Exam Review

Completion

Complete each statement.

1. The maximum value of the function $y = 3\cos\left(\dfrac{\pi}{6}x\right) - 9$ is ________________ .

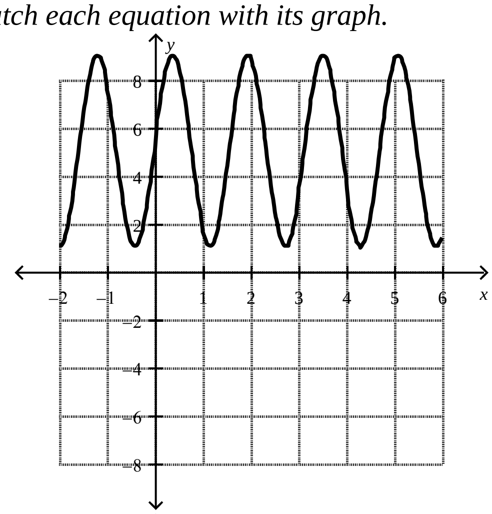
2. The period of the function $y = -8\sin\left(\dfrac{\pi}{3}(x-9)\right) - 3$ is ________________ .

3. If $f(x) = \sqrt{x-4}$ and $g(x) = x^2 - 3$, then the domain of the function $h(x) = f(g(x))$ is ________________.

Matching

Match each equation with its graph.

A



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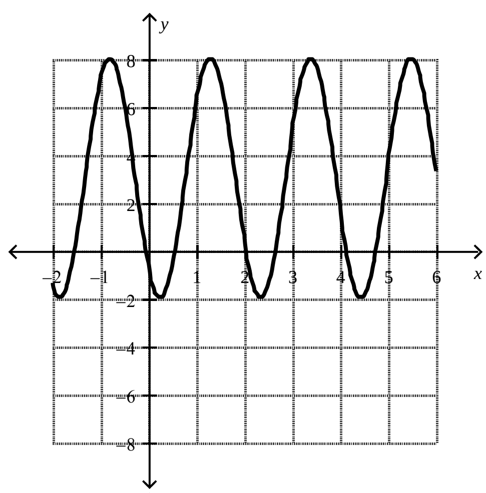

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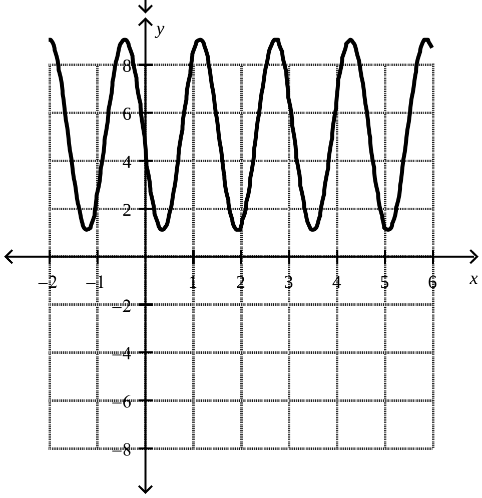

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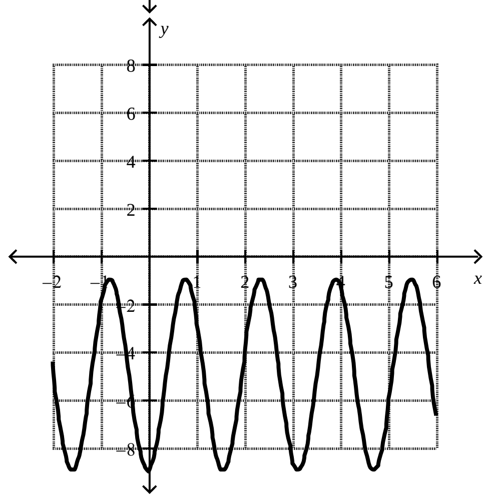
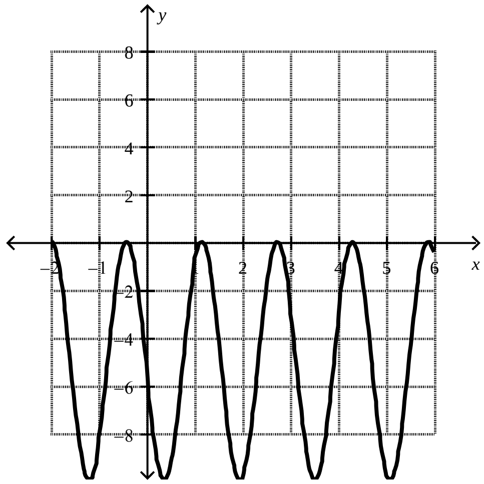


C



1. $y = -4\sin\left[4\left(x + \frac{\pi}{4}\right)\right] + 5$

2. $y = -5\sin\left[4\left(x - \frac{\pi}{2}\right)\right] - 5$

3. $y = 4\cos\left[4\left(x - \frac{\pi}{4}\right)\right] - 5$

4. $y = 5\cos\left[-3\left(x + \frac{\pi}{4}\right)\right] + 3$

Match the single logarithm in simplest form with the correct equivalent expression.

A $\log_7 s - \log_7 u + 3\log_7 x$

B $8\log_7 s - \log_7 u - 3\log_7 x$

C $8/3\log_7 u - 8\log_7 s - 3\log_7 x$

D $8/3\log_7 u - 8\log_7 s + 3\log_7 x$

E $8\log_7 s - 8/3\log_7 u + 3\log_7 x$

F $8\log_7 s + 8/3\log_7 u + 3\log_7 x$

5. $\log_7 \frac{u^{8/3}}{s^8 x^3}$

6. $\log_7 \frac{sx^3}{u}$

7. $\log_7 \frac{u^{8/3}x^3}{s^8}$

8. $\log_7 \frac{s^8 x^3}{u^{8/3}}$

9. $\log_7 \dfrac{s^8}{ux^3}$

Short Answer

1. Determine the equation, in standard form, of each parabola after being transformed from $f(x) = x^2$ by the given translations.
a) 4 units to the right and 3 units up
b) 2 units to the left and 1 unit up
c) 2 units down and 7 units to the left

2. Determine the equation of the function *g*(*x*) after the indicated reflection.
a) $f(x) = (x - 1)^2 + 2$, in the *x*-axis
b) $f(x) = |x| + 1$, in the *y*-axis

3. For each *g*(*x*), describe, in the appropriate order, the combination of transformations that must be applied to the base function $f(x) = \sqrt{x}$.
a) $g(x) = -\sqrt{2(x + 1)} - 2$
b) $g(x) = 2\sqrt{x - 3} - 4$
c) $g(x) = -\frac{1}{2}\sqrt{5 - x} + 1$

4. For each function *f*(*x*),
i) determine $f^{-1}(x)$
ii) graph *f*(*x*) and its inverse
a) $f(x) = \frac{5}{2}x - 3$
b) $f(x) = 3(x - 2)^2 - 3$

5. **a)** Determine the inverse of the function $f(x) = x^2 - 4x + 5$.
b) Determine the domain and range of the function and its inverse.
c) Is the inverse a function? Explain.

6. A child swings on a playground swing set. If the length of the swing's chain is 3 m and the child swings through an angle of $\frac{\pi}{9}$, what is the exact arc length through which the child travels?

7. Determine the exact value of $\dfrac{\cot\left(\frac{\pi}{3}\right) + \csc\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4}\right)}$.

8. Given that $\sin x = \cos\left(\frac{\pi}{5}\right)$ and that *x* lies in the first quadrant, determine the exact measure of angle *x*.

9. Without using a calculator, determine two angles between 0° and 360° that have a cosecant of $-\frac{2}{\sqrt{3}}$. Include an explanation of how you determined the two angles.

10. Given a circle of diameter 21 cm, determine the arc length subtended by a central angle of 1.2 radians.

11. Find the exact value of $\left[\cos\left(\frac{5\pi}{6}\right)\right]^2 - \left[\sin\left(\frac{5\pi}{6}\right)\right]^2$.

12. Angles A and B are located in the first quadrant. If $\sin A = \frac{\sqrt{2}}{2}$ and $\cos B = \frac{\sqrt{3}}{2}$, determine the exact value of sec A + sec B.

13. Determine the exact measures for all angles where $\tan\theta = -\sqrt{3}$ in the domain $-180° \le \theta \le 180°$.

14. A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

15. Solve $6\sin^2 x - 5\cos x - 2 = 0$ on the interval $x \in [0, 2\pi]$.

16. A Ferris wheel of diameter 18.5 m rotates at a rate of 0.2 rad/s. If passengers board the lowest car at a height of 3 m above the ground, determine a sinusoidal function that models the height, *h*, in metres, of the car relative to the ground as a function of the time, *t*, in seconds.

17. The water level at an ocean inlet has a depth, *d*, in metres, that varies with the time, *t*, in hours after midnight, according to the equation $d = 5\sin\left[2\pi\left(\frac{t-4}{12.4}\right)\right] + 6$. What is the water depth at 2:30 a.m., to the nearest hundredth of a metre?

18. Describe the transformations that, when applied to the graph of *y* = cos *x*, result in the graph of $y = -2\cos\left[\frac{1}{8}\left(x - \frac{\pi}{3}\right)\right] + 1$.

19. A pebble is embedded in the tread of a rotating bicycle wheel of diameter 60 cm. If the wheel rotates at 4 revolutions per second, determine a relationship between the height, *h*, in centimetres, of the pebble above the ground as a function of time, *t*, in seconds.

