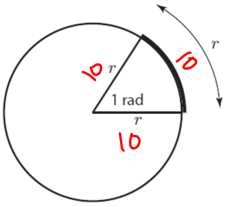
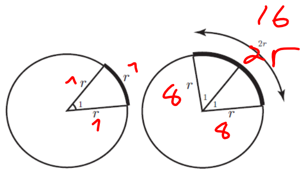
Radian Measure

https://www.youtube.com/watch?v=cTDCveFm_-o

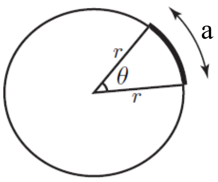
A radian is the angle subtended by an arc of length r (radius)





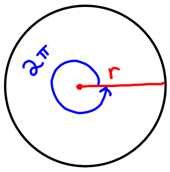
An angle of 2 rad subtends an arc of 2r...
7 rads subtends an arc of???

Use the above information to develop a formula to connect arc length, radius and the measure of an angle in radian measure...



$$a = \theta r$$

Dec 4-9:40 PM



Arc Length = θr

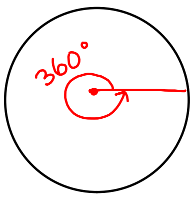
whole circle = $2\pi r$

$C = 2\pi r$

Mar 15-2:37 PM

$C = 2\pi r$

6 full radians



3.14 rad = π = 180°

6.4
0.33
0.35
0.23
0.22
0.29
0.29
~~0.28~~

$\boxed{360° = 2\pi}$

$6.28 = 2\pi$

Mar 15-11:16 AM

HW p176

7f) $7.8 + 2\pi$

$7.8 + 2(3.14...) = 14.1$

$7.8 - 2\pi$

$1.52 - 2\pi$

-4.8

8b) $\frac{5\pi}{2} - 2\pi$ $\quad -\frac{9\pi}{2}$

$\frac{5\pi}{2} - \frac{4\pi}{2}$

$\frac{\pi}{2} - \frac{4\pi}{2}$

$-\frac{3\pi}{2} - \frac{4\pi}{2}$

$-\frac{7\pi}{2} - \frac{4\pi}{2}$

$-\frac{11}{2}$

not coterminal

Mar 16-2:13 PM

9a) $135° + 360°n$

$135° - 360°n$

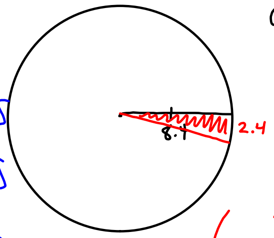
$\boxed{135° \pm 360°n}$

b) $-\frac{\pi}{2} \pm 2\pi n$

Mar 16-2:19 PM

$2\pi r$

6.46
6.54
6.47
$\boxed{6.29}$
6.17
$\boxed{6.27}$
6.36
6.30
6.26
6.21
$\boxed{6.289}$



$\frac{2.4}{8.4}$

$6.\underline{28}$

Mar 13-2:42 PM

$6.\underline{2}\,\underline{8}$ radians

2π

$2(3.14) = 6.28$

Mar 13-10:19 AM

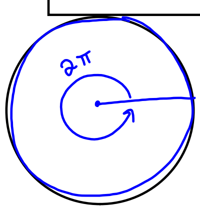
Radians ↔ Degrees

Use this formula to develop a conversion factor between radians and degrees..

arc length $a = \theta r$ or $\theta = \frac{a}{r}$ ← arc length, ← radius

in radians

$1° = \frac{\pi}{180}$ radians	$1 \text{ rad} = \frac{180}{\pi}$



arc length

$C = 2\pi r$

Feb 20-9:32 PM

$360° = 2\pi$

$\boxed{180° = \pi}$ ← conversion factor

$90° = \frac{\pi}{2}$

$45° = \frac{\pi}{4}$

exact ↓ approx

$60° \times \frac{\pi}{\cancel{180°}} = \frac{60\pi}{180} = \frac{\pi}{3}$ radians 1.05

$135° \times \frac{\pi}{180°} = \frac{135\pi}{180} = \frac{3\pi}{4}$ radians 2.36

Radians to Degrees

$\boxed{180° = \pi}$

$\frac{5\cancel{\pi}}{6} \times \frac{180°}{\cancel{\pi}} = 150°$

$-\frac{2\cancel{\pi}}{3} \times \frac{180}{\cancel{\pi}} = -120°$

$3.74 \text{ radians} \times \frac{180°}{\pi} \doteq 214.3°$

Mar 15-2:42 PM

$360° = 2\pi$

$\boxed{180° = \pi}$

$90° = \pi/2$

$45° = \pi/4$

$60° \times \frac{\pi}{180°} = \frac{60\pi}{180} = \frac{\pi}{3}$

$\cancel{30°} \times \frac{\pi}{\cancel{180}_6} = \frac{\pi}{6}$

exact approx

$135° \times \frac{\pi}{180} = \frac{135\pi}{180} = \frac{3\pi}{4} \doteq 2.36$ radians

$300° \times \frac{\pi}{180°} = \frac{300\pi}{180} = \frac{5\pi}{3} \doteq 5.24$ radians

Mar 15-11:37 AM

Radians to Degrees

$\frac{\cancel{\pi}}{3} \times \frac{180°}{\cancel{\pi}} = 60°$

$\frac{7\cancel{\pi}}{13} \times \frac{180°}{\cancel{\pi}} = \frac{7(180)}{13} = 96.9°$

$4.72 \text{ rad} \times \frac{180}{\pi} = 270.4°$

Mar 15-11:47 AM

Textbook P175

#2 all

#3 e

° → rad

ex $90° \times \frac{\pi}{180} = \frac{90\pi}{180} = \frac{\pi}{2}$

#4 all

#5 e

rad → °

ex $\frac{\pi}{4} \times \frac{180}{\pi} = \frac{180}{4} = 45°$

$\sin 30° = 0.5$ $\sin\left(\frac{\pi}{6}\right) = 0.5$

Mar 15-11:51 AM

Degrees to Radians

a) $47^\circ\left(\frac{\pi}{180^\circ}\right) \doteq 0.82\text{ rad}$

b) $120^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{12\cancel{0}\pi}{18\cancel{0}} = \boxed{\frac{2\pi}{3}} \doteq 2.09\text{ rad}$

exact ☺

Radians to Degrees

a) $5.34\text{ rad}\left(\frac{180^\circ}{\pi}\right) \doteq 306^\circ$

b) $\frac{3\cancel{\pi}}{4}\text{ rad}\left(\frac{180}{\cancel{\pi}}\right) = 135^\circ$

Mar 13-2:58 PM

Practice

Try **Degrees to Radians**

a) $87^\circ\left(\frac{\pi}{180^\circ}\right) \doteq 1.52\text{ radians}$

b) $120^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{12\cancel{0}\pi}{18\cancel{0}} = \frac{2\pi}{3} \doteq 2.09\text{ radians}$

exact

✓

Radians to Degrees

a) $5.32\text{ radians}\left(\frac{180^\circ}{\pi}\right) \doteq 305^\circ$

b) $\frac{3\cancel{\pi}}{4}\text{ radians}\left(\frac{180^\circ}{\cancel{\pi}}\right) = 135^\circ$

Mar 13-10:33 AM

Strategies for converting between angle units...

Unitary Method

$360^\circ = 2\pi$

$1^\circ = \frac{2\pi}{360}$ - - - - - - - →

$= \frac{\pi}{180}$

$60^\circ =$

Proportion Method

$180^\circ = \pi$

$60^\circ = x$

Feb 20-9:43 PM

Converting radians to degrees...

Unit Analysis

$\frac{5\pi}{4} = \left(\frac{5\cancel{\pi}}{4}\right)\left(\frac{180^\circ}{\cancel{\pi}}\right)$

$= \frac{5(180^\circ)}{4}$

$= 225^\circ$

Why does $\left(\frac{180^\circ}{\pi}\right)$ have value 1?

$\boxed{\pi\text{ rad} = 180^\circ}$

Feb 20-9:54 PM

When each of the following angles is converted from degrees to radians the answer can be expressed as a multiple of π (note that it may be a fractional multiple). In each case state the multiple (e.g for an answer of $\frac{4\pi}{5}$ the multiple is $\frac{4}{5}$).

a) 90^o b) 360^o c) 60^o d) 45^o
e) 120^o f) 15^o g) 135^o h) 270^o

Feb 20-9:57 PM

When each of the following angles is converted from degrees to radians the answer can be expressed as a multiple of π (note that it may be a fractional multiple). In each case state the multiple (e.g for an answer of $\frac{4\pi}{5}$ the multiple is $\frac{4}{5}$).

a) 90^o b) 360^o c) 60^o d) 45^o
e) 120^o f) 15^o g) 135^o h) 270^o

exact approx

a) $90^\circ \cdot \frac{\pi}{180^\circ} = \frac{90\pi}{180} = \frac{\pi}{2} \doteq 1.57$

b) $360^\circ \cdot \frac{\pi}{180} = \frac{360\pi}{180} = 2\pi \doteq 6.28$

c) $60^\circ \cdot \frac{\pi}{180} = \frac{60\pi}{180} = \frac{\pi}{3} \doteq 1.05$

d) $45^\circ \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4} \doteq 0.79$

e) $120^\circ \cdot \frac{\pi}{180} = \frac{120\pi}{180} = \frac{2\pi}{3} \doteq 2.09$

f) $15^\circ \cdot \frac{\pi}{180} = \frac{15\pi}{180} = \frac{\pi}{12} \doteq 0.26$

g) $135^\circ \cdot \pi/180 = \frac{135\pi}{180} = \frac{3\pi}{4} \doteq 2.36$

h) $270^\circ \cdot \pi/180 = \frac{270\pi}{180} = \frac{3\pi}{2} \doteq 4.71$

Feb 20-9:57 PM

Convert each of the following angles from radians to degrees.

a) $\frac{\pi}{2}$ radians b) $\frac{3\pi}{4}$ radians c) π radians d) $\frac{\pi}{6}$ radians
e) 5π radians f) $\frac{4\pi}{5}$ radians g) $\frac{7\pi}{4}$ radians h) $\frac{\pi}{10}$ radians

$\boxed{\pi = 180}$

e) $5\cancel{\pi} \cdot \frac{180}{\cancel{\pi}}$ 900°

f) $\frac{4\cancel{\pi}}{5} \cdot \frac{180}{\cancel{\pi}}$ 144°

g) $\frac{7\cancel{\pi}}{4} \cdot \frac{180}{\cancel{\pi}}$ 315°

h) $\frac{\cancel{\pi}}{10} \cdot \frac{180}{\cancel{\pi}}$ 18°

Feb 20-9:58 PM

Convert each of the following angles from degrees to radians giving your answer to 2 decimal places.

a) 17^o b) 49^o c) 124^o d) 200^o

0.86 2.16 3.49

$17^\circ \left(\frac{\pi}{180}\right) = \cancel{0.297}$
$= 0.30$

$\cancel{\frac{17\pi}{180}}$

Feb 20-9:59 PM

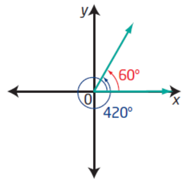
Convert each of the following angles from radians to degrees, giving your answer to 1 decimal place.
a) 0.6 radians b) 2.1 radians c) 3.14 radians d) 1 radian

a) $0.6\text{rad}\left(\frac{180}{\pi}\right)$
$34.4°$
b) $2.1\text{rad}\left(\frac{180}{\pi}\right)$
$123.8°$
c) $179.9°$
d) $57.3°$

Feb 20-9:59 PM

Coterminal Angles

When you sketch an angle of 60° and an angle of 420° in standard position, the terminal arms coincide. These are **coterminal angles**.



coterminal angles
- angles in standard position with the same terminal arms
- may be measured in degrees or radians
- $\frac{\pi}{4}$ and $\frac{9\pi}{4}$ are coterminal angles, as are 40° and −320°

$60°+360°=420°$
$60°-360°=-300°$
$60°+360+360=780°$

Strategy for finding coterminal angles in either radians or degrees?

