = (\sqrt{y - 3})^2$

$x^2 - 8x + 16 = y - 3$

$\boxed{x^2 - 8x + 19 = y}$

Feb 11-12:27 AM

1b) $f(x) = x - 3$ $g(x) = x + 3$

$f(g(x)) = (x+3) - 3 = x$

$g(f(x)) = (x - 3) + 3 = x$

Sep 28-2:29 PM

2e) $f(x) = \frac{x}{1-x}$

$x = \frac{y}{(1-y)}$

$x(1-y) = y$

$x - xy = y$

$x = y + xy$

$x = y(1+x)$

$\frac{x}{1+x} = y$

Sep 28-2:32 PM

f) $y = \frac{2x-1}{3x+2}$

$x = \frac{2y-1}{(3y+2)}$

$x(3y+2) = 2y-1$

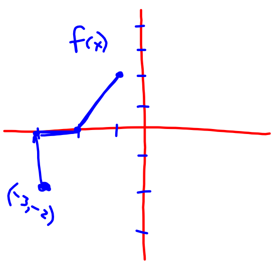
$3xy + 2x = 2y - 1$

$3xy - 2y = -1 - 2x$

$y(3x-2) = -1-2x$

$y = \frac{-1-2x}{3x-2}$

Sep 28-2:35 PM



Determine $f^{-1}(x)$ if $f(x) = 3\sqrt{x-5}+8$

Feb 13-8:34 AM

Practice Problems...

Pages 51 - 55
#2, 3, 5, 6, 8, 9, 11, 15, 18, 20, 21

Feb 13-8:40 AM

Combination of Functions

- Two functions f and g can be combined to form new functions
  - $f+g$,
  - $f-g$,
  - fg, and
  - f/g

  just as we add, subtract, multiply, and divide real numbers.
- This is summarized in the following table:

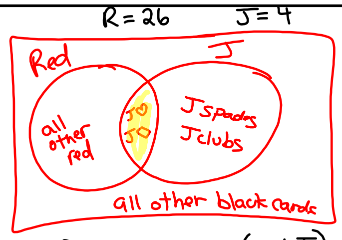
Algebra of Functions Let f and g be functions with domains A and B. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows:

$(f+g)(x) = f(x) + g(x)$	domain $= A \cap B$
$(f-g)(x) = f(x) - g(x)$	domain $= A \cap B$
$(fg)(x) = f(x)g(x)$	domain $= A \cap B$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	domain $= \{x \in A \cap B \mid g(x) \neq 0\}$

"AND" intersection

Sep 13-10:12 PM

R=26 J=4



$n(R\cap J) = 2$ $n(R\cup J) = 28$

Sep 28-2:43 PM

$\cap$ intersection "AND"

$\cup$ "Union" "OR"

Sep 28-2:47 PM

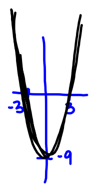
- ***Review of Intersection and Union of two sets:***

$f(x) = \sqrt{x+4}$ $g(x) = \sqrt{x^2-9}$

Let A represent the domain of f and B the domain of g.

A: $x+4 \geq 0$
$x \geq -4$

B: $x^2-9 \geq 0$
$(x-3)(x+3) \geq 0$
$x \leq -3$ $x \geq 3$



I. Intersection: $A \cap B$

$-4 \leq x \leq -3$, $x \geq 3$

II. Union: $A \cup B$

$x \in \mathbb{R}$

Sep 13-10:20 PM

Example

- If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{4 - x^2}$, find the functions $f + g$, $f - g$, fg, and f/g.

**Also examine the domain of each of these new functions

Sep 13-10:34 PM

Compositions of Functions

When the input in a function is another function, the result is called a **composite function.** If

$f(x) = 3x + 2$ and $g(x) = 4x - 5$

then $f[g(x)]$ is a composite function. The statement $f[g(x)]$ is read " f of g of x" or "the composition of f with g." $f[g(x)]$ can also be written as

$(f \circ g)(x)$ or $f \circ g(x)$ $= f(g(x))$

The symbol between f and g is a small open circle. When replacing one function with another, be very careful to get the order correct because compositions of functions are not necessarily commutative (as you'll see).

Feb 11-12:32 AM

Example 1

If $f(x) = 3x + 2$ and $g(x) = 4x - 5$, find each of the following.

1. $f[g(4)]$
2. $g \circ f(4)$
3. $f[g(x)]$
4. $(g \circ f)(x)$

Feb 11-12:32 AM

Example 2

If $f(x) = 3x^2 + 2x + 1$ and $g(x) = 4x - 5$, find each of the following:

1. $f[g(x)]$
2. $g[f(x)]$

Feb 11-12:34 AM

Warm Up

If $f(x)=\frac{4}{x-1}$ and $g(x)=2x$, then the solution set of $f\left(g(x)\right)=g\left(f(x)\right)$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

Feb 11-12:40 AM

P54 P508

#12a) #14

#15a)

Test review → start page56

le lundi 28 Septembre 2015

Sep 28-3:10 PM

Check Up

Given the three functions....