20. A girl jumps rope such that the height, *h*, in metres, of the middle of the rope can be approximated by the equation $h = 0.7\sin(72t + 9) + 0.75$, where *t* is the time, in seconds.
a) What is the amplitude of this function?
b) How many revolutions of the rope does the girl make in 1 min?

21. Find the exact value of $\cos\frac{2\pi}{9}\cos\frac{\pi}{18} + \sin\frac{2\pi}{9}\sin\frac{\pi}{18}$.

22. Prove the identity $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$.

23. What is the solution for $2\cos x - \sqrt{3} = 0$ for $0 \le x \le 2\pi$?

24. Solve $\sec^2\theta - 2\tan\theta - 3 = 0$. State the general solution to the nearest degree.

25. For the function $y = \frac{1}{2}(3)^{x-2}$,

a) describe the transformations of the function when compared to the function $y = 3^x$
b) sketch the graph of the given function and $y = 3^x$ on the same set of axes
c) state the domain, the range, and the equation of the asymptote

26. Write the equation for the function that results from each transformation or set of transformations applied to the base function $y = 5^x$.
a) reflect in the *y*-axis
b) shift 3 units to the right
c) shift 1 unit down and 4 units to the left
d) reflect in the *x*-axis and shift 2 units down

27. Solve for *n*: $9^{n-1} = \left(\frac{1}{3}\right)^{4n-1}$

28. Solve for *x*: $3^x = 9^{x^2 - \frac{1}{2}}$

29. Evaluate $\log_2 64 + \log_3 27 \times \log_4 \frac{1}{256}$.

30. If $\log_4(m - n) = 0$ and $\log_4(m + n) = 2$, determine the values of *m* and *n*.

31. A certain type of exponential growth can be described by the equation $N = N_0 10^{kt}$, where N_0 is the initial amount; *k* is the doubling time, in years; and *N* is the amount after time, *t*, in years, has passed. Suppose that the population of a small town doubles every 25 years. How long does it take to triple, to the nearest tenth of a year?

32. Solve the equation $6^{3x+1} = 2^{2x-3}$. Leave your answer in exact form.

33. Solve for *x*.
$2\log_4(x+4) - \log_4(x+12) = 1$

34. Given the functions $f(x) = x + 1$ and $g(x) = x^2 + 3x + 1$, determine a simplified equation for $h(x) = f(x) + g(x)$.

For the following question(s), assume that x is in radians, if applicable.

35. Given the functions $f(x) = \cos x$ and $g(x) = 2^x + 5$, determine the value of $g(f(\pi))$.

36. Given the functions $f(x) = x^2 - 7$ and $g(x) = 2 - x^3$, what is the value of $f(g(2))$?

FREE RESPONSE: Show all work for each of the following in the space provided.

[Value = ??]

1. Consider the function $f(x) = 2(x-1)^2 - 3$.
a) Determine the equation of each function.
i) $-f(x)$
ii) $f(-x)$
iii) $-f(-x)$
b) Graph all four functions from part a) on the same set of axes.
c) From the graph, determine the pairs of equations that can be represented as translations of each other.
d) Describe the translation that can be applied to each pair of functions you determined in part c) to generate the same graph.

2. The base function $f(x) = \sqrt{x}$ is reflected in the *x*-axis, stretched horizontally by a factor of 2, compressed vertically by a factor of $\frac{1}{3}$, and translated 3 units to the left and 5 units down.
a) Write the equation of the transformed function $g(x)$.
b) Graph the original function and the transformed function on the same set of axes.
c) Which transformations must be done first but in any order?
d) Which transformations must be done last but in any order?

3. Consider the function $f(x) = -2\sqrt{x+1} - 4$.
a) State the domain and range of the function.
b) Use this information to determine the domain and range of the inverse of the function.
c) Determine the inverse of the function.
d) Graph the function and its inverse on the same set of axes (include the line $y = x$ to verify the inverse).

4. The equation $y = \frac{9}{5}x + 32$ can be used to convert between Celsius and Fahrenheit temperatures, where *x* is the temperature in degrees Celsius and *y* is the temperature in degrees Fahrenheit.
a) Determine the inverse of this equation. What does it represent? What do the variables represent?
b) Graph the original and inverse functions on the same set of axes.
c) Which temperature is the same in Celsius and Fahrenheit? Explain how you know.

5. The volume of a drinking cup can be approximated by the formula $V = \frac{A_{top} + A_{bottom}}{2}h$, where the top and bottom of the cup are circular and *h* is the height. If the cup has a bottom diameter of 6 cm, a height of 10 cm, and sides that slope outward at an angle of 0.09, determine the cup's volume.

6. The point (–5, 7) is located on the terminal arm of ∠A in standard position.
a) Determine the primary trigonometric ratios for ∠A.
b) Determine the primary trigonometric ratios for ∠B with the same sine as ∠A, but different signs for the other two primary trigonometric ratios.
c) Use a calculator to determine the measures of ∠A and ∠B, to the nearest degree.

7. **a)** Without using a calculator, determine two angles between 0° and 360° that have a sine ratio of $-\frac{1}{2}$.
b) Use a calculator and a diagram to verify your answers to part a).

8. Consider ∠A such that cos A = $\frac{12}{13}$.
a) In which quadrant(s) is this angle? Explain.
b) If the sine of the angle is negative, in which quadrant is the angle? Explain.
c) Sketch a diagram to represent the angle in standard position, given that the condition in part b) is true.
d) Find the coordinates of a point on the terminal arm of the angle.
e) Write exact expressions for the other two primary trigonometric ratios for the angle.

9. **a)** For the given trigonometric ratio, determine two other angles that have the same value.
i) sin 45°
ii) tan 300°
iii) cos 120°
b) Explain how you determined the angles in part a).

10. The point P(–3, –6) lies on the terminal arm of an angle in standard position.
a) Which primary trigonometric ratios are positive and which are negative?
b) Which reciprocal trigonometric ratios are positive and which are negative?
c) Determine exact values for the primary trigonometric ratios.
d) Determine exact values for the reciprocal trigonometric ratios.

11. A bicycle tire revolves at 150 rpm (revolutions per minute). What is its angular velocity, in radians per second, rounded to two decimal places?

12. A group of marketing students are creating a strategy to sell ceiling fans. They determine that the number of sales will vary according to the month of the year. The formula $s = 9.2 + 2.4\cos\left[\frac{\pi}{6}(t-6)\right]$ gives the expected sales, in thousands, according to the month, *t*, where *t* = 1 represents January, *t* = 2 is February, and so on.
a) In which month are sales of fans expected to reach 11 600?
b) In which month are sales expected to be the least?
c) Does this formula seem reasonable? Explain.

13. Sketch the graph of $y = \frac{1}{2}\sin\left[\pi(x-1)\right] + 3$ for two cycles, where angles are in radians.

14. Sketch the graph of $y = 3\cos\left(\frac{1}{2}x - 2\pi\right) - 1$ for $-2\pi \le x \le 2\pi$.

15. At a certain ocean bay, the maximum height of the water is 4 m above mean sea level at 8:00 a.m. The height is at a maximum again at 8:24 p.m. Assuming that the relationship between the height, *h*, in metres, and the time, *t*, in hours, is sinusoidal, determine the height of the water above mean sea level, to the nearest tenth of a metre, at 10:00 a.m.

16. A windmill has blades that are 20 m in length, and the centre of their circular motion is a point 23 m above the ground. The blades have a frequency of 4 revolutions per minute when in operation.
a) Use a sinusoidal function to model the height above the ground of the tip of one blade as a function of time.
b) Graph the function over three complete cycles.
c) How far above the ground is the tip of the blade after 10 s?

17. A sinusoidal function has an amplitude of 2, a period of 180°, and a maximum at (0, 4).
a) Represent this function with an equation using a sine function.
b) Represent this function with an equation using a cosine function.
c) Explain how these two functions are related.

18. Consider the function $f(x) = \frac{1}{2}\sin\left[3(x - 30°)\right] + 4$.
a) Determine the amplitude, the period, the phase shift, and the vertical shift of the function with respect to $y = \sin x$.
b) What are the minimum and maximum values of the function?
c) Determine the *x*-intercepts of the function in the interval $0° \le x \le 360°$.
d) Determine the *y*-intercept of the function.

19. Prove the identity $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$.

20. Prove the identity $\dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \tan\theta$.

21. Prove the identity $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} = 1 - \tan\theta$.

22. Prove the identity $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{2}{\sin^2\theta}$.

23. What is the solution to the equation $\cos 2x + 2 = \sin x$ for $0° \le x \le 360°$?

24. Solve $4\sin^4 x + 3\sin^2 x - 1 = 0$ over the domain $0° \le x \le 360°$.

25. Jeff buys a new vehicle for $35 000. It is known that the vehicle will depreciate by 20% of its current value every year.
a) Write an equation to relate the depreciated value, *V*, of the vehicle to the age, *t*, in years, of the vehicle.
b) Use the equation to determine the value of the vehicle 2 years after Jeff buys it.
c) Approximately how long will it take the vehicle to depreciate to $3000?

26. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.
a) Write an equation to relate the amount of cobalt-60 remaining and the number of half-life periods.
b) What amount will be present in 10.6 years?
c) How many years will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount?

27. **a)** Rewrite the function $y = 2^{-2x+4} + 6$ in the form $y = a(2)^{b(x-h)} + k$.
b) Describe the transformations that must be applied to the graph of $y = 2^x$ to obtain the graph of the given function.
c) Graph the function.
d) Determine the equation of the function that results after the graph in part c) is reflected in the *x*-axis.
e) Graph the function from part d).