Degrees → add/subtract 360's
Radians → " " 2π

$\frac{\pi}{4}+2\pi = \frac{\pi}{4}+\frac{8\pi}{4} = \frac{9\pi}{4}$

$\frac{\pi}{4}-2\pi = \frac{\pi}{4}-\frac{8\pi}{4} = \frac{-7\pi}{4}$

ex $\frac{5\pi}{7} = \frac{19\pi}{7} = -\frac{9\pi}{7}$

ex $\frac{12\pi}{17}$ $+\frac{46\pi}{17}$ $-\frac{22\pi}{17}$

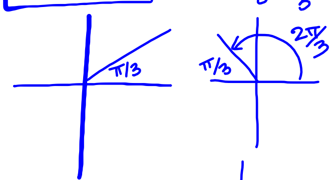
Feb 20-10:01 PM

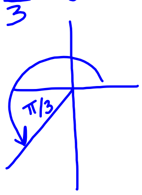
Special angles

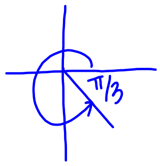
$\boxed{60° = \frac{\pi}{3}}$

$\pi - \pi/3$
$\frac{3\pi}{3}-\frac{\pi}{3} = \frac{2\pi}{3}$

$\pi + \pi/3$
$\frac{3\pi}{3}+\frac{\pi}{3} = \frac{4\pi}{3}$







$2\pi - \frac{\pi}{3}$
$\frac{6\pi}{3}-\frac{\pi}{3} = \frac{5\pi}{3}$

Mar 22-2:35 PM

P176 #14

1rev / 15sec

14 a) $a = \theta r$
$= \frac{5\pi}{3}(5\text{ m})$
$= \frac{25\pi}{3}\text{ m}$

14 b) Area of Sector
$A = \frac{\theta}{2\pi}\cdot\pi r^2$
$= \frac{1}{2}r^2\theta$
$= \frac{1}{2}(5\text{m})^2\left(\frac{5\pi}{3}\right)$
Area $= \frac{125\pi}{6}\text{ m}^2$

c) $V_a = \frac{2\pi}{15\text{sec}}$
$= \frac{2\pi}{15}\text{ rad/sec}$

$V_a = \frac{\theta}{t}$
$V_a(t) = \theta$
$\frac{2\pi}{15}/s\,(120\text{sec}) = \theta$
$16\pi = \theta$

1st → Finish W.S
2nd → Start review
p215 review
p218 practice test

Apr 8-1:10 PM

$360° = 2\pi$

$\boxed{180° = \pi}$

$90° = \pi/2$

$45° = \pi/4$

5e) $300°\left(\frac{\pi}{180°}\right) = \frac{30\cancel{0}\pi}{18\cancel{0}} = \boxed{\frac{5\pi}{3}} \doteq 5.24$

exact

6i) $1025°\left(\frac{\pi}{180°}\right) \doteq 17.890$

7e) $\frac{17\cancel{\pi}}{\cancel{6}}\left(\frac{\overset{30°}{\cancel{180°}}}{\cancel{\pi}}\right) = 510°$

Mar 14-10:03 AM

For each angle in standard position, determine one positive and one negative angle measure that is coterminal with it.

a) $270°$ **b)** $-\frac{5\pi}{4}$

$270° + 360°$

$630°$

$270° - 360°$

$-90°$

$-\frac{5\pi}{4} + 2\pi$

$-\frac{5\pi}{4} + \frac{8\pi}{4}$

$+\frac{3\pi}{4}$

$-\frac{5\pi}{4} - 2\pi$

$-\frac{5\pi}{4} - \frac{8\pi}{4}$

$-\frac{13\pi}{4}$

Feb 20-10:02 PM

Determine a ~~negative~~ principal angle co-terminal with each of the following angles:

$0 < \theta < 360°$

or

$0 < \theta < 2\pi$

1) $476895° \div 360$

$1324.\boxed{708\bar{3}} \times 360°$

$255°$

2) $\frac{\overset{17892}{\cancel{35784\pi}}}{5} \cdot \frac{1}{\cancel{2\pi}}$

$\frac{17892}{5}$

$3578.\boxed{4} \times 2\pi$

$0.4 \times 2\pi$

$\frac{\cancel{4}}{\underset{5}{\cancel{10}}} \cdot \cancel{2}\pi$

$\frac{4\pi}{5}$

Feb 25-1:37 PM

Hw page 175

#1, 2a, 3a, 4a, 5a, 7, 8, 9

Mar 14-10:32 AM

P176

8a) $\frac{5\pi}{6}$ $\quad$ $\frac{17\pi}{6}$

$\frac{5\pi}{6} + 2\pi$

$\frac{5\pi}{6} + \frac{12\pi}{6}$

$\frac{17\pi}{2}$

b) $\frac{5\pi}{2}$, $-\frac{9\pi}{2}$

$\frac{5\pi}{2} - 2\pi$

$\frac{5\pi}{2} - \frac{4\pi}{2}$

$+\frac{\pi}{2} - 2\pi$

$+\frac{\pi}{2} - \frac{4\pi}{2}$

$-3\pi/2 - 2\pi$

$-\frac{3\pi}{2} - \frac{4\pi}{2}$

$-\frac{7\pi}{2}$

Feb 20-10:53 AM

9. a) $135° + 360°n$ $\rightarrow +n \; +$ $\rightarrow -n \; -$

$135° \pm 360°n$

General Rule

b) $-\frac{\pi}{2} + 2\pi$

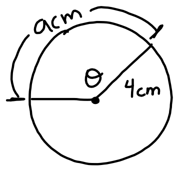
$-\frac{\pi}{2} + \frac{4\pi}{2}$

$\frac{3\pi}{2} \pm 2\pi n$

Feb 20-11:04 AM

13. $a = \theta r$

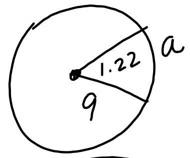
a)



$\theta = \frac{a}{r}$

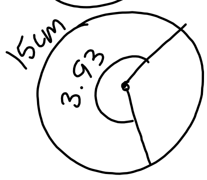
$\theta = \frac{9}{4}$ radians

b)



$a = 1.22(9\,ft)$

c)

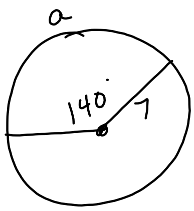


$a = \theta r$

$\frac{a}{\theta} = r$

$\frac{15}{3.93} = r$

d)



$a = \theta r$

$a = \left(140° \cdot \frac{\pi}{180°}\right)(7m)$

$a = 17.1\,m$

Feb 20-11:09 AM

Decimals to Fractions

1) Terminating Decimals

ex $0.8 \rightarrow \frac{8}{10} = \frac{4}{5}$

$0.221 \rightarrow \frac{221}{1000}$

$8.221 \rightarrow \frac{8221}{1000}$

2) Repeating Decimals

ex $0.\overline{4} = \frac{4}{9}$

ex $0.353535... = \frac{35}{99}$

ex. $0.\overline{158} = \frac{158}{999}$

3) Mixed of terminating and Repeating

ex $0.833333 = \frac{83-8}{90}$

$= \frac{75}{90} = \frac{5}{6}$

ex $0.12\overline{345} = \frac{12345-12}{99900}$

Feb 20-11:13 AM

Add/Subtract	Multiply/Dividing
(Common denominator)	
ex $\frac{3}{7}+\frac{5}{4}=\frac{12}{28}+\frac{35}{28}=\frac{47}{28}$	ex $\frac{3}{7}\cdot\frac{5}{4}=\frac{15}{28}$
ex $\frac{3\pi}{4}+\frac{5\pi}{6}=\frac{9\pi}{12}+\frac{10\pi}{12}=\frac{19\pi}{12}$	ex $\frac{4}{9}\div\frac{7}{2}$ $\frac{4}{9}\cdot\frac{2}{7}=\frac{8}{63}$

Feb 20-11:23 AM

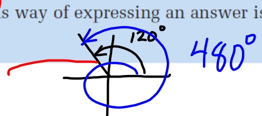
Coterminal Angles in General Form

Any given angle has an infinite number of angles coterminal with it, since each time you make one full rotation from the terminal arm, you arrive back at the same terminal arm. Angles coterminal with any angle θ can be described using the expression

$\boxed{\theta \pm (360°)n \text{ or } \theta \pm 2\pi n,}$

where n is a natural number. This way of expressing an answer is called the **general form**.

$\theta + 360n$



general form

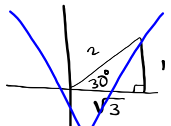
- an expression containing parameters that can be given specific values to generate any answer that satisfies the given information or situation
- represents all possible cases

Let's use the following two angles...