$f(x)=1-x$ $g(x)=\sqrt{x+1}$ $h(x)=x^2+5$

Evaluate each of the following:

1. $(f\circ g)(3)$ $g(3)=\sqrt{3+1}=2$ $f(2)=1-(2)=-1$ $\boxed{(f\circ g)(3)=-1}$
2. $(g\circ h)(0)$
3. $(g\circ g\circ f)(-7)$
4. $(h\circ g\circ h\circ f)(-1)$ $f(-1)=2$ $h(2)=9$ $g(9)=\sqrt{10}$ $h(\sqrt{10})=15$
5. $(f\circ h\circ g)(m)$
6. $f\left(h\left(\sqrt{9x^4-1}\right)\right)$. $h(x)=x^2+5$

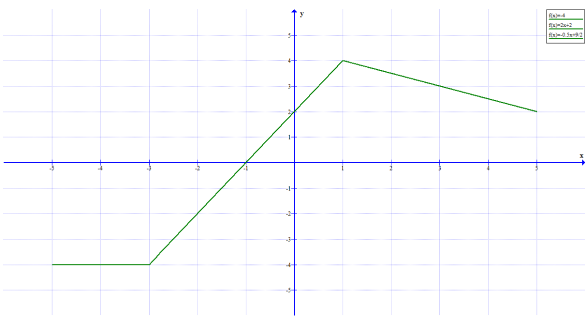
$h(\sqrt{9x^4-1})=(\sqrt{9x^4-1})^2+5$
$=9x^4-1+5$
$=9x^4+4$

$f(x)=1-x$
$f(9x^4+4)=1-(9x^4+4)$
$=-9x^4-3$

Sep 18-8:29 AM

Given the graph of f(x) shown below, evaluate the following:

$$\frac{3f(1)-5\left[f(3)-7f(0)\right]}{2f(-2)-3f(-4)}$$



Feb 13-3:25 PM

Unit Test: Monday

Notes
- ⇒ Sketch piecewise function
- ⇒ function Notation
  - ⇒ combinations: Domain (Intersection of Domains)
  - ⇒ compositions:
- ⇒ Catalog of essential functions

Functions: Transformations

→ Translations, Reflections, Stretches

$y = af[b(x-h)] + k$ ← Vertical Translation

- a: Vert. stretch, Reflect in x-axis
- b: Reflect in y-axis, Horizontal stretch $\left(\frac{1}{b}\right)$
- h: Horizontal Translation

Mapping:

$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

⇒ Inverse Functions
- Switch "x" & "y" (Domain & Range)
- Sketch Inverses from a given graph (Reflects in line $y=x$)
- One-one function (Horizontal line)
- Switch to inverse algebraically

ie. $f(x) = x + 7$

$x = y + 7$

$x - 7 = y$

$f^{-1}(x) = x - 7$

Feb 14-9:07 AM

Chapter Review from textbook...

Pages 56-57
#2, 3, 6, 8, 9, 10, 11, 14, 15, 16

Practice Test
Pages 58-59
All questions

Feb 14-9:24 AM

WARM-UP

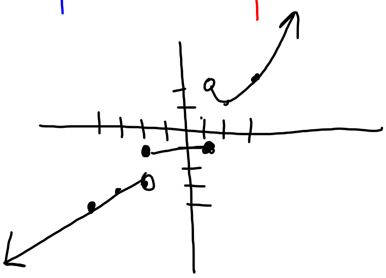
$$f(x) = \begin{cases} \frac{1}{2}x - 2 & \text{if } x < -2 \\ -1 & \text{if } -2 \le x \le 1 \\ (x-2)^2 + 1 & \text{if } x > 1 \end{cases}$$

$\frac{1}{2}(-4) - 2$
$-2 - 2$
-4

x	y
-4	-4
-3	-3.5
-2	-3

x	y
-2	-1
-1	-1
0	-1
1	-1

x	y
1	2
2	1
3	2



Sep 28-1:28 PM

Test topics

Functions – vertical Line Test – every $\boxed{x}$ has one $\boxed{y}$

one-to-one Function – horizontal line test

Domain, Range
graphs + equations; graphs only

Set Notation vs. Interval Notation

$[3, 10)$ $3 \le x < 10$

$(-2, \infty)$ $x > -2$

Composite Functions

$f \circ g(x) \leftrightarrow (f \circ g)(x) \leftrightarrow f(g(x))$

Transformations

$y = a\,f(b(x-h)) + k$

$y - k = af(b(x-h))$

$(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

R, S, T

Inverse Functions
p56 #1-17 (don't graph)

Sep 29-2:35 PM