28. Solve the equation $\sqrt[3]{256^2} \times 16^x = 64^{x-3}$.

29. Dye spilled in a bucket of water mixes with the water at a rate defined by $R = -236\log\left(\frac{V}{80}\right)$, where *R* is the intensity of the dye and *V* is the volume of dye spilled, in millilitres. What is the intensity of the mixture if 50 mL of dye was spilled? Round your answer to two decimal places.

30. The magnitude of an earthquake is defined as $M = \log\left(\frac{A}{A_0}\right)$, where *A* is the amplitude of the ground motion and A_0 is the amplitude corrected for the distance from the actual earthquake that would be expected for a "standard earthquake." On March 2, 2012, an earthquake with an amplitude $10^{5.1}$ times A_0 was recorded in Norman Wells, Northwest Territories.

a) What was the earthquake's magnitude on the Richter scale?
b) How does the earthquake in Norman Wells compare to the earthquake off Vancouver Island in 1946 that measured 7.3 on the Richter scale?

31. The stellar magnitude scale compares the brightness of stars using the equation $m_2 - m_1 = \log\left(\frac{b_1}{b_2}\right)$, where m_1 and m_2 are the apparent magnitudes (how bright the stars appear in the sky) of the two stars being compared, and b_1 and b_2 are their brightness (how much light they emit).
a) The brightest appearing star in our sky, Sirius, has an apparent magnitude of –1.5. How much brighter does Sirius appear than Betelgeuse, whose apparent magnitude is 0.12? Round your answer to the nearest whole number.
b) The Sun appears about 1.3×10^{10} times as bright in the sky as does Sirius. What is the apparent magnitude of the Sun, to the nearest tenth?

32. Show that $\frac{1}{\log_a b} = \log_b a$.

33. Show that $3\log\sqrt{x} + 2\log x - \frac{1}{2}\log x = 3\log x$.

34. A 200-g sample of a radioactive substance is placed in a chamber to be tested. After 3 h, 140 g of the sample remains.
a) Determine the half-life of this substance, to the nearest hundredth of an hour.
b) Graph the amount of the substance remaining as a function of time.

35. Explain the steps used to solve the equation $\log_2\sqrt{x^2 - 8x} = \log_2 3$.

36. Solve the equation $\log\sqrt[3]{x^2 + 48x} = \frac{2}{3}$.

37. For his dream car, Bruce invested $18 000 at 7.8% interest, compounded monthly, for 5 years. After the 5 years, he still did not have enough money. How much longer will he have to invest the money at 5% interest, compounded daily, to have a total of $35 000? Round to the nearest tenth of a year.

38. Given the functions $f(x) = x^2 + 3x + 2$ and $g(x) = x + 1$, determine a simplified equation for $k(x) = f(g(x))$.

39. Given the functions $f(x) = \frac{1}{1 - x^2}$ and $g(x) = \sin x$, determine the equation for $h(x) = f(g(x))$.

40. Given the functions $f(x) = \sqrt{x}$ and $g(x) = x + 1$, determine all possible values of *x* for which $f(g(x)) = g(f(x))$.

Exam Review
Answer Section

COMPLETION

1. ANS: –6

DIF: Average

2. ANS: 6

DIF: Average

3. ANS: $\{x \mid x \leq -\sqrt{7} \text{ or } x \geq \sqrt{7}, x \in \mathbb{R}\}$

DIF: Average

MATCHING

1. ANS: A DIF: Average
2. ANS: C DIF: Average
3. ANS: E DIF: Average
4. ANS: D DIF: Average

5. ANS: C DIF: Easy
6. ANS: A DIF: Easy
7. ANS: D DIF: Easy
8. ANS: E DIF: Easy
9. ANS: B DIF: Easy

SHORT ANSWER

1. ANS:

a) $g(x) = (x-4)^2 + 3$

$= x^2 - 8x + 16 + 3$

$= x^2 - 8x + 19$

b) $g(x) = (x+2)^2 + 1$

$= x^2 + 4x + 4 + 1$

$= x^2 + 4x + 5$

c) $g(x) = (x+7)^2 - 2$

$= x^2 + 14x + 49 - 2$

$= x^2 + 14x + 47$

DIF: Average

2. ANS:

a) $g(x) = -f(x)$

$$= -(x-1)^2 - 2$$

b) $g(x) = f(-x)$

$$= |-x| + 1$$

$$= |x| + 1$$

DIF: Average

3. ANS:

a) a reflection in the *x*-axis, a horizontal compression by a factor of $\frac{1}{2}$, and then a translation of 1 unit to the left and 2 units down

b) a vertical stretch by a factor of 2, and then a translation of 3 units to the right and 4 units down

c) reflections in the *x*-axis and the *y*-axis, a vertical compression by a factor of $\frac{1}{2}$, and then a translation of 5 units to the right and 1 unit up

DIF: Difficult +

4. ANS:

a) i)

$$y = \frac{5}{2}x - 3$$

$$x = \frac{5}{2}y - 3$$

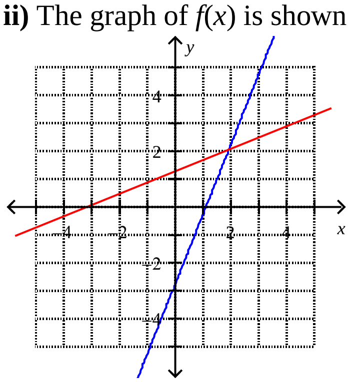
$$x + 3 = \frac{5}{2}y$$

$$2x + 6 = 5y$$

$$y = \frac{2}{5}x + \frac{6}{5}$$

$$f^{-1}(x) = \frac{2}{5}x + \frac{6}{5}$$

ii) The graph of $f(x)$ is shown in blue and the graph of $f^{-1}(x)$ is shown in red.



b) i) $y = 3(x-2)^2 - 3$

$$x = 3(y-2)^2 - 3$$

$$x + 3 = 3(y-2)^2$$

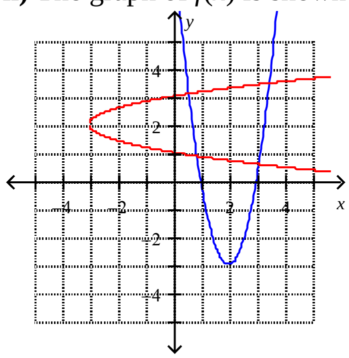
$$\frac{x+3}{3} = (y-2)^2$$

$$\pm\sqrt{\frac{x+3}{3}} = y - 2$$

$$y = 2 \pm \sqrt{\frac{x+3}{3}}$$

$$f^{-1}(x) = 2 \pm \sqrt{\frac{x+3}{3}}$$

ii) The graph of $f(x)$ is shown in blue and the graph of $f^{-1}(x)$ is shown in red.



DIF: Difficult

5. ANS:

a) First, rewrite the quadratic function in vertex form.

$$f(x) = x^2 - 4x + 5$$

$$= (x^2 - 4x + 4 - 4) + 5$$

$$= (x-2)^2 + 1$$

Then, determine the inverse.

$$x = (y-2)^2 + 1$$

$$x - 1 = (y-2)^2$$

$$\pm\sqrt{x-1} = y - 2$$

$$2 \pm \sqrt{x-1} = y$$

$$f^{-1}(x) = 2 \pm \sqrt{x-1}$$

b) For $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \geq 1,\ y \in \mathbb{R}\}$

For $f^{-1}(x)$: domain $\{x \geq 1,\ x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

c) The inverse is not a function because it fails the vertical line test.

DIF: Average

6. ANS:

$$a = \theta r$$

$$= \frac{\pi}{9}(3)$$

$$a = \frac{\pi}{3}$$

The child travels through an arc length of $\frac{\pi}{3}$ m.

DIF: Easy

7. ANS:

$$\frac{\cot\left(\frac{\pi}{3}\right)+\csc\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\frac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}+\frac{1}{\sin\left(\frac{\pi}{3}\right)}}{\cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{\frac{\cos\left(\frac{\pi}{3}\right)+1}{\sin\left(\frac{\pi}{3}\right)}}{\cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{\cos\left(\frac{\pi}{3}\right)+1}{\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)}$$

$$= \frac{\frac{1}{2}+1}{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$$

$$= \frac{\frac{3}{2}}{\frac{\sqrt{6}}{4}}$$

$$= \left(\frac{3}{2}\right)\left(\frac{4}{\sqrt{6}}\right)$$

$$= \frac{6}{\sqrt{6}}$$

$$\frac{\cot\left(\frac{\pi}{3}\right)+\csc\left(\frac{\pi}{3}\right)}{\cos\left(\frac{\pi}{4}\right)} = \sqrt{6}$$

DIF: Average

8. ANS:

$$\sin x = \cos\left(\frac{\pi}{5}\right)$$

$$\sin x = \cos\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$$

$$\sin x = \sin\left(\frac{3\pi}{10}\right)$$

$$x = \frac{3\pi}{10}$$

DIF: Average

9. ANS:

Since $\csc\theta = -\frac{2}{\sqrt{3}}$, $\sin\theta = -\frac{\sqrt{3}}{2}$. Since $\sin 60° = \frac{\sqrt{3}}{2}$, the reference angle is 60°. The ratio is negative in quadrants III and IV.This means that the angle can be found by looking for reflections of 60° that lie in these quadrants.

quadrant III: 180° + 60° = 240°

quadrant IV: 360° – 60° = 300°

DIF: Average

10. ANS:

$$r = \frac{d}{2}$$

$$= \frac{21}{2}$$

$$= 10.5$$

$$a = r\theta$$

$$a = (10.5)(1.2)$$

$$= 12.6$$

The arc length is 12.6 cm.