$\theta = 70°$ $\qquad$ $\theta = \frac{5\pi}{6}$

$70° \pm 360° n$ $\qquad$ $\frac{5\pi}{6} \pm 2\pi n$

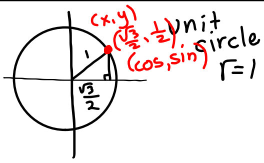
Feb 20-10:05 PM



$X = \sqrt{3}$

$y = 1$

$r = 2$



$\sin 30° = \frac{1}{2}$ $\qquad$ $x = \frac{\sqrt{3}}{2}$

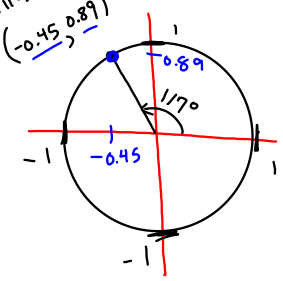
$\cos 30° = \frac{\sqrt{3}}{2}$ $\qquad$ $y = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ $\qquad$ $r = 1$

Mar 19-2:28 PM

$(\cos 117, \sin 117)$

$(-0.45, 0.89)$



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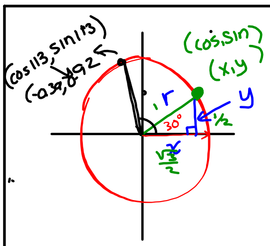
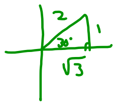


Special angles
30°, 60°, 45°

$\sin\theta = \frac{opp}{hyp}$

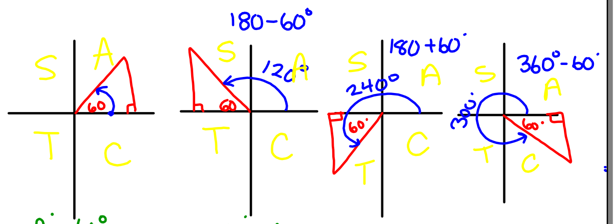
$\sin\theta = \frac{y}{r}$



$\sin 30° = \frac{1}{2}$

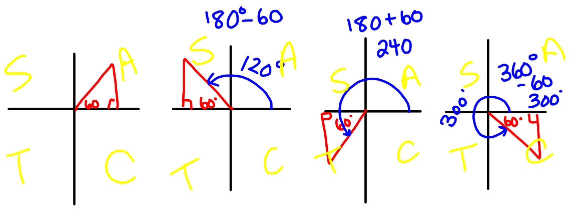
$\cos 30° = \frac{\sqrt{3}}{2}$

Mar 19-11:04 AM



sin 60°	sin 120°	sin 240°	sin 300°
0.866	0.866	−0.866	−0.866

Mar 19-2:38 PM



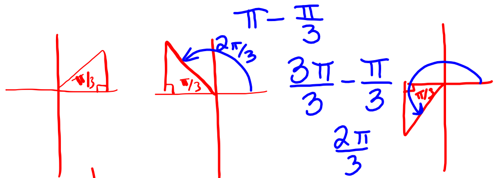
sin 60° = 0.866

sin 120° = 0.866

sin 240° = −0.866

sin 300° = −0.866

Mar 19-11:31 AM



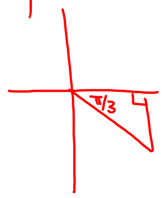
$\pi - \frac{\pi}{3}$

$\frac{3\pi}{3} - \frac{\pi}{3}$

$\frac{2\pi}{3}$

$\pi + \frac{\pi}{3}$

$\frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$



$2\pi - \frac{\pi}{3}$

$\frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

Mar 19-11:37 AM

TextBook p201
#1, 6, 8, 9

S	A
T	C

$45^\circ \quad \frac{}{4}$

$30^\circ \quad \frac{}{6}$

$60^\circ \quad \frac{}{3}$

$90^\circ \quad \frac{}{2}$

$270^\circ \quad \frac{}{2}$

Mar 19-11:51 AM

$45^\circ \quad \frac{}{4}$

$30^\circ \quad \frac{}{6}$

$60^\circ \quad \frac{}{3}$

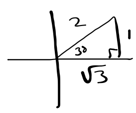
$90^\circ \quad \frac{}{2}$

$270^\circ \quad \frac{}{2}$

Mar 19-2:08 PM

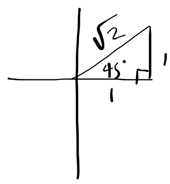
Mar 19-2:21 PM

$\sin 30^\circ = \frac{1}{2}$

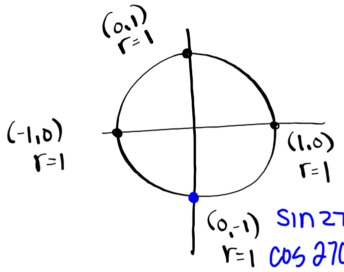


$\cos 60^\circ = \frac{1}{2}$

$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$



Mar 19-2:53 PM



$\sin 270° = \frac{-1}{1} = -1$

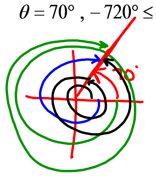
$\cos 270° = \frac{0}{1} = 0$

$\tan 270° = \frac{-1}{0}$ undefined

Mar 19-2:56 PM

What if we are given a restricted domain?

$\theta = 70°$, $-720° \le \theta \le 1080°$



430°

−290°

−650°

790°

$\theta = \frac{5\pi}{6}$, $-2\pi \le \theta \le 8\pi$

$\frac{5\pi}{6} + 2\pi = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

$\frac{5\pi}{6} + 4\pi = \frac{5\pi}{6} + \frac{24\pi}{6} = \frac{29\pi}{6}$

$\frac{5\pi}{6} + 6\pi = \frac{5\pi}{6} + \frac{36\pi}{6} = \frac{41\pi}{6}$

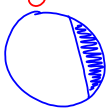
$\frac{5\pi}{6} - 2\pi = \frac{5\pi}{6} - \frac{12\pi}{6} = \frac{-7\pi}{6}$

Feb 20-10:10 PM

Sector → bound by 2 radii and an arc

"pizza"

Segment → bound by a chord and an arc



"garlic finger"

Measurement of area of sector of a circle in Radian

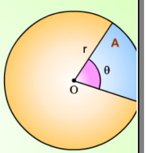
In general, if the angle of a sector, θ, is measured in degree,

then the **area of the sector**, $A = \frac{\theta}{360} \times \pi r^2$

If θ is measured in **radians**,

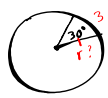
then the area of the sector $A = \frac{\theta}{2\cancel{\pi}} \times \cancel{\pi} r^2$ 2π rad = 360°

∴ $A = \frac{1}{2} r^2 \theta$



Oct 5-1:03 PM

9.



$a = \theta r$

$r = \frac{a}{\theta}$

$r = \frac{3}{\pi/6}$

$= 3 \cdot \frac{6}{\pi}$

$= \frac{18}{\pi} \approx 5.73$

$30° \cdot \frac{\pi}{180}$

$\frac{\pi}{6}$

10. $a = \theta r$

$= (13\cancel{0}° \cdot \frac{\pi}{18\cancel{0}})(15\text{cm})$

$\frac{13\pi}{\cancel{18}\,6} \cdot \cancel{15}^{5}\text{ cm} = \frac{65\pi}{6}\text{ cm}$

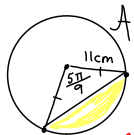
11. $\theta = \frac{a}{r}$

$\theta = \frac{5\ \cancel{cm}}{6\ \cancel{cm}}$

$\theta = \frac{5}{6}$ radians

$\frac{5}{\cancel{6}} \cdot \frac{\cancel{180}^{30}°}{\pi}$

$\frac{150°}{\pi}$



$A_{segment} = A_{sector} - A_{triangle}$

$105.6 - 59.6$

46.0

$A_{sector} = \frac{\theta}{2\pi} \cdot \pi r^2$

$= \frac{\frac{5\pi}{9} \cdot (11\text{cm})^2}{2}$

$\frac{605\pi}{18}\text{cm}^2$

105.6

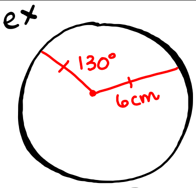
$A_{triangle} = \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2}(11\text{cm})^2 \sin(\frac{5\pi}{9})$

$= 59.6$

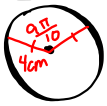
Feb 21-10:56 AM

ex



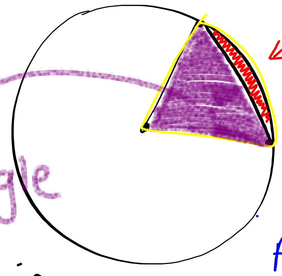
$$A_{sector} = \frac{\theta}{360} \cdot \pi r^2$$
$$= \frac{130}{360} \cdot \pi (6cm)^2$$
exact → $= 13\pi \, cm^2$
approx → $\doteq 40.8 \, cm^2$

ex



$$A_{sector} = \frac{\theta}{2\pi} \cdot \pi r^2$$
$$= \frac{\frac{9\pi}{10}}{2\cancel{\pi}} \cdot \cancel{\pi}(4cm)^2$$
$$= \frac{9\pi}{10} \cdot \frac{1}{\cancel{2}} \cdot \cancel{16}^{8} cm^2$$
exact $= \frac{72\pi}{10}$
$$= \frac{36\pi}{5} \doteq 22.6$$

Apr 10-3:08 PM



$A = \frac{1}{2} bh$

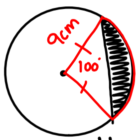
new

$A = \frac{1}{2} r^2 \sin\theta$

$A = \frac{1}{2} r^2 \theta$

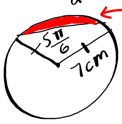
Mar 17-2:37 PM

Area of a Segment



$$A_{segment} = A_{sector} - A_{triangle}$$
$$= \frac{\theta}{360} \cdot \pi r^2 - \frac{1}{2} r^2 \sin\theta$$
$$= \frac{100}{360} \cdot \pi (9cm)^2 - \frac{1}{2}(9cm)^2 \sin 100°$$
$$= 70.7 - 39.9$$
$$30.8$$

$A_{triangle} = \frac{bh}{2}$
$= \frac{1}{2} r^2 \sin\theta$

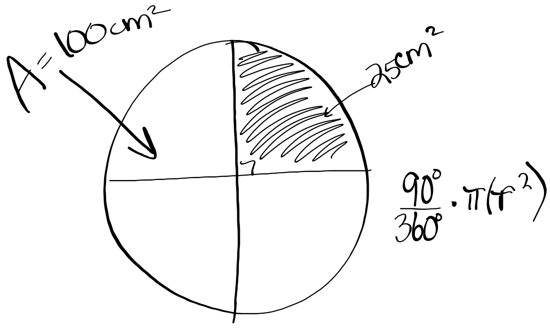


segment

$A_{sector} - A_{triangle}$
$$\frac{\frac{5\pi}{6}}{2\pi} \cdot \pi (7cm)^2 - \frac{1}{2}(7cm)^2 \sin\left(\frac{5\pi}{6}\right)$$
$$64.1 \quad - \quad 12.25$$
$$51.9$$

Feb 20-11:42 AM

Mar 16-10:18 AM



$\frac{90^\circ}{360^\circ} \cdot \pi(r^2)$

Oct 5-2:43 PM

$V_a = \omega$ $V = V_a \cdot r$ or $V = \omega r$

7b) $d = 198m$, $r = 99m$

$V_a = \frac{1 rev}{20 min} = \frac{2\pi}{20 min} = \frac{\pi}{10}$ radians/min

$= \frac{\pi}{10 \cancel{min}} \cdot \frac{1 \cancel{min}}{60 sec} = \frac{\pi}{600}$ rad/s

$V = V_a \cdot r$

$= \frac{\pi}{600 s} \cdot 99m$ (200, 33)

$= \frac{33\pi}{200}$ m/s

b) 60 rpm, $r = 325mm$, $r = 0.325m$

$\boxed{a = d} = ?$

$t = 1 min$

$V_a = \frac{\theta}{t}$ $\theta = V_a \cdot t$

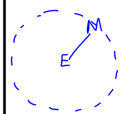
$V_a = \frac{60 rev}{1 min} = \frac{120\pi}{1 min}$

$\theta = 120\pi$

$d = a = \theta D$

$= 120\pi(0.325m)$

$= 39\pi$ m



$r = 385000 km$

$V_a = \frac{1 rev}{27.2 days} \times \frac{1 day}{24 hours} = \frac{2\pi}{652.8}$ rad/h