DIF: Average

11. ANS:

$$\left[\cos\left(\frac{5\pi}{6}\right)\right]^2 - \left[\sin\left(\frac{5\pi}{6}\right)\right]^2 = \left(-\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} - \frac{1}{4}$$

$$= \frac{1}{2}$$

DIF: Difficult

12. ANS:

$\sin A = \frac{\sqrt{2}}{2}$ $\cos B = \frac{\sqrt{3}}{2}$

$\angle A = \frac{\pi}{4}$ $\angle B = \frac{\pi}{6}$

$$\begin{aligned}
\sec A + \sec B &= \sec\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{6}\right) \\
&= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{3}} \\
&= \frac{2\sqrt{3} + 2\sqrt{2}}{\sqrt{6}} \\
&= \frac{2\sqrt{18} + 2\sqrt{12}}{6} \\
&= \frac{2 \times 3\sqrt{2} + 2 \times 2\sqrt{3}}{6} \\
&= \frac{3\sqrt{2} + 2\sqrt{3}}{3}
\end{aligned}$$

DIF: Average

13. ANS:

The tangent ratio is negative in quadrants II and IV. In quadrant II for the domain $0° \le \theta \le 180°$, $\theta = 120°$. In quadrant IV for the domain $-180° \le \theta \le 0°$, $\theta = -60°$.

DIF: Difficult

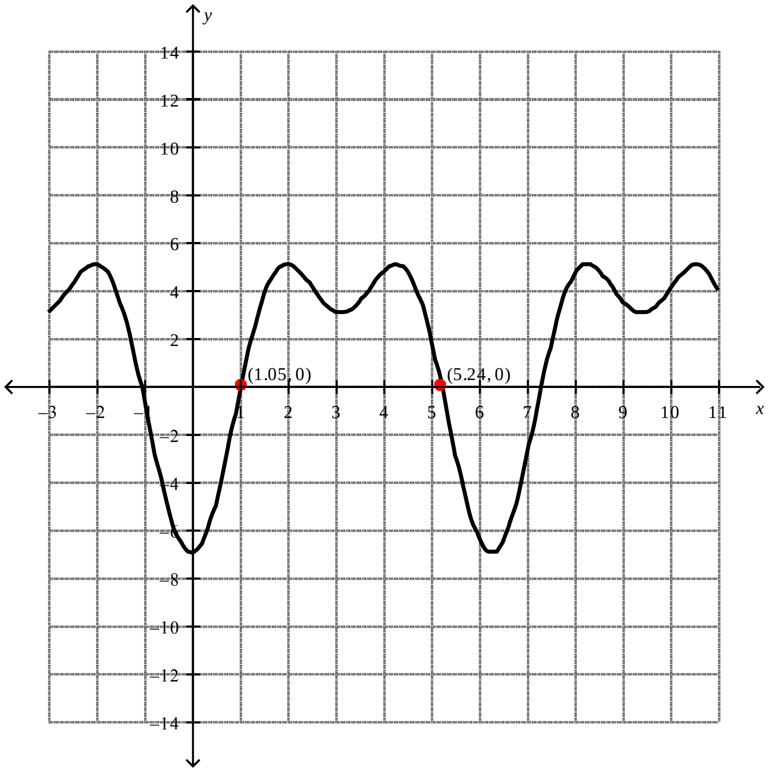
14. ANS:

$\frac{5\pi}{6}$

DIF: Easy

15. ANS:

Graph $6\sin^2 x - 5\cos x - 2 = 0$ using graphing technology.



The zeros occur at approximately 1.0 and 5.2 or exactly $\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

DIF: Difficult +

16. ANS:

The amplitude is half the diameter of the Ferris wheel, or 9.25 m. The highest point is 18.5 + 3 or 21.5 m above the ground, so the vertical displacement is 21.5 – 9.25 or 12.25 m. The wheel rotates at a rate of 0.2 rad/s, which means it takes 5 s/rad. One full rotation of the wheel is equivalent to 2π radians. Therefore, it will take $2\pi\left(5\frac{\text{s}}{\text{rad}}\right) = 10\pi$ seconds for the Ferris wheel to do one full rotation. The period is 10π.

period = $\frac{2\pi}{|b|}$

$10\pi = \frac{2\pi}{|b|}$

$|b| = \frac{2\pi}{10\pi}$

$|b| = \frac{1}{5}$

The value of *b* is thus $\frac{1}{5}$.

There is no phase shift if a cosine function with a negative value for *a* is used. Thus, the sinusoidal function that models the height of the car is

$h = -9.25\cos\left(\frac{t}{5}\right) + 12.25$

DIF: Average

17. ANS:

Substitute t = 2.5 into the equation, since 2:30 a.m. is 2.5 hours after midnight.

$$d = 5\sin\left[2\pi\,\frac{(t-4)}{12.4}\right] + 6$$

$$= 5\sin\left(2\pi\,\frac{(2.5-4)}{12.4}\right) + 6$$

$$\approx 2.56$$

The water depth is approximately 2.56 m.

DIF: Easy

18. ANS:

a reflection in the x-axis, a vertical stretch by a factor of 2, a horizontal stretch by a factor of 8, a phase shift of $\frac{\pi}{3}$ to the right, and a vertical translation of 1 unit up

DIF: Average

19. ANS:

Solutions may vary. Sample solution:

The amplitude is half the diameter, or 30 cm. The maximum height of the pebble is 60 cm, so the vertical displacement must be 30 cm. The wheel rotates at 4 revolutions per second, so the period is $\frac{1}{4}$ s. Thus, the value of b is $\frac{2\pi}{\frac{1}{4}}$ or 8π.

Thus, the relationship between the height of the pebble above the ground and time is

$h = 30\sin(8\pi t) + 30$

DIF: Easy

20. ANS:

a) 0.7 m

b) Since b = 72 and period = $\frac{2\pi}{b}$, then period = $\frac{2\pi}{72}$ or $\frac{\pi}{36}$ s. The number of revolutions of the rope is the reciprocal of the period, or $\frac{36}{\pi}$, or 11.46 rev/s. Multiply by 60 to get 688 revolutions/min.

DIF: Difficult

21. ANS:

$$\cos\frac{2\pi}{9}\cos\frac{\pi}{18}+\sin\frac{2\pi}{9}\sin\frac{\pi}{18}=\cos\left(\frac{2\pi}{9}-\frac{\pi}{18}\right)$$
$$=\cos\left(\frac{4\pi}{18}-\frac{\pi}{18}\right)$$
$$=\cos\left(\frac{3\pi}{18}\right)$$
$$=\cos\left(\frac{\pi}{6}\right)$$
$$=\frac{\sqrt{3}}{2}$$

DIF: Average

22. ANS:

L.S. $=\tan^2\theta-\sin^2\theta$ R.S. $=\sin^2\theta\tan^2\theta$

$$=\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta$$
$$=\frac{\sin^2\theta-\sin^2\theta\cos^2\theta}{\cos^2\theta}$$
$$=\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$
$$=\frac{\sin^2\theta(\sin^2\theta)}{\cos^2\theta}$$
$$=\sin^2\theta\tan^2\theta$$

L.S. = R.S.

DIF: Average

23. ANS:

$$2\cos x-\sqrt{3}=0$$
$$2\cos x=\sqrt{3}$$
$$\cos x=\frac{\sqrt{3}}{2}$$
$$x=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$
$$=\frac{\pi}{6}$$

Since cosine is also positive in quadrant IV, another solution is $2\pi-\frac{\pi}{6}=\frac{11\pi}{6}$.

DIF: Easy

24. ANS:

$$\sec^2\theta - 2\tan\theta - 3 = 0$$
$$(1+\tan^2\theta) - 2\tan\theta - 3 = 0$$
$$\tan^2\theta - 2\tan\theta - 2 = 0$$

Use the quadratic formula.

$$\tan\theta = \frac{2 \pm \sqrt{12}}{2}$$
$$= 1 \pm \sqrt{3}$$
$$\approx 2.732 \text{ or } -0.732$$
$$\theta = \tan^{-1}(2.732) \text{ or } \tan^{-1}(-0.732)$$
$$\approx 70° \text{ or } -36°$$

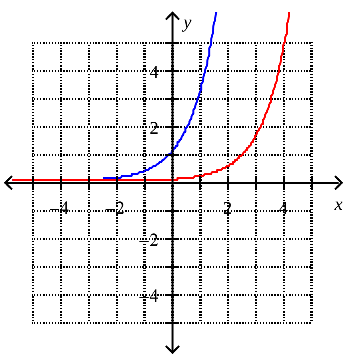
Since the period for $\tan\theta$ is 180°, a positive solution corresponding to –36° is –36° + 180° or 144°. The general solution is $70° + 180°n$ and $144° + 180°n$, where $n \in \mathrm{I}$.

DIF: Difficult

25. ANS:

a) a vertical compression by a factor of $\frac{1}{2}$ and a translation of 2 units to the right

b) The graph of $y = 3^x$ is shown in blue and the graph of $y = \frac{1}{2}(3)^{x-2}$ is shown in red.



c) domain $\{x \mid x \in \mathrm{R}\}$, range $\{y \mid y > 0, y \in \mathrm{R}\}$, $y = 0$

DIF: Average

26. ANS:

a) $y = 5^{-x}$

b) $y = 5^{x-3}$

c) $y = 5^{x+4} - 1$

d) $y = -5^x - 2$

DIF: Average

27. ANS:

$$9^{n-1} = \left(\frac{1}{3}\right)^{4n-1}$$
$$\left(3^2\right)^{n-1} = \left(3^{-1}\right)^{4n-1}$$
$$3^{2n-2} = 3^{1-4n}$$
Equate the exponents:
$$2n - 2 = 1 - 4n$$
$$6n = 3$$
$$n = \frac{1}{2}$$

DIF: Average

28. ANS:
$$3^x = 9^{x^2 - \frac{1}{2}}$$
$$3^x = 3^{2\left(x^2 - \frac{1}{2}\right)}$$
Equate the exponents:
$$x = 2x^2 - 1$$
$$0 = 2x^2 - x - 1$$
$$0 = (2x+1)(x-1)$$
$$x = -\frac{1}{2}, \qquad x = 1$$

DIF: Difficult

29. ANS:
$$\log_2 64 + \log_3 27 \times \log_4\left(\frac{1}{256}\right) = 6 + 3(-4)$$
$$= 6 - 12$$
$$= -6$$

DIF: Difficult

30. ANS:
Solve the system of equations:
$m - n = 4^0$ and $m + n = 4^2$
$$m - n = 1$$
$$m + n = 16$$
Add these equations to find *m*:
$$2m = 17$$
$$m = 8.5$$

Subtract the first equation from the second to find *n*:

$$2n = 15$$

$$n = 7.5$$

DIF: Difficult

31. ANS:

First solve for *k*.

$$N = N_0 10^{kt}$$

$$2N_0 = N_0 10^{k(25)}$$

$$2 = 10^{25k}$$

$$\log 2 = 25k \log 10$$

$$k = \frac{\log 2}{25}$$

Solve for *t* when $N = 3N_0$.

$$3N_0 = N_0 10^{\left(\frac{\log 2}{25}\right)t}$$

$$3 = 10^{\left(\frac{\log 2}{25}\right)t}$$

$$\log 3 = \left(\frac{\log 2}{25}\right)t$$

$$t = \left(\frac{25}{\log 2}\right)\log 3$$

$$t \approx 39.62$$

It will take 39.6 years for the population to triple.

DIF: Difficult +

32. ANS:

$$6^{3x+1} = 2^{2x-3}$$

$$\log(6^{3x+1}) = \log(2^{2x-3})$$

$$(3x+1)\log 6 = (2x-3)\log 2$$

$$3x\log 6 + \log 6 = 2x\log 2 - 3\log 2$$

$$x(3\log 6 - 2\log 2) = -3\log 2 - \log 6$$

$$x = \frac{-(3\log 2 + \log 6)}{3\log 6 - 2\log 2}$$

DIF: Average

33. ANS:

$$2\log_4(x+4)-\log_4(x+12)=1$$

$$\log_4(x+4)^2-\log_4(x+12)=1$$

$$\log_4\frac{(x+4)^2}{(x+12)}=\log_4 4^1$$

$$\frac{(x+4)^2}{(x+12)}=4$$

$$(x+4)^2=4x+48$$

$$x^2+8x+16=4x+48$$

$$x^2+4x-32=0$$

$$(x+8)(x-4)=0$$

$$x=-8,\ x=4$$

Since x = –8 is an extraneous root, the solution is x = 4.

DIF: Average

34. ANS:

$$h(x)=f(x)+g(x)$$

$$=x+1+x^2+3x+1$$

$$=x^2+4x+2$$

DIF: Easy

35. ANS:

$$g(f(x))=g(\cos x)$$

$$=2^{\cos x}+5$$

$$g(f(\pi))=2^{(\cos\pi)}+5$$

$$=2^{-1}+5$$

$$=\frac{1}{2}+5$$

$$=\frac{11}{2}\text{ or }5.5$$

DIF: Difficult

36. ANS:

$$g(2)=2-(2)^3$$

$$=-6$$

$$f(-6)=(-6)^2-7$$

$$=29$$

$$f(g(2))=29$$

DIF: Average

PROBLEM

1. ANS:

a) i) $-f(x) = -[2(x-1)^2 - 3]$

$= -2(x-1)^2 + 3$

ii) $f(-x) = 2(-x-1)^2 - 3$

$= 2(-1)^2(x+1)^2 - 3$

$= 2(x+1)^2 - 3$

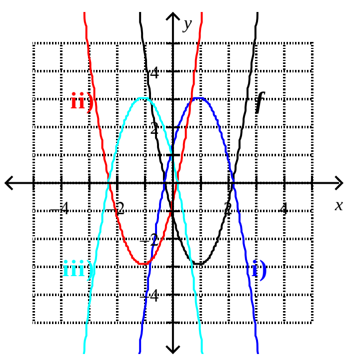
iii) $-f(-x) = -[2(-x-1)^2 - 3]$

$= -[2(-1)^2(x+1)^2 - 3]$

$= -[2(x+1)^2 - 3]$

$= -2(x+1)^2 + 3$

b)



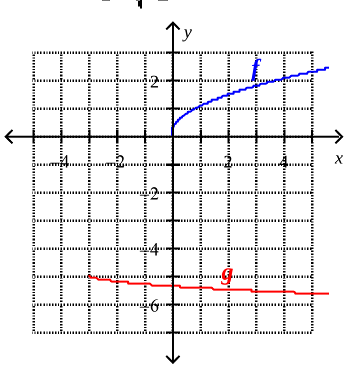
c) The graphs of *f*(*x*) and *f*(–*x*) are horizontal translations of each other. This is also true of $-f(x)$ and $-f(-x)$.
d) In both pairs, one curve is a horizontal translation of 2 units left or right of the other.

DIF: Difficult

2. ANS:

a) $g(x) = -\frac{1}{3}\sqrt{\frac{1}{2}(x+3)} - 5$

b)



c) The reflection, horizontal stretch, and vertical compression must be done first, but can be done in any order.
d) The translations to the left and down must be done last, but can be done in any order.

DIF: Difficult +

3. ANS:
a) domain $\{x \geq -1, x \in \mathrm{R}\}$, range $\{y \leq -4, y \in \mathrm{R}\}$
b) domain $\{x \leq -4, x \in \mathrm{R}\}$, range $\{y \geq -1, y \in \mathrm{R}\}$
c)

$$y = -2\sqrt{x+1} - 4$$

$$x = -2\sqrt{y+1} - 4$$

$$x + 4 = -2\sqrt{y+1}$$

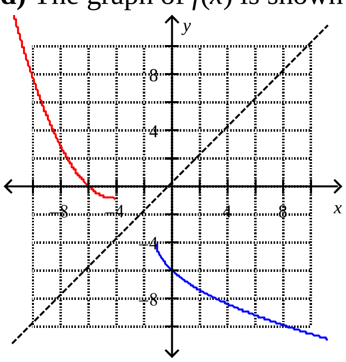
$$-\frac{(x+4)}{2} = \sqrt{y+1}$$

$$\left(-\left(\frac{x+4}{2}\right)\right)^2 = y + 1$$

$$y = \left(\frac{x+4}{2}\right)^2 - 1$$

$$f^{-1}(x) = \frac{1}{4}(x+4)^2 - 1$$

d) The graph of $f(x)$ is shown in blue and the graph of $f^{-1}(x)$ is shown in red.



DIF: Difficult

4. ANS:
a)

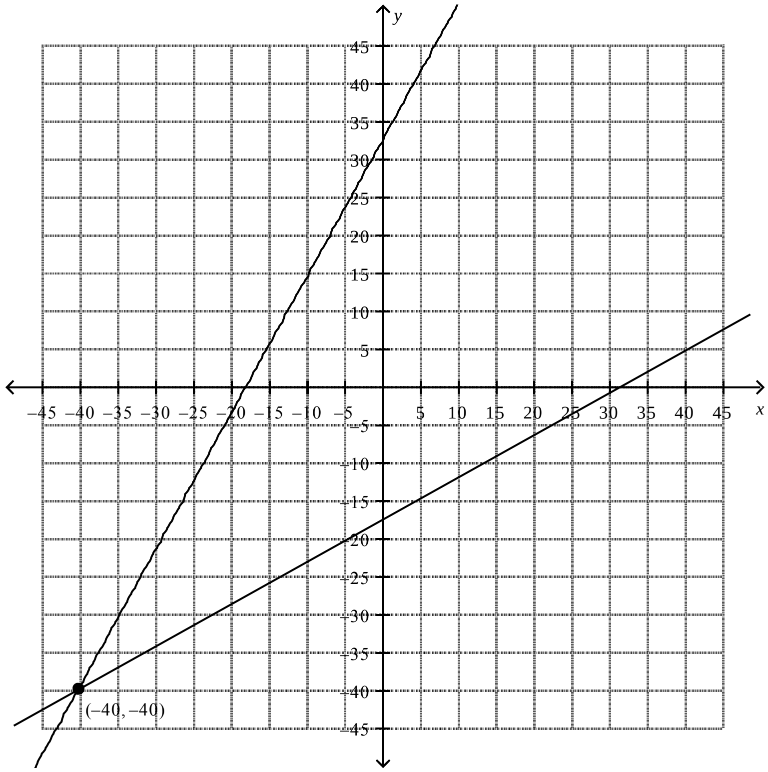
$$x = \frac{9}{5}y + 32$$

$$x - 32 = \frac{9}{5}y$$

$$y = \frac{5}{9}(x - 32)$$

The inverse represents the equation to convert a temperature from degrees Fahrenheit to degrees Celsius. The variable *x* represents the temperature in degrees Fahrenheit and the variable *y* represents the temperature in degrees Celsius.