$V = V_a \cdot r$

$\frac{2\pi}{652.8 h} \cdot 385000 km$

~~$\frac{240625\pi}{204}$ km/h~~

≈ 3706 km/h

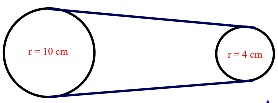
Apr 8-10:57 AM

Applying our knowledge of rotations and radians... $a = \theta r$

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians? $3 \cdot 2\pi = 6\pi$

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger whell rotate?



a) large wheel
$a = \theta r$
$= \left(\frac{2\pi}{3}\right)(10cm)$
$= \frac{20\pi}{3} cm$ same

Small wheel
$\theta = \frac{a}{r}$
$\theta = \frac{\frac{20\pi}{3} cm}{4 cm}$
$\frac{20\pi}{12}$
$\frac{5\pi}{3}$

b) large wheel
$a = \theta r$
$= 6\pi \cdot 10cm$
$= 60\pi$ cm

small wheel
$\theta = \frac{60\pi \cancel{cm}}{4 \cancel{cm}}$
$\theta = 15\pi$

c) Small
$a = \left(\frac{-15\pi}{\cancel{4}}\right)(\cancel{4}cm)$
$= -15\pi$ cm

Large $\theta = \frac{-15\pi \cancel{cm}}{10 \cancel{cm}}$
$= \frac{-3\pi}{2}$

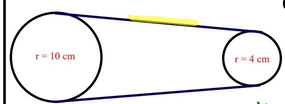
Nov 20-11:50 AM

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians? $3 \cdot 2\pi = 6\pi$

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger whell rotate?



a) Big Wheel
$a = \theta r$
$= \frac{2\pi}{3} \cdot 10cm$
$= \frac{20\pi}{3}$ cm
same

Small Wheel
$a = \frac{20\pi}{3}$ cm
$r = 4cm$
$\boxed{a = \theta r}$
$\frac{a}{r} = \theta$
$\frac{\frac{\cancel{20}\pi}{3} \cancel{cm}}{\cancel{4} \cancel{cm}} = \theta$ (5)
$\frac{5\pi}{3} = \theta$

b) Big Wheel
$a = \theta r$
$= 6\pi \cdot 10cm$
$= 60\pi$ cm

Small Wheel
$\theta = \frac{a}{r}$
$= \frac{60\pi \cancel{cm}}{4 \cancel{cm}}$
$= 15\pi$

Nov 20-11:50 AM

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?

(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?

(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger whell rotate?

θ ?

a) Big wheel $a = \theta r$, $a = \frac{2\pi}{3}(10cm)$, $a = \frac{20\pi}{3} cm$

Small wheel $a = \frac{20\pi}{3} cm$, $r = 4 cm$, $\theta = ?$

$a = \theta r$, $\frac{20\pi}{3} \cdot \frac{1}{4} = \theta$, $\frac{20\pi}{12} = \theta$, $\frac{5\pi}{3} = \theta$

b) Big Wheel

$a = \theta r$

$= (3 \cdot 2\pi)(10cm)$

$= 6\pi(10cm)$

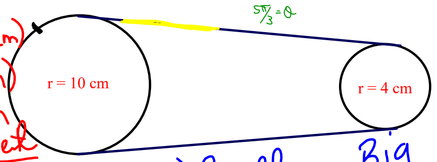
$= 60\pi\ cm$

Small Wheel

$a = \theta r$

$\theta = \frac{a}{r} = \frac{60\pi\ \cancel{cm}}{4\ \cancel{cm}}$

$= 15\pi$



c) Small

$a = \theta r$

$= -\frac{15}{\cancel{4}}\pi(\cancel{4})$

$= -15\pi\ cm$

Big

$\theta = \frac{a}{r}$

$= \frac{-15\pi}{10}$

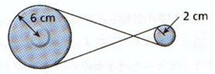
$= -3/2\,\pi$

$= -1.5\pi$

$-\frac{3\pi}{2}$ radians

Nov 20-11:50 AM

Two flywheels are connected by a belt, as shown in the diagram below. The larger one has a radius of 6 cm and the smaller one has a radius of 2 cm.

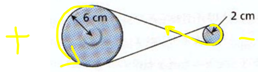


(a) If the small wheel rotates –300°, then through how many radians does the large wheel rotate?

(b) If the large wheel rotates $\frac{7\pi}{6}$ radians, what distance would a point on the circumference of the small wheel rotate?

Feb 26-1:33 PM

Two flywheels are connected by a belt, as shown in the diagram below. The larger one has a radius of 6 cm and the smaller one has a radius of 2 cm.



(a) If the small wheel rotates –300°, then through how many radians does the large wheel rotate?

(b) If the large wheel rotates $\frac{7\pi}{6}$ radians, what distance would a point on the circumference of the small wheel rotate?

a) Small

$a = \theta r$

$a = \frac{300\pi}{180}(2cm)$

$a = \frac{5\pi}{3}(2cm)$

$a = \frac{10\pi}{3}$

Large

$a = \frac{10\pi}{3}$

$r = 6$

$\theta = \frac{a}{r}$

$= \frac{\frac{10\pi}{3}}{6} = \frac{10\pi}{18} = \frac{5\pi}{9}$

b) Large

$a = \frac{7\pi}{\cancel{6}}(\cancel{6})$

$a = 7\pi\ cm$

Same

Feb 26-1:33 PM

7b) $V = \frac{d}{t}$ $V_a = \frac{\theta}{t}$ $V = V_a \cdot r$

$r = 99m$

$V_a = \frac{1 rev}{20 min} = \frac{2\pi}{20 min} = \frac{\pi}{10}$ rad/min

$V_a = \frac{\pi}{10 min} \cdot \frac{1 min}{60 sec} = \frac{\pi}{600}/s$

$V = V_a \cdot r$

$\frac{\pi}{\cancel{600}\,s} \cdot \cancel{99}\,m$

$\frac{33\pi}{200}\,m/s$

c) $V_a = \frac{60 rev}{1 min} = \frac{120\pi}{1 min}$ $V_a = \frac{\theta}{t}$

$r = 325 mm$

$d = ?$

$t = 1 min$

$a = \theta \cdot r$

$= 120\pi \cdot 0.325$

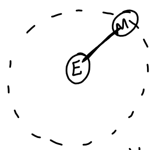
$= 39\pi$

$V_a \cdot t = \theta$

$\frac{120\pi}{\cancel{min}} \cdot 1\cancel{min} = \theta$

$120\pi = \theta$

d)



$r = 385000\ Km$

$V_a = \frac{1 rev}{27.2 days}$

$V_a = \frac{2\pi}{27.2 \cancel{days}} \times \frac{1\ \cancel{day}}{24 h} = \frac{2\pi}{652.8}$

$= \frac{5\pi}{1632}\ h$

$V = V_a \cdot r$

$= \frac{5\pi}{1632\ h} \cdot 385000\ Km$

$= \frac{240625\pi}{204}\ Km/h \approx 3706\ Km/h$

Apr 8-2:12 PM

Linear Velocity

$\boxed{V = \frac{d}{t}}$ $\boxed{V = V_a \cdot r}$ $\boxed{V = \omega r}$

Angular Velocity

$\boxed{V_a = \frac{\theta}{t} \xrightarrow{\div 2\pi} \frac{\text{revolutions}}{t}}$

rpm → revolutions per minute

$\boxed{a = \theta r}$

$\boxed{\text{distance} = \text{arc length}}$

$$A_{segment} = A_{sector} - A_{triangle} = \frac{\theta}{2\pi} \cdot \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

Apr 9-2:24 PM

Angular Velocity: Gear & Pulley

1. Small r=3cm 120rpm
Large r=7.5cm

a) $V_a = \frac{\cancel{120}^{4}\ \cancel{rev}}{1\ \cancel{min}} \cdot \frac{2\pi}{1\ \cancel{rev}} \cdot \frac{1\ \cancel{min}}{\cancel{60}\ sec}$

4π rad/sec

b) $\boxed{V = V_a \cdot r}$
$= \frac{4\pi}{sec} \cdot 3cm$
$= 12\pi\, cm/sec$

c) $V = 12\pi\, cm/sec$

d) $V_a = \frac{V}{r}$
$= \frac{12\pi\, \cancel{cm}/sec}{7.5\cancel{cm}}$
$= \frac{\frac{8\pi}{5} \text{"rad"}}{sec}$

e) $V_a = \frac{\frac{8\pi}{5} rad}{1\ sec} \cdot \frac{1 rev}{2\pi} \cdot \frac{\cancel{60}\ sec}{1 min}$
48 rpm

3. $r = 4cm$
$\theta = 120°$

a) $120° \cdot \frac{\pi}{180} = \frac{2\pi}{3}$ radians

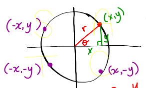
b) $a = \theta \cdot r$
$= \frac{2\pi}{3} \cdot 4cm$
$= \frac{8\pi}{3} cm$

c) $V_a = \frac{120°}{\frac{1}{2}\ sec}$
OK $240°/sec$
OR
$V_a = \frac{\frac{2\pi}{3} rad}{\frac{1}{2} sec}$
$\frac{4\pi}{3}$ rad/sec

d) $V = V_a \cdot r$
$= \frac{\frac{4\pi}{3}}{sec} \cdot 4cm$
$= \frac{16\pi\, cm}{3}/sec$

Feb 24-11:00 AM

PreCalc 11 Review - Special angles



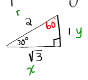
$\boxed{x^2 + y^2 = r^2}$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$ $\csc\theta = \frac{hyp}{opp} = \frac{r}{y}$

$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$ $\sec\theta = \frac{hyp}{adj} = \frac{r}{x}$

$\tan\theta = \frac{opp}{adj} = \frac{y}{x}$ $\cot\theta = \frac{adj}{opp} = \frac{x}{y}$

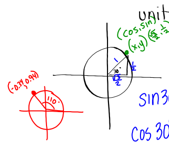
Special angles



$x^2 + y^2 = r^2$
$(\sqrt{3})^2 + (1)^2 = 2^2$
$3 + 1 = 4$
$4 = 4$

sin 30°
cos 30°

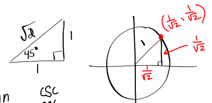
sin 60°



$\sin 30° = \frac{\frac{1}{2}}{1} = \frac{1}{2}$

$\cos 30° = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

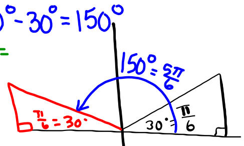


sin csc
cos sec
tan cot

Feb 25-10:57 AM

$180° - 30° = 150°$

$\pi - \frac{\pi}{6} =$
$\frac{6\pi}{6} - \frac{\pi}{6}$
$\frac{5\pi}{6}$



$180° + 30°$
$\pi + \frac{\pi}{6}$

$360° - 30°$
$2\pi - \frac{\pi}{6}$

#8 $r^2 = x^2 + y^2$

$\boxed{\text{unit circle } r = 1}$

Feb 25-11:53 AM

Today → test review worksheet

extra practice: gears & pulley worksheet

p215 #1-7, 12-15, 17, 20, 21

#7c $5 \text{ revolutions} \xrightarrow{\times 2\pi} 10\pi \text{ radians}$ ($\div 2\pi$)