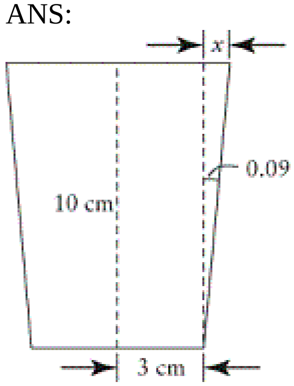
b)



c) The temperature that is the same in Celsius and Fahrenheit is −40°. This is the invariant point of the original function and its inverse.

DIF: Average

5. ANS:



First, find *x*.

$$\frac{x}{10} = \tan 0.09$$

$$x = 10\tan 0.09$$

$$A_{\text{bottom}} = \pi r^2 \qquad A_{\text{top}} = \pi r^2$$

$$= \pi(3)^2 \qquad = \pi(3 + x)^2$$

$$= 9\pi \qquad = \pi(3 + 10\tan 0.09)^2$$

$$\approx 15.23\pi$$

$$V = \left(\frac{A_{top} + A_{bottom}}{2}\right)h$$

$$= \left(\frac{15.23\pi + 9\pi}{2}\right)(10)$$

$$\approx 381$$

The volume of the cup is approximately 381 cm^3, or 381 mL.

DIF: Difficult

6. ANS:

$\angle$A is in quadrant II. Therefore, only the sine ratio will be positive.
Use the Pythagorean theorem.

$$r^2 = x^2 + y^2$$

$$= (-5)^2 + 7^2$$

$$= 25 + 49$$

$$= 74$$

$$r = \sqrt{74}$$

Therefore, $\sin A = \frac{7}{\sqrt{74}}$, $\cos A = -\frac{5}{\sqrt{74}}$, and $\tan A = -\frac{7}{5}$.

b) The quadrant in which the sine ratio is still positive, but the cosine and tangent ratios change from negative to positive, is quadrant I. In this quadrant, all three primary trigonometric ratios are positive.

$\sin B = \frac{7}{\sqrt{74}}$, $\cos B = \frac{5}{\sqrt{74}}$, and $\tan B = \frac{7}{5}$.

c) Use the fact that $\angle$B is the reference angle for $\angle$A.

$$\angle B = \sin^{-1}\frac{7}{\sqrt{74}}$$

$$\angle B \approx 54°$$

$$\angle A = 180° - 54°$$

$$= 126°$$

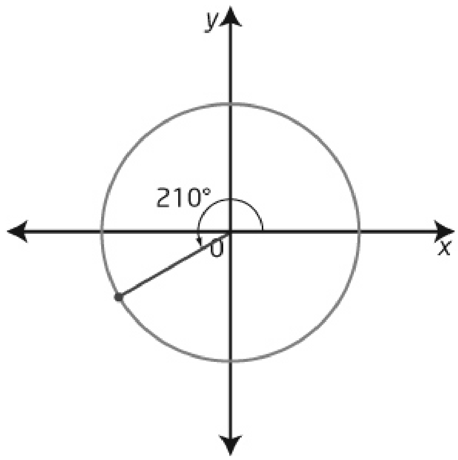
DIF: Average

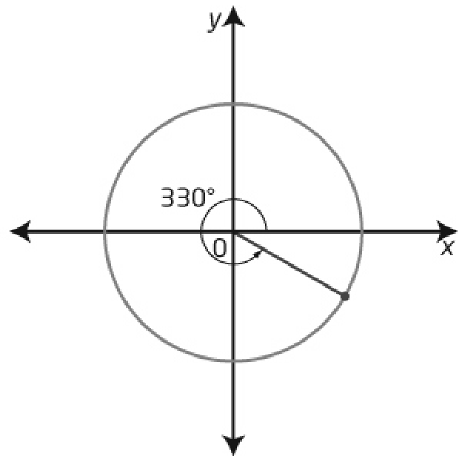
7. ANS:

a) Since sin 30° = $\frac{1}{2}$, the reference angle is 30°. The sine ratio is negative in quadrants III and IV. Look for reflections of the 30° angle in these quadrants.

quadrant III: 180° + 30° = 210°
quadrant IV: 360° – 30° = 330°

b) Using a calculator, sin 210° = $-\frac{1}{2}$ and sin 330° = $-\frac{1}{2}$.





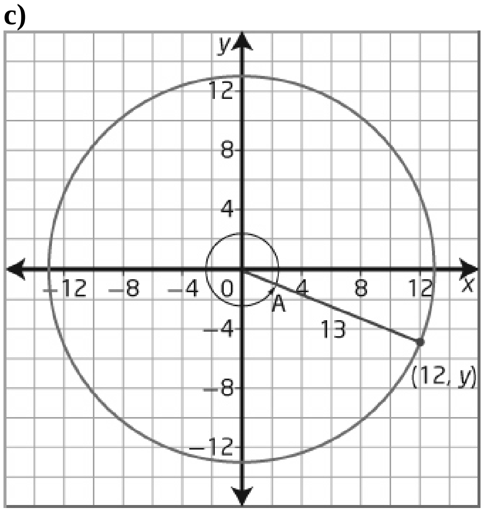
DIF: Average

8. ANS:

a) Since the cosine ratio is positive, the angle is in quadrant I or IV.

b) If the sine ratio is negative, the angle is in quadrant IV.

c)



d) Use the Pythagorean theorem.

$$r^2 = x^2 + y^2$$

$$13^2 = 12^2 + y^2$$

$$y^2 = 169 - 144$$

$$y^2 = 25$$

$$y = \pm 5$$

Therefore, a point on the terminal arm is (12, –5).

e) $\sin A = -\dfrac{5}{13}$, $\tan A = -\dfrac{5}{12}$

DIF: Average

9. ANS:

a) Answers may vary. Sample answers:

i) –315° and 405°
ii) –60° and 120°
iii) 240° and –240°
b) In all cases, locate the given angle on a Cartesian plane, and then identify the reference angle. Then, locate the other quadrant where the given trigonometric ratio has the same sign as the given ratio. Then, any angle that is co-terminal to the two angles in the diagram has the same trigonometric ratio as that given.

DIF: Average

10. ANS:
a) Since the point on the terminal arm lies in quadrant III, the tangent ratio is positive, and the sine and cosine ratios are negative.
b) The cotangent ratio is positive, and the cosecant and secant ratios are negative.
c) $r^2 = x^2 + y^2$

$$\begin{aligned} &= (-3)^2 + (-6)^2 \\ &= 9 + 36 \\ &= 45 \\ r &= \sqrt{45} \\ &= 3\sqrt{5} \end{aligned}$$

$\sin P = \dfrac{y}{r} = \dfrac{-6}{3\sqrt{5}} = -\dfrac{2}{\sqrt{5}}$

$\cos P = \dfrac{x}{r} = \dfrac{-3}{3\sqrt{5}} = -\dfrac{1}{\sqrt{5}}$

$\tan P = \dfrac{y}{x} = \dfrac{-6}{-3} = 2$

d) Using the answers in part c), take the reciprocal of each primary trigonometric ratio to write the reciprocal trigonometric ratios.
$\csc P = -\dfrac{\sqrt{5}}{2}$, $\sec P = -\sqrt{5}$, and $\cot P = \dfrac{1}{2}$

DIF: Average

11. ANS:
$$\frac{150 \text{ revolutions}}{\text{minute}} = \frac{150(2\pi)}{60 \text{ s}} \approx 15.71 \text{ rad/s}$$
The angular velocity is 15.71 rad/s.

DIF: Average

12. ANS:
a) Solve the equation $11.6 = 9.2 + 2.4\cos\left[\dfrac{\pi}{6}(t-6)\right]$ for *t*.

$$11.6 = 9.2 + 2.4\cos\left[\frac{\pi}{6}(t-6)\right]$$

$$2.4 = 2.4\cos\left[\frac{\pi}{6}(t-6)\right]$$

$$1 = \cos\left[\frac{\pi}{6}(t-6)\right]$$

$$\frac{\pi}{6}(t-6) = 0 \text{ or } \frac{\pi}{6}(t-6) = 2\pi$$

$$t = 6 \text{ or } t = 18$$

These both represent June. Sales will be 11 600 in June. Since the cosine function is equal to 1 at this point, this is the maximum number of sales.

b) The minimum sales will occur when the cosine function is equal to -1, which occurs when

$$\frac{\pi}{6}(t-6) = \pi$$

$$t - 6 = 6$$

$$t = 12$$

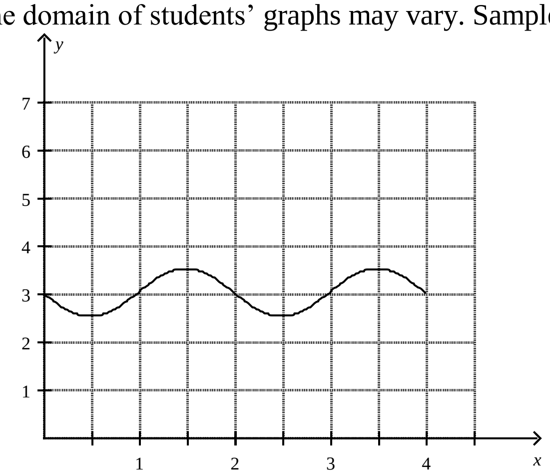
The minimum sales are in December.

c) Yes. You would expect the most sales to occur at the beginning of summer and the least sales in a cold month like December.

DIF: Average

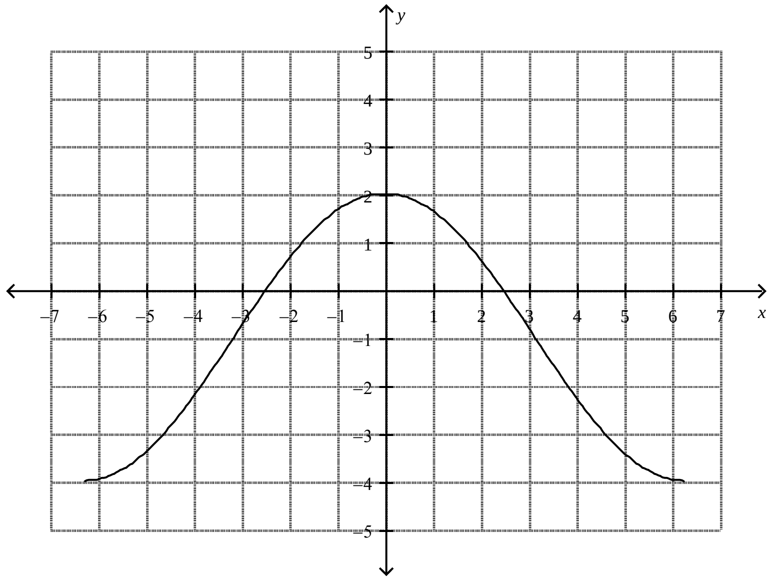
13. ANS:

The domain of students' graphs may vary. Sample graph:



DIF: Average

14. ANS:



DIF: Average

15. ANS:

The period is 12 h 24 min, or 12.4 h.

$$12.4 = \frac{2\pi}{b}$$

$$b = \frac{\pi}{6.2}$$

An equation that models the depth of the water is $d = 4\cos\left(\frac{\pi}{6.2}t\right)$.

At 10:00 a.m., $t = 2$. Substitute $t = 2$ into the equation.

$$d = 4\cos\left(\frac{\pi}{6.2}t\right)$$

$$= 4\cos\left[\frac{\pi}{6.2}(2)\right]$$

$$\approx 2.1$$

The depth of the water at 10 a.m. is 2.1 m.

DIF: Average

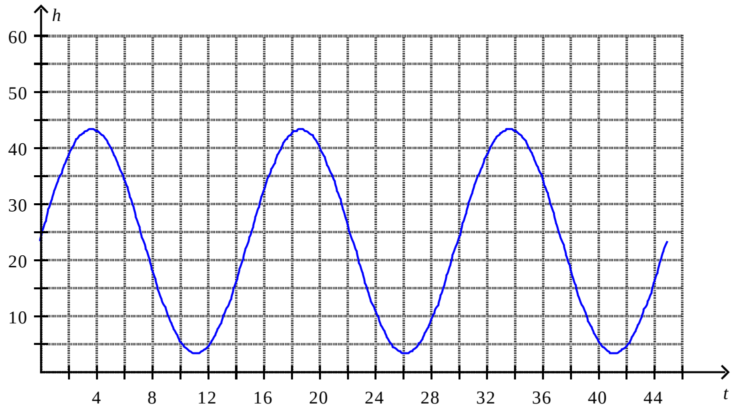
16. ANS:

Solutions may vary. Sample solution:

a) The amplitude is 20 m, and the vertical displacement is 23 m. The frequency of the blades is 4 revolutions per minute, so the period is 0.25 min or 15 s. Thus, $b = \frac{2\pi}{15}$, and the sinusoidal function is

$$h = 20\sin\left(\frac{2\pi}{15}t\right) + 23.$$

b)



c) Substitute t = 10 into the equation.

$$h = 20\sin\left(\frac{2\pi}{15}t\right) + 23$$

$$= 20\sin\left(\frac{2\pi}{15}(10)\right) + 23$$

$$\approx 5.7$$

The tip of the blade is approximately 5.7 m above the ground at t = 10 s.

DIF: Average

17. ANS:

a) The function $y = 2\sin 2x + 2$ has an amplitude of 2, a period of 180°, and a maximum of 4, but the maximum occurs at x = 45°. For the maximum to occur at (0, 4), shift the function 45° to the left:
$y = 2\sin[2(x + 45°)] + 2$

b) The function $y = 2\cos 2x + 2$ has an amplitude of 2, a period of 180°, and a maximum of 4 at x = 0.

c) A phase shift of a sine function of 45° to the left will create a coincident cosine function, as long as the period of the sine function is 180°.

DIF: Average

18. ANS:

a) amplitude: $\frac{1}{2}$; period: $\frac{360°}{3}$ or 120°; phase shift: 30° to the right; vertical shift: 4 units up

b) minimum: $-\frac{1}{2} + 4$ or $\frac{7}{2}$, maximum: $\frac{1}{2} + 4$ or $\frac{9}{2}$

c) The function does not cross the x-axis, so there are no x-intercepts.

d) Substitute x = 0 in the function:

$$f(x) = \frac{1}{2}\sin[3(x - 30°)] + 4$$

$$f(0) = \frac{1}{2}\sin[3(0 - 30°)] + 4$$

$$= \frac{1}{2}\sin(-90°) + 4$$

$$= -\frac{1}{2} + 4$$

$$= \frac{7}{2}$$

The *y*-intercept is $\frac{7}{2}$.

DIF: Average

19. ANS:

$$\text{L.S.} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$\begin{aligned}\text{R.S.} &= \sec 2\theta - \tan 2\theta \\ &= \frac{1}{\cos 2\theta} - \frac{\sin 2\theta}{\cos 2\theta} \\ &= \frac{1 - \sin 2\theta}{\cos 2\theta} \\ &= \frac{1 - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} \\ &= \frac{\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} \\ &= \frac{(\cos\theta - \sin\theta)(\cos\theta - \sin\theta)}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)} \\ &= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\end{aligned}$$

L.S. = **R.S.**

Therefore, $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \sec 2\theta - \tan 2\theta$.

DIF: Difficult

20. ANS:

$$\text{L.S.} = \frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$$

$$= \frac{1 - (1 - 2\sin^2\theta) + 2\sin\theta\cos\theta}{1 + (2\cos^2\theta - 1) + 2\sin\theta\cos\theta}$$

$$= \frac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta(\sin\theta + \cos\theta)}{2\cos\theta(\sin\theta + \cos\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta$$

$$\text{L.S.} = \tan\theta$$

L.S. = R.S

Therefore, $\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} = \tan\theta.$

DIF: Average

21. ANS:

$$\text{L.S.} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\cos\theta(\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta}$$

$$\text{R.S.} = 1 - \tan\theta$$

$$= 1 - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta}$$

L.S. = R.S.

DIF: Average

22. ANS:

$$\text{L.S.} = \frac{1}{1 + \cos\theta} + \frac{1}{1 - \cos\theta}$$

$$= \frac{1 - \cos\theta + 1 + \cos\theta}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{2}{1 - \cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

$$\text{R.S.} = \frac{2}{\sin^2\theta}$$

L.S. = R.S.

DIF: Average

23. ANS:

$$\cos 2x + 2 = \sin x$$
$$1 - 2\sin^2 x + 2 = \sin x$$
$$2\sin^2 x + \sin x - 3 = 0$$
$$(2\sin x + 3)(\sin x - 1) = 0$$
$$\sin x = -\frac{3}{2} \text{ or } \sin x = 1$$
$$\text{no solution or } x = 90°$$

The solution is $x = 90°$.

DIF: Easy

24. ANS:

$$4\sin^4 x + 3\sin^2 x - 1 = 0$$
$$(4\sin^2 x - 1)(\sin^2 x + 1) = 0 \quad \text{Divide both sides by } \sin^2 x + 1 \text{ because it is always positive.}$$
$$4\sin^2 x - 1 = 0$$
$$\sin x = \pm\frac{1}{2}$$

sin x is positive in quadrants I and II and negative in quadrants III and IV. The solution is $x = 30°, 150°, 210°, 330°$.

DIF: Average

25. ANS:

a) $V = 35\,000(0.80)^t$

b) $V = 35\,000(0.80)^t$

$$= 35\,000(0.80)^2$$
$$= 22\,400$$

The value of the vehicle after 2 years is $22 400.

c) $V = 35\,000(0.80)^t$

$$3000 = 35\,000(0.80)^t$$

Use systematic trial. When $t = 11$, $V = 3006.48$. Therefore, after approximately 11 years, the vehicle will be worth $3000.

DIF: Average

26. ANS:

a) $A = 60\left(\frac{1}{2}\right)^n$, where A is the amount of cobalt-60 remaining, in milligrams, and n is the number of half-life periods.

b) 10.6 years equals 2 half-life periods, since $5.3 \times 2 = 10.6$.

$$A = 60\left(\frac{1}{2}\right)^n$$

$$= 60\left(\frac{1}{2}\right)^2$$

$$= \frac{60}{4}$$

$$= 15$$

15 mg will be present in 10.6 years.

c) $12.5\% = 0.125$

$$= \frac{1}{8}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$$3 = n$$

It will take 5.3 × 3, or 15.9 years, for the amount of cobalt-60 to decay to 12.5% of its initial amount.