Angular Velocity

$V_a = \dfrac{\theta}{t}$ or $\dfrac{\text{revolutions}}{t}$ ($\times 2\pi$)

distance = arc length

Linear

$V = \dfrac{d}{t}$

$\boxed{V = V_a \cdot r}$

$\boxed{a = \theta r}$

Apr 9-11:20 AM

$g(x) = \sqrt{x}$

$a = 3$ $h = -2$

$b = -\frac{7}{2}$ $k = -6$

$y = 3f\left(-\frac{7}{2}(x+2)\right) - 6$

$y = 3\sqrt{-\frac{7}{2}(x+2)} - 6$

$(x, y) \to \left(-\frac{2}{7}x - 2,\ 3y - 6\right)$

$(4, 2) \to \left(-\frac{2}{7}(4) - 2,\ 3(2) - 6\right)$

$\to \left(-\frac{8}{7} - \frac{14}{7},\ 6 - 6\right)$

$\left(-\frac{22}{7}, 0\right)$

Mar 20-10:27 AM

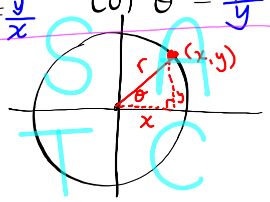
Review Pre-Calc 11

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$ I, II

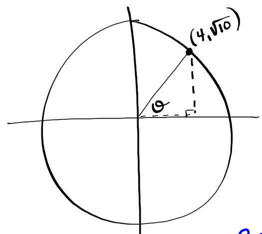
$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$ I, IV

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$ I, III

$r^2 = x^2 + y^2$



Mar 16-2:41 PM



$r^2 = x^2 + y^2$

$r^2 = (4)^2 + (\sqrt{10})^2$

$r^2 = 16 + 10$

$r^2 = 26$

$r = \sqrt{26}$

$\sin\theta = \frac{\sqrt{10}}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{\sqrt{260}}{26} = \frac{2\sqrt{65}}{26} = \frac{\sqrt{65}}{13}$

$\cos\theta = \frac{4}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{4\sqrt{26}}{26} = \frac{2\sqrt{26}}{13}$

$\tan\theta = \frac{\sqrt{10}}{4}$

$\csc\theta = \frac{\sqrt{26}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{260}}{10} = \frac{2\sqrt{65}}{10}$

$\frac{\sqrt{63}}{5}$

$\sec\theta = \frac{\sqrt{26}}{4}$

$\cot\theta = \frac{4}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{4\sqrt{10}}{10} = \frac{2\sqrt{10}}{5}$

Mar 16-2:49 PM

Example

Refer to Figure 8. Suppose we have a circle of radius 10cm and an arc of length 15cm. Suppose we want to find (a) the angle θ, (b) the area of the sector OAB, (c) the area of the minor segment (shaded).

a) $\theta = ?$

$r = 10cm$

$a = 15cm$

$a = \theta r$

$\frac{15cm}{10cm} = \theta$

$\frac{3}{2} rad = \theta$

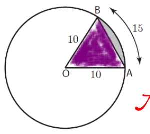


Figure 8. The shaded area is called the minor segment.

b) Area of Sector

$A_{sector} = \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(10cm)^2\left(\frac{3}{2}\right)$

$= \frac{3}{4}(\cancel{100}^{25} cm^2)$

$75cm^2$

c) $A_{segment} = A_{sector} - A_{Triangle}$

$= 75cm^2 - \frac{1}{2} r^2 \sin\theta$

$= 75cm^2 - \frac{1}{2}(10cm)^2 \sin\left(\frac{3}{2}\right)$

$= 75cm^2 - 49.9cm^2$

$25.1 cm^2$

Feb 20-10:17 PM

1 rotation = 360° = 2π

	Degrees	radians
0.5	180°	π
0.43	154.8°	$0.86\pi = 2.7011$

$\frac{1 \text{ rotation}}{360°} = \frac{0.43 \text{ rotations}}{x°}$

$x = 0.43(360°)$

0.43 of 360°

0.43×360

$\frac{2\pi}{1 \text{ rotation}} = \frac{x}{0.43 \text{ rotations}}$

$2\pi(0.43) = x$

Oct 5-2:44 PM

Practice Problems...

Pages 175 - 178
#3, 4, 5, 6, 7, 9, 11, 12, 13

Feb 25-1:41 PM

Check-Up...

Arrange the following angles in descending order:

340° $\quad 4.28\,rad \quad \frac{9\pi}{5} \quad (10\pi)°$

Determine a negative angle co-terminal with each of the following:

$(i)\ \frac{5881\pi}{3}$

Feb 25-2:20 PM

Check-Up...

Arrange the following angles in descending order.

340° 4.28 rad $\frac{9\pi}{5}$ $(10\pi)^\circ$

1: $340\left(\frac{\pi}{180}\right) = 5.93$ 3: 4.28 rad 2: 5.65 4: $31.4° \times \frac{\pi}{180} = 0.54$ rad

Determine a negative angle co-terminal with each of the following:

(i) $\frac{5881\pi}{3} \cdot \frac{1}{2\pi}$

980.16666... $\times 2\pi$

2 ways

$\frac{16-1}{90} = \frac{15}{90} = \frac{1}{6}$

$\frac{1}{6} \cdot 2\pi$

$\pi/3$

0.3333π

$\frac{1\pi}{3}$

$\frac{\pi}{3} - 2\pi$

$\frac{\pi}{3} - \frac{6\pi}{3}$

$-\frac{5\pi}{3}$

(ii) $\frac{29784\pi}{5} \cdot \frac{1}{2\pi}$

2978.4 $\times 2\pi$

0.8π

$\frac{4}{10} \cdot 2\pi$

$\frac{4\pi}{5}$

$\frac{4}{5}\pi - 2\pi$

$\frac{4}{5}\pi - \frac{10\pi}{5}$

$-\frac{6\pi}{5}$

Feb 25-2:20 PM

11 f) $\frac{7\pi}{3}$ $-2\pi \le \theta \le 4\pi$

$-\frac{6\pi}{3} \le \theta \le \frac{12\pi}{3}$

$\frac{7\pi}{3} - 2\pi$

$\frac{7\pi}{3} - \frac{6\pi}{3}$

$\frac{\pi}{3}$

$\frac{\pi}{3} - \frac{6\pi}{3}$

$-\frac{5\pi}{3}$

$\frac{\pi}{3} + \frac{6\pi}{3}$

$\frac{7\pi}{3}$

~~$\frac{7\pi}{3} + \frac{6\pi}{3}$~~

~~$\frac{13\pi}{3}$~~

#13. a) arc length $a = 9$

radius $r = 4$

angle (in rad) $\theta = ?$

$a = \theta r$

$\frac{a}{r} = \theta$

$\frac{9}{4} = \theta$

2.25 rad $= \theta$

c) $a = 15$ $\boxed{a = \theta r}$

$\theta = 3.93$

$r = \frac{a}{\theta} = \frac{15}{3.93} = 3.82$

Mar 17-1:05 PM

$0.35 \rightarrow \frac{35}{100}$ $0.457 = \frac{457}{1000}$

$0.44444 \rightarrow \frac{4}{9}$ $0.\overline{23} = \frac{23}{99}$

$0.16666666 \rightarrow \frac{16-1}{90} = \frac{15}{90}$

$0.10\overline{35} \rightarrow \frac{1035 - 10}{9900} = \frac{1025}{9900}$

Mar 17-9:50 AM

Solutions

9. $\frac{18}{\pi}$

10. $\frac{65\pi}{6}$

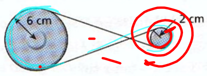
11. $\frac{150}{\pi}$ 47.75°

13. 120π

14. 3 seconds

Apr 11-3:00 PM

Two flywheels are connected by a belt, as shown in the diagram below. The larger one has a radius of 6 cm and the smaller one has a radius of 2 cm.



(a) If the small wheel rotates –300°, then through how many radians does the large wheel rotate?

(b) If the large wheel rotates $\frac{7\pi}{6}$ radians, what distance would a point on the circumference of the small wheel rotate?

a) Small

$a = \theta r$

$= \left(300° \cdot \frac{\pi}{\cancel{180}\,90}\right)(\cancel{2}\text{cm})$

$= \frac{10\pi}{3}$

Big

$\theta = \frac{a}{r}$

$= \frac{\left(\frac{10\pi}{3}\text{cm}\right)}{6\text{ cm}}$

$= \frac{10\pi}{18}$

$= \frac{5\pi}{9}$

b) Big Wheel

$\theta = \frac{7\pi}{6}$ $r = 6\text{cm}$

$a = \left(\frac{7\pi}{\cancel{6}}\right)(\cancel{6}\text{cm})$

$a = 7\pi$ cm

Feb 26-1:33 PM

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time

$\omega = v_a = \frac{\theta}{t}$ $\theta = \frac{a}{r}$

θ = angle (radians)

v_a = angular velocity

a = arc length

t = time

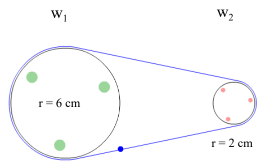
r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute). $\times 2\pi$ Find the roller's angular velocity.

$V_a = \frac{\theta}{t} = \frac{4400\pi}{1\text{ min}}$ radian/min

Ex. (a) If wheel 1 rotates 40 radians, how far has the belt traveled?

(b) Given the 40 rad rotation of wheel 1, what was the angle of rotation for wheel 2?



Dec 7-9:32 PM

Ex. A small electrical motor turns at 2200 rpm.

(a) Express the angular velocity in rad/s.

(b) Find the distance a point 0.8cm from the center of rotation travels in 0.008 s. t

V_a

a) $V_a = \frac{440\cancel{0}\pi}{1\cancel{\text{min}}} \cdot \frac{1\cancel{\text{min}}}{6\cancel{0}\text{ sec}} = \frac{220\pi}{3}$ rad/sec

b) $V_a = \frac{\theta}{t}$

$\theta = V_a \cdot t$

$= \frac{220\pi}{3}\frac{\text{rad}}{\cancel{s}} \cdot 0.008\cancel{s}$

$\theta = \frac{44\pi}{75}$

$a = \theta r$

$= \frac{44\pi}{75} \cdot 0.8\text{cm}$

$= \frac{176\pi}{375}$

$= 1.47$cm

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

Dec 7-10:40 PM

Ex. A small electrical motor turns at 2200 rpm.

$\theta = 2200 \cdot 2\pi$

$\theta = 4400\pi$

(a) Express the angular velocity in rad/s.

(b) Find the distance a point 0.8cm from the center of rotation travels in 0.008 s.

t= r=

$V_a = \frac{\theta}{t} = \frac{440\cancel{0}\pi}{1\text{ min}} \cdot \frac{1\text{ min}}{6\cancel{0}\text{ sec}}$

$= \frac{220\pi}{3}$ rad/sec

b) $V = V_a \cdot r$

$= \frac{220\pi}{3\text{ sec}} \cdot 0.8\text{cm}$

$= \frac{176\pi}{3}$ cm/sec

Formulas....