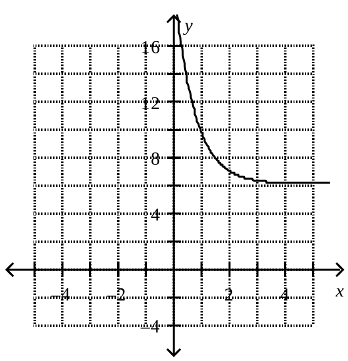
DIF: Difficult

27. ANS:

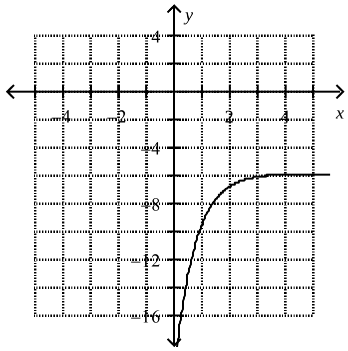
a) $y = 2^{-2(x-2)} + 6$

b) Reflect in the *y*-axis, compress horizontally by a factor of $\frac{1}{2}$, and translate 2 units to the right and 6 units up.

c)



d) $y = -2^{-2(x-2)} - 6$

e) 

DIF: Average

28. ANS:

$$\sqrt[3]{256^2} \times 16^x = 64^{x-3}$$

$$\left(2^8\right)^{\frac{2}{3}} \times 2^{4x} = 2^{6x-18}$$

$$2^{4x+\frac{16}{3}} = 2^{6x-18}$$

$$4x + \frac{16}{3} = 6x - 18$$

$$-2x = -\frac{70}{3}$$

$$x = \frac{35}{3}$$

DIF: Average

29. ANS:

Substitute $V = 50$ into the formula $R = -236\log\left(\frac{V}{80}\right)$ and solve for R.

$R \doteq 48.17$

DIF: Average

30. ANS:

a) $M = \log\left(\frac{10^{5.1}A_0}{A_0}\right)$

$M = \log 10^{5.1}$

$M = 5.1$

The earthquake in Norman Wells measured 5.1 on the Richter scale.

b) $\dfrac{\text{Vancouver Island amplitude}}{\text{Norman Wells amplitude}} = \dfrac{10^{7.3}A_0}{10^{5.1}A_0}$

$= \dfrac{10^{7.3}}{10^{5.1}}$

≈ 158

The earthquake off Vancouver Island was about 158 times as strong as the earthquake off Norman Wells.

DIF: Average

31. ANS:

Let m_1 represent the apparent magnitude of Sirius, b_1 represent the brightness of Sirius, m_2 represent the apparent magnitude of the Sun, and b_2 represent the brightness of the Sun.

a) $m_2 - m_1 = \log\left(\dfrac{b_1}{b_2}\right)$

$0.12 + 1.5 = \log\left(\dfrac{b_1}{b_2}\right)$

$1.62 = \log\left(\dfrac{b_1}{b_2}\right)$

$10^{1.62} = \dfrac{b_1}{b_2}$

$\dfrac{b_1}{b_2} \approx 41.69$

Sirius is approximately 42 times as bright as the Sun.

b) $m_1 - m_2 = \log\left(\dfrac{b_2}{b_1}\right)$

$-1.5 - m_2 = \log(1.3 \times 10^{10})$

$m_2 \approx -1.5 - 10.11$

$m_2 \approx -11.61$

The apparent magnitude of the Sun is –11.6.

DIF: Difficult

32. ANS:

$\mathbf{L.S.} = \dfrac{1}{\log_a b}$ $\qquad\qquad$ $\mathbf{R.S.} = \log_b a$

$= 1 \div \log_a b$

$= 1 \div \dfrac{\log b}{\log a}$

$= 1 \times \dfrac{\log a}{\log b}$

$= \log_b a$

L.S. = **R.S.**

Thus, $\dfrac{1}{\log_a b} = \log_b a$.

DIF: Difficult +

33. ANS:

$\mathbf{L.S.} = 3\log\sqrt{x} + 2\log x - \log\sqrt{x}$ $\qquad\qquad$ $\mathbf{R.S.} = 3\log x$

$= \log x^{\frac{3}{2}} + \log x^2 - \log x^{\frac{1}{2}}$

$= \log\left[\dfrac{x^{\frac{3}{2}}x^2}{x^{\frac{1}{2}}}\right]$

$= \log x^{\frac{3}{2}+2-\frac{1}{2}}$

$= \log x^3$

$= 3\log x$

L.S. = **R.S.**

Thus, $3\log\sqrt{x} + 2\log x - \dfrac{1}{2}\log x = 3\log x$.

DIF: Average

34. ANS:

a) Let *h* represent the half-life of the substance.

$$140 = 200\left(\frac{1}{2}\right)^{\frac{3}{h}}$$

$$0.7 = 0.5^{\frac{3}{h}}$$

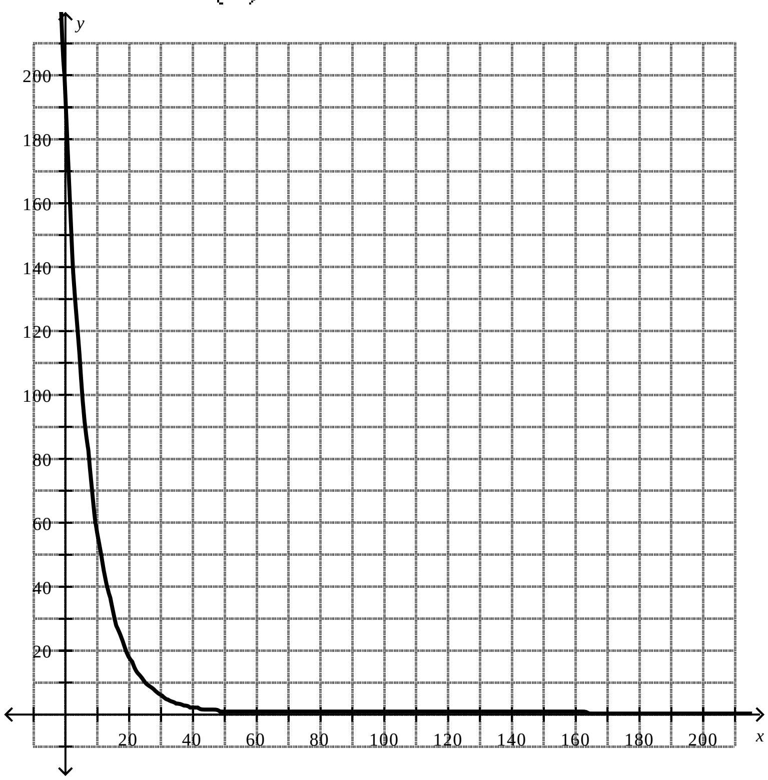
$$\log 0.7 = \frac{3}{h}\log 0.5$$

$$h = \frac{3\log 0.5}{\log 0.7}$$

$$h \approx 5.83$$

The half-life is 5.83 h.

b) Graph $y = 200\left(\frac{1}{2}\right)^{\frac{x}{5.83}}$.



DIF: Difficult

35. ANS:

$\log_2 \sqrt{x^2 - 8x} = \log_2 3$

$\sqrt{x^2 - 8x} = 3$	Since the bases are the same, equate the logarithmic arguments.
$x^2 - 8x = 9$	Square both sides.
$x^2 - 8x - 9 = 0$	Express the quadratic equation in standard form.
$(x - 9)(x + 1) = 0$	Solve for x.

$x = 9$ or $x = -1$

Check the values for extraneous roots.
In this case, both values are possible.

DIF: Difficult

36. ANS:

$$\log \sqrt[3]{x^2 + 48x} = \frac{2}{3}$$

$$\log(x^2 + 48x)^{\frac{1}{3}} = \frac{2}{3}$$

$$\frac{1}{3}\log(x^2 + 48x) = \frac{2}{3}$$

$$\log(x^2 + 48x) = 2$$

$$\log(x^2 + 48x) = \log 100$$

$$x^2 + 48x = 100$$

$$x^2 + 48x - 100 = 0$$

$$(x + 50)(x - 2) = 0$$

$$x = -50 \text{ or } x = 2$$

Check the values for extraneous roots.
In this case, both values are possible and solve the equation, so they are both valid.

DIF: Difficult

37. ANS:

For the first 5 years, the investment is compounded monthly for a total of 5 × 12, or 60, periods.

$$A = 18\ 000(1 + 0.0065)^{60}$$

$$= 26\ 552.12$$

Solve for the remaining time—compounded daily for *n* years is 365*n* periods.

$$35\ 000 = 26\ 552.12\left(1 + \frac{0.05}{365}\right)^{365n}$$

$$\frac{35\ 000}{26\ 552.12} = \left(\frac{365.05}{365}\right)^{365n}$$

$$\log\left(\frac{35\ 000}{26\ 552.12}\right) = 365n\log\left(\frac{365.05}{365}\right)$$

$$n \approx 5.53$$

Bruce will need invest for approximately 5.5 more years.

DIF: Difficult

38. ANS:

$$\begin{aligned} k(x) &= (x+1)^2 + 3(x+1) + 2 \\ &= x^2 + 2x + 1 + 3x + 3 + 2 \\ &= x^2 + 5x + 6 \end{aligned}$$

DIF: Average

39. ANS:

$$\begin{aligned} h(x) &= \frac{1}{1 - \sin^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

DIF: Average

40. ANS:

$$\begin{aligned} f(g(x)) &= g(f(x)) \\ f(x+1) &= g\left(\sqrt{x}\right) \\ \sqrt{x+1} &= \sqrt{x} + 1 \\ \left(\sqrt{x+1}\right)^2 &= \left(\sqrt{x} + 1\right)^2 \\ x + 1 &= x + 2\sqrt{x} + 1 \\ 0 &= 2\sqrt{x} \\ x &= 0 \end{aligned}$$

DIF: Difficult