$a = \theta r$

$V_a = \frac{\theta}{t}$

$V = \frac{d}{t}$

$V = V_a(r)$

$d = Vt$

$= \frac{176\pi\text{ cm}}{3\ \cancel{\text{sec}}} \cdot 0.008\cancel{\text{sec}} = \frac{176\pi}{375}$ cm

$\doteq 1.5$ cm

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

$\theta = 3 \cdot 2\pi = 6\pi$

b)

$r = 5$m

$V_a = ?$ rad/s

$V_a = \frac{\theta}{t}$

$= \frac{6\pi\text{ rad}}{1\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}}$

$= \frac{\pi}{10}$ rad/sec

b) $V = V_a \cdot r$

$= \frac{\pi}{10\text{ s}} \cdot 5\text{m}$

$= \frac{\pi}{2}$ m/s

$d = V \cdot t$

$= \frac{\pi\text{ m}}{2\ \cancel{s}} \cdot 6.5\cancel{\text{secs}}$

$= \frac{6.5\pi}{2}$ m

$= \frac{13\pi}{4}$ m

$\doteq 10.2$ m

Dec 7-10:40 PM

Ex. A bicycle wheel has a radius of 36 cm and is turning at 4.8m/s. Determine the angular velocity of this wheel?

$\boxed{a=\theta r}$ $\boxed{V=\frac{d}{t}}$ $\boxed{V_a=\frac{\theta}{t}}$

$r = 0.36m$

$V = 4.8 m/s$

$V_a =$

$* \boxed{V = (V_a) r} *$

$\frac{V}{r} = V_a$

$\frac{4.8\cancel{m}/s}{0.36\cancel{m}} = V_a$

$13.\bar{3}\ \text{rad}/s = V_a$

Dec 8-8:38 PM

Linear Velocity

$V = \frac{d}{t}$

Angular Velocity

$\omega = V_a = \frac{\theta}{t}$

$V = (V_a)(r)$

$V = \omega r$

Apr 8-9:55 AM

10d) $3 = 6p^2 - 7p$

$\boxed{0 = 6p^2 - 7p - 3}$

$-9 \times 2 = -18$

$-9 + 2 = -7$

$0 = \cancel{(6p - 9)}\cancel{(6p + 2)}$

$0 = (2p - 3)(3p + 1)$

$2p - 3 = 0$
$2p = 3$
$p = 3/2$

$3p + 1 = 0$
$3p = -1$
$p = -1/3$

10e) $3x^2 + 9x = 30$

$3x^2 + 9x - 30 = 0$

$3(x^2 + 3x - 10) = 0$

$5 \times -2 = -10$

$5 + -2 = 3$

$3(x + 5)(x - 2) = 0$

$x + 5 = 0$
$x = -5$

$x = 2$

Mar 17-1:05 PM

$A_{sector} = \frac{\theta}{2\cancel{\pi}} \cdot \cancel{\pi} r^2$

$= \frac{1}{2} \theta r^2$

Practice Problems...

Page 176 - 179
#14, 15, 16, 19, 20 a, b , 21, 22, 23, ~~23~~, ~~18~~, ~~26~~, 27

Feb 26-1:34 PM

Warm Up

A basketball rolling across the floor completes 75 revolutions per minute. The linear velocity of the basketball is 2.5 m/s. Find the radius of the basketball and its angular velocity.

$V_a = \frac{75 \text{ rev}}{\text{min}} = \frac{150\pi}{1\,\cancel{\text{min}}} \cdot \frac{1\,\cancel{\text{min}}}{60 \text{ sec}} = \frac{15\cancel{0}\pi}{6\cancel{0} s} = \frac{5\pi}{2} \text{ rad/s}$

$\boxed{V = V_a \cdot r}$

$r = \frac{V}{V_a} = \frac{\cancel{2.5} m/\cancel{s}}{\cancel{2.5}\pi/\cancel{s}} = \frac{1m}{\pi}$

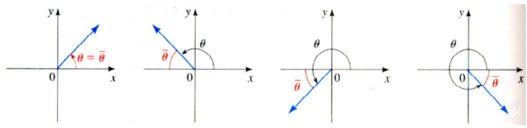
$r = 0.32m$

Nov 21-4:24 PM

Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

The picture below illustrates this concept.



What is the significance of reference angles?

Feb 26-2:36 PM

P176 #13c

$a = 15$

$\theta = 3.93$

$a = \theta r$

$\frac{a}{\theta} = r$

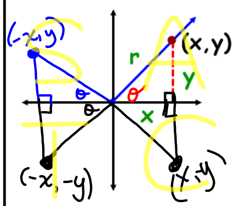
$\frac{15}{3.93} = r$

$3.82 = r$

Mar 17-9:56 AM

Angles on the Cartesian Plane

- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.
- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$
"Primary"	"Reciprocal"

Jan 31-6:49 PM

$(3,-4)$ $c^2=a^2+b^2$
$r^2=x^2+y^2$
$r^2=(3)^2+(-4)^2$
$r^2=25$
$r=5$

S A
T

$\sin\theta=\frac{-4}{5}$ $\csc\theta=\frac{-5}{4}$
$\cos\theta=\frac{3}{5}$ $\sec\theta=\frac{5}{3}$
$\tan\theta=\frac{-4}{3}$ $\cot\theta=\frac{-3}{4}$

$\csc\theta=\frac{7}{\sqrt{3}}$

$r^2=x^2+y^2$
$7^2=x^2+(\sqrt{3})^2$
$49=x^2+3$
$46=x^2$
$x=\sqrt{46}$

$\sin\theta=\sqrt{3}/7$
$\cos\theta=\frac{\sqrt{46}}{7}$
$\tan\theta=\frac{\sqrt{3}}{\sqrt{46}}$
$\csc\theta=7/\sqrt{3}$
$\sec\theta=\frac{7}{\sqrt{46}}$
$\cot\theta=\frac{\sqrt{46}}{\sqrt{3}}$

$\sin\theta<0$, $\tan\theta>0$ Quad III

S A
csc
T C
cot sec

$\cos\theta>0$ $\sec\theta>0$

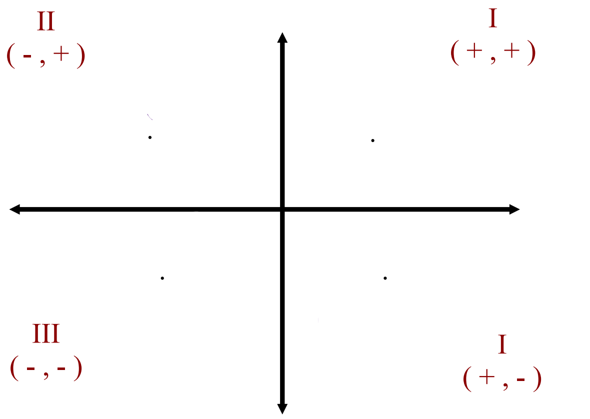
S A
T C

Quad I or IV

Mar 17-1:24 PM

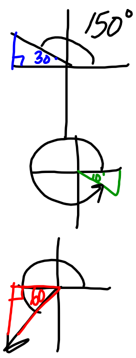
TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



Sep 7-10:31 PM

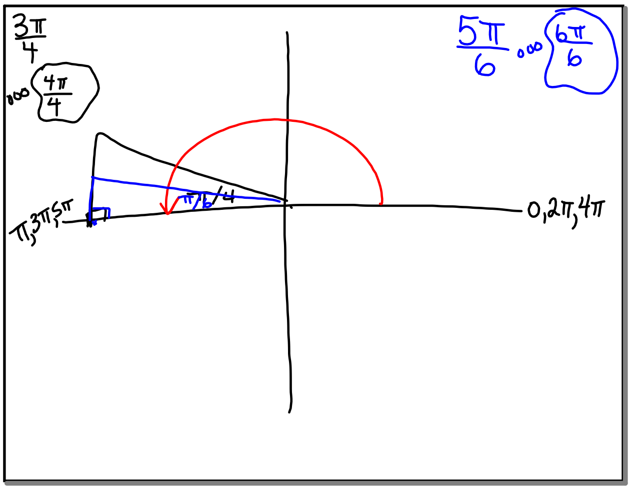
Rotation Angle	Refence Angle
150°	30° Quad II
350°	10° Quad IV
240°	60° Quad III
$\frac{3\pi}{4}$ rad	$\frac{\pi}{4}$ in Quad II



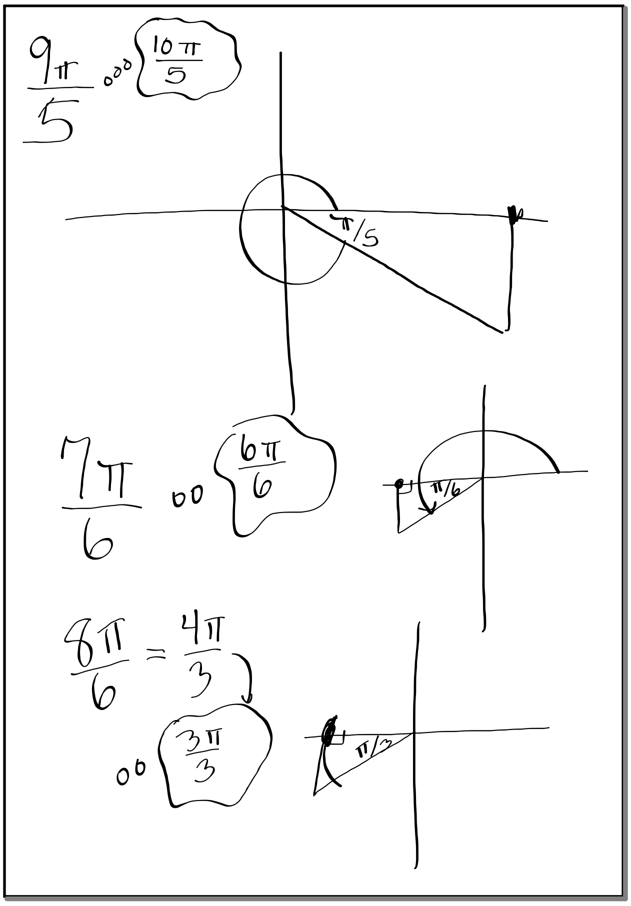
Mar 17-10:07 AM

$\pi/4 \rightarrow 45^\circ$
$\pi/3 \rightarrow 60^\circ$
$\pi/6 \rightarrow 30^\circ$
$\pi/2 \rightarrow 90^\circ/270^\circ$

Mar 17-10:13 AM



Mar 17-10:14 AM



Mar 17-10:18 AM

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta$

Feb 26-2:55 PM

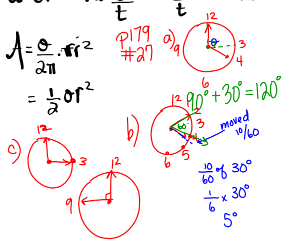
Homework p176-179

$a = \theta r$ $\quad V_a = \frac{\theta}{t}$ $\quad V = \frac{d}{t}$ $\quad V = V_a \cdot r$

$A = \frac{\theta}{2\pi} \cdot \pi r^2$

$= \frac{1}{2}\theta r^2$

p179 #27



$\frac{10}{60}$ of 30°

$\frac{1}{6} \times 30°$

5°

p178 #20.

#14. $V_a = \frac{1\text{ rev}}{15\text{ sec}} = \frac{2\pi}{15\text{ sec}}$

a) $a = \theta r$

$= \frac{5\pi}{3} \cdot 5m$

$= \frac{25\pi}{3} m$

$\doteq 8.\overline{3}\,\pi = 26.2m$

b) $A = \frac{\theta}{2\pi} \cdot \pi r^2$

$= \frac{1}{2}\theta r^2$

$= \frac{1}{2}\left(\frac{5\pi}{3}\right) \cdot (5m)^2$

$= \frac{125\pi}{6} m^2$

$\sim 65.4 m^2$

c) $V_a = \frac{2\pi}{15\text{ min}}$

$V_a = \frac{\theta}{t}$

$V_a \cdot t = \theta$

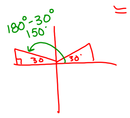
$\frac{2\pi}{\cancel{15}\ \cancel{\text{sec}}} \cdot 2\text{min} \cdot \frac{\cancel{60\text{sec}}\ 4}{1\text{min}}$

$16\pi = \theta$

19. $\frac{50°}{360°} = \frac{5}{36} \times \frac{400}{\cancel{400}\ 80} = \frac{1}{}$

$= \frac{500}{9}$

$\doteq 55.6$ grad.



Mar 24-9:45 AM

Example

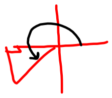
Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$\tan\theta = \frac{-4}{-2\sqrt{3}}$

$\tan\theta = \frac{2}{\sqrt{3}}$

$\theta = \tan^{-1}(2/\sqrt{3})$

$\theta = 0.857$



$\pi + 0.857$

3.9987

Feb 26-2:56 PM

Solve for θ

$-2\pi < \theta < 2\pi$

$\cos\theta = 0.752$

$\theta = \cos^{-1}(0.752)$

$\theta = 0.7197$ radians

$\theta = 2\pi - 0.7197$

$\theta = 5.563$

Mar 20-11:22 AM

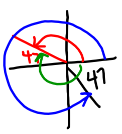
Degrees

$-360° < \theta < 360°$

$\tan\theta = -1.0592$

$\theta = \tan^{-1}(-1.0592)$

$\theta = -47°$



$\theta = -47°$

$\theta = 133°$

$\theta = 313°$

$\theta = -227°$

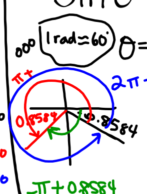
Radians

$-2\pi \leq \theta \leq 2\pi$

$\sin\theta = -0.7568$

1 rad ≈ 60°

$\theta = -0.8584$



$\theta = -0.8584$

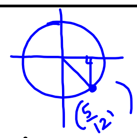
$\theta = 4.0$

$\theta = 5.425$

$\theta = -2.283$

Mar 20-11:30 AM

The point $P(\theta) = \left(\frac{5}{12}, y\right)$ lies on the terminal arm of the unit circle and is in Quad IV



a) Find y b) Find $\tan\theta$ c) Find $\csc\theta$

$y = -\frac{\sqrt{}}{2}$ because in Quad IV

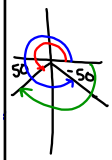
Mar 20-11:47 AM

Degrees

Solve $-360° \le \theta \le 360°$

Given $\sin\theta = -0.7660$

$\theta = \sin^{-1}(-0.7660)$



$\theta \doteq 50°$

$\theta \doteq 50°$

$\theta = 230°$

$\theta = 310°$

$\theta = -130°$

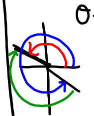
Radians

Solve $-2\pi \le \theta \le 2\pi$

* radian mode *

$\tan\theta = -0.8280$

$\theta = -0.6916$



$-\pi - 0.6916$

$\theta = -0.6916$

$\theta = 2.45$

$\theta = 5.5916$

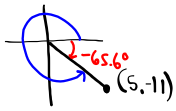
$\theta = -3.8332$

Mar 20-2:11 PM

Find θ when the Point $(5,-11)$ is on the terminal arm and find all 6 trig ratios

reference angle (in degrees) $65.6°$

rotation angle $294.4°$



$\sin\theta = \frac{-11}{\sqrt{146}}$ $\csc\theta = \frac{-\sqrt{146}}{11}$

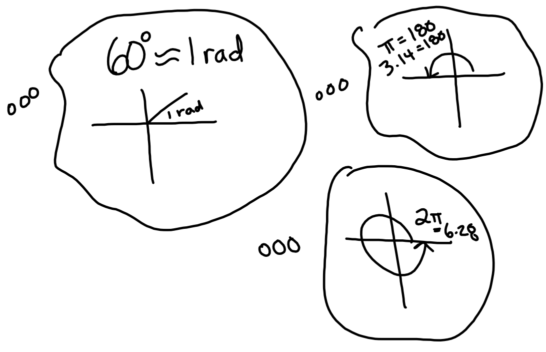
$\cos\theta = \frac{5}{\sqrt{146}}$ $\sec\theta = \frac{\sqrt{146}}{5}$

$\tan\theta = \frac{-11}{5}$ $\cot\theta = \frac{-5}{11}$

$r^2 = (5)^2 + (-11)^2$
$= 25 + 121$
$r^2 = 146$
$r = \sqrt{146}$

$\sin\theta = \frac{-11}{\sqrt{146}} \cdot \frac{\sqrt{146}}{\sqrt{146}}$
$= -\frac{11\sqrt{146}}{146}$

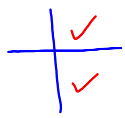
Mar 20-2:34 PM



Mar 20-2:28 PM

p202 #3, 8, 10

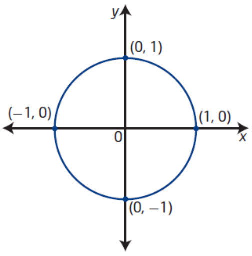
3. | | |
|---|---|
| s | a |
| t | c |



Quad I, IV

Mar 20-11:52 AM

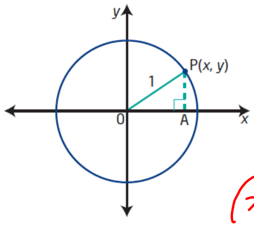
Unit Circle



unit circle

- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

Feb 26-2:21 PM



$x^2+y^2=r^2$

$x^2+y^2=25$ if $r=5$

$(x-3)^2+(y+4)^2=49$

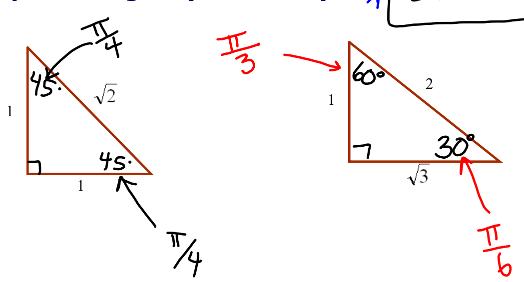
The equation of the unit circle is $x^2 + y^2 = 1$.

Determine the equation of a circle with centre at the origin and radius 6.

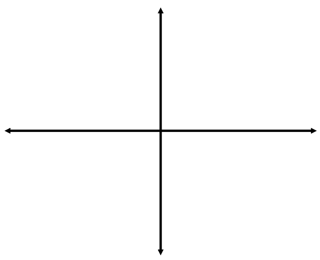
Feb 26-2:26 PM

Special Angles (in radians)

$* \boxed{\sin 30^\circ = \frac{1}{2}}$

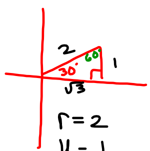


Quadrantal Angles



Feb 26-2:27 PM

$\boxed{\text{memorize} \quad \sin 30^\circ = \frac{1}{2}}$



$r=2$

$y=1$

$x^2=r^2-y^2$

$x^2=2^2-1^2$

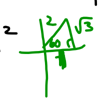
$x^2=4-1$

$x=\sqrt{3}$

$\sin\frac{\pi}{6}=\frac{1}{2}$ $\quad \csc\frac{\pi}{6}=\frac{2}{1}=2$

$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ $\quad \sec\frac{\pi}{6}=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

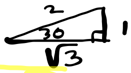
$\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$ $\quad \cot\frac{\pi}{6}=\frac{\sqrt{3}}{1}=\sqrt{3}$



$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

Mar 24-10:34 AM

30°



$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

* $\sin 30° = \frac{1}{2}$

$\csc 30° = \frac{2}{1} = 2$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sec 30° = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot 30° = \frac{\sqrt{3}}{1} = \sqrt{3}$

60

$\sin 60° = \frac{\sqrt{3}}{2}$

$\csc 60° = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

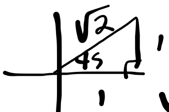
$\cos 60° = \frac{1}{2}$

$\sec 60° = 2$

$\tan 60° = \frac{\sqrt{3}}{1}$

$\cot 60° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

45°



$\sin 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\csc 45 = \sqrt{2}$

$\cos 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

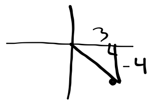
$\sec 45 = \sqrt{2}$

$\tan 45 = 1/1 = 1$

$\cot 45 = 1$

Mar 18-12:58 PM

(3,-4)



$c^2 = a^2 + b^2$
$r^2 = x^2 + y^2$
$r^2 = (3)^2 + (-4)^2$
$r^2 = 25$
$r = 5$

$\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$\csc\theta = -\frac{5}{4}$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$

$\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{-4}{3}$

$\cot\theta = -\frac{3}{4}$

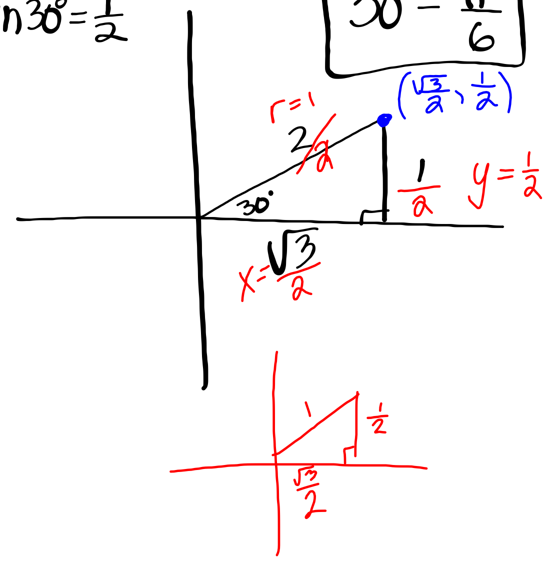
Mar 17-10:23 AM

Quad I		Quad II		Quad III		Quad IV	
		180-30 =	$\pi - \pi/6$	180+30	$\pi + \pi/6$	360-30	$2\pi - \frac{\pi}{6}$
30°	$\frac{\pi}{6}$	150°	$\frac{5\pi}{6}$	210°	$\frac{7\pi}{6}$	330°	$\frac{11\pi}{6}$
60°	$\frac{\pi}{3}$	120°	$\frac{2\pi}{3}$	240°	$\frac{4\pi}{3}$	300°	$\frac{5\pi}{3}$
45°	$\frac{\pi}{4}$	135°	$\frac{3\pi}{4}$	225°	$\frac{5\pi}{4}$	315°	$\frac{7\pi}{4}$

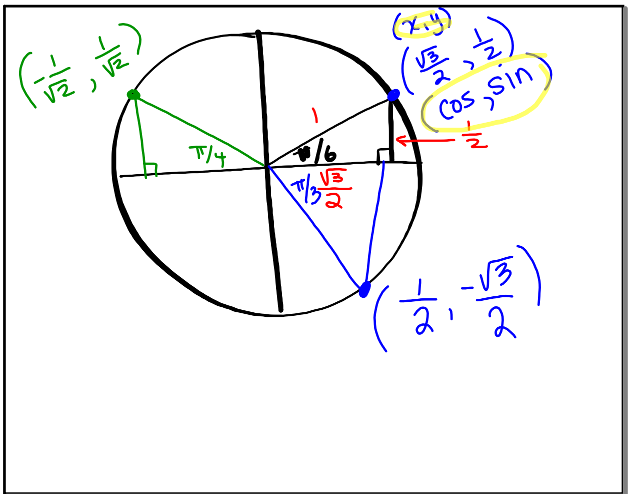
Mar 18-1:09 PM

$\sin 30° = \frac{1}{2}$

$\boxed{30° = \frac{\pi}{6}}$

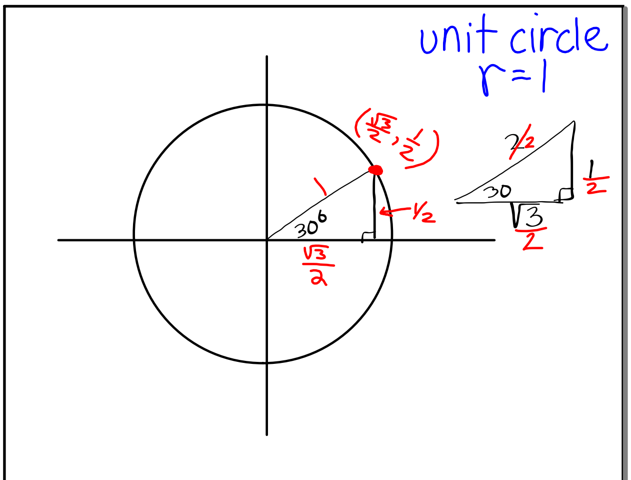


Mar 17-10:28 AM



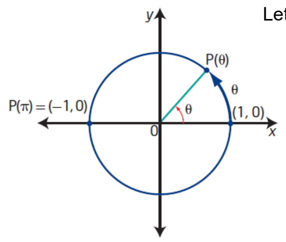
Mar 24-10:45 AM

Mar 17-10:28 AM



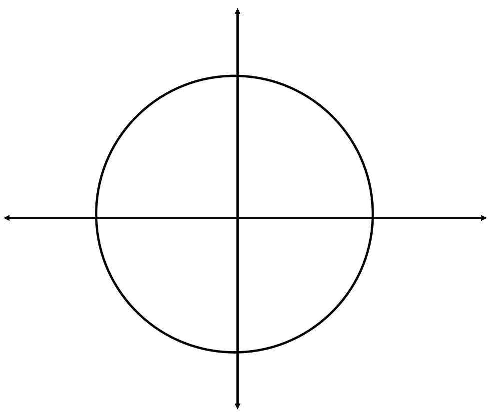
Mar 18-1:21 PM

Special Angles on the Unit Circle:



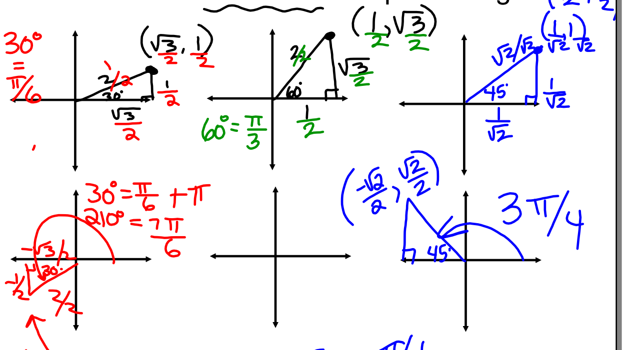
Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles for all multiples of $\pi/4$ between 0 and 2π



Feb 26-2:35 PM

Coordinates on unit circle for special angles



$$\pi - \pi/4$$
$$\frac{4\pi}{4} - \frac{\pi}{4}$$
$$3\pi/4$$

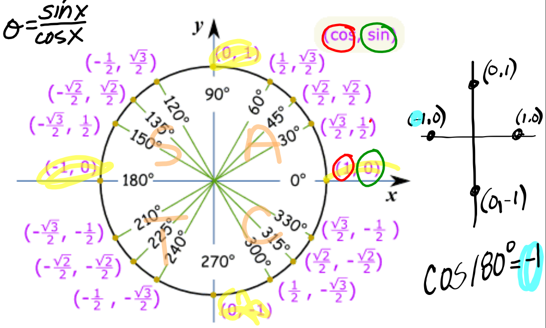
Mar 19-1:22 PM

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Mar 18-10:02 AM

Unit Circle of Special Angles in Degrees

$$\tan\theta = \frac{\sin x}{\cos x}$$



This is lovely...so what is it used for????

Feb 26-2:32 PM

1. c) $\cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ S | A / T | C

d) $\cot \frac{7\pi}{6} = +\sqrt{3}$

$\frac{\pi}{6} = 30°$
$\cot 30° = \sqrt{3}$

e) $\csc 210° = -2$

S | A / T | C $210° = 30$

Feb 26-11:00 AM

$\sin 30° = \frac{1}{2}$ $\sin\theta = \frac{y}{r}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\cos\theta = \frac{x}{r}$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r}$

$\frac{y}{\cancel{r}} \cdot \frac{\cancel{r}}{x}$

$= \frac{y}{x}$

$\tan 30° = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}$

Mar 20-11:05 AM

$c^2 = a^2 + b^2$

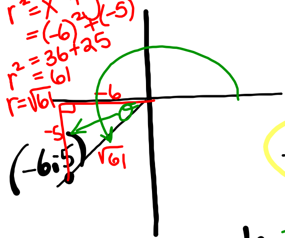
$r^2 = x^2 + y^2$

$= (-6)^2 + (-5)^2$

$r^2 = 36 + 25$

$r^2 = 61$

$r = \sqrt{61}$



$\sin\theta = \frac{-5}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = -\frac{5\sqrt{61}}{61}$ $\csc\theta = -\frac{\sqrt{61}}{5}$

$\cos\theta = \frac{-6}{\sqrt{61}} = -\frac{6\sqrt{61}}{61}$ $\sec\theta = -\frac{\sqrt{61}}{6}$

$\tan\theta = \frac{5}{6}$ $\cot\theta = \frac{6}{5}$

$\theta = \tan^{-1}\left(\frac{5}{6}\right)$

$\theta = 40°$

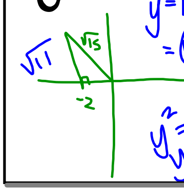
Rotation angle 220°

reference angle 40°

$\sec\theta = \frac{\sqrt{15}}{2}$ in Quad II ($\sqrt{15} \leftarrow r$, $2 \leftarrow x$)

$r = ?\ \sqrt{15}$

$\theta =$



$y^2 = r^2 - x^2$

$= (\sqrt{15})^2 - (-2)^2$

$15 - 4$

$y^2 = 11$

$y = \sqrt{11}$

$\sin\theta = \frac{\sqrt{11}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{165}}{15}$ $\csc\theta = \frac{\sqrt{15}}{\sqrt{11}}$

$\cos\theta = \frac{-2}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{-2\sqrt{15}}{15}$

$\tan\theta = -\frac{\sqrt{11}}{2}$ $\cot\theta = \frac{-2}{\sqrt{11}}$

$= -\frac{2\sqrt{11}}{11}$

Apr 3-2:50 PM

P186 ⇒ 2, 3, 5 (unit circle r=1)

a, c only

202 → #3

CAST rule

#2 $\left(-\frac{3}{4}, \frac{1}{4}\right)$ $x^2 + y^2 = r^2$

$\left(-\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 \neq 1^2$

$\frac{9}{16} + \frac{1}{16}$

$10/16$

3c) $\left(-\frac{7}{8}, y\right)$ Quad III $r = 1$

$\left(\frac{7}{8}, \frac{\sqrt{15}}{8}\right)$

$\left(-\frac{7}{8}\right)^2 + y^2 = 1^2$

$y^2 = 1 - \frac{49}{64}$

$y^2 = \frac{64}{64} - \frac{49}{64}$

$\sqrt{y^2} = \sqrt{\frac{15}{64}}$ $y = \frac{\sqrt{15}}{8}$

$\left(-\frac{7}{8}, -\frac{\sqrt{15}}{8}\right)$

$(-x, -y)$

#5 $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\theta = ?$ $0 \le \theta \le 2\pi$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{\cancel{\sqrt{2}}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{\sqrt{2}}} = 1$

$\tan\theta = 1$ using radians

using degrees

S A

T C

$\theta = \tan^{-1}(1)$ $\theta = \tan^{-1}(1)$

$\theta = 45°, 225°$ $\theta = 0.785398, 3.927$

$= \frac{9\pi}{4}$ $\frac{\pi}{4}$ $\frac{9\pi}{4}$

P202 #3

d) $\sin\theta > 0$ $\cot\theta < 0$ Quad II

S A

T C

Mar 17-10:35 AM

$\tan\theta = 1$

degrees radians

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°, 225°$

S A

T C

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

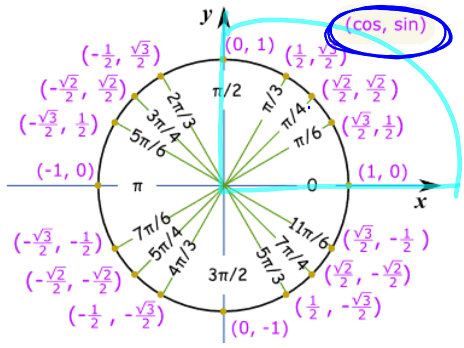
$\frac{\pi}{4}$ ✓ $\theta = 0.7854$

$+\pi$

$5\pi/4$ ✓ $\theta = 3.9270$

Mar 18-12:48 PM

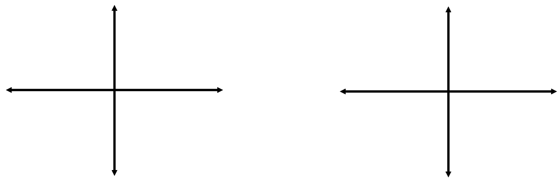
Unit Circle of Special Angles in Radians



Feb 26-2:35 PM

Have a look at

ex. $\sin 690°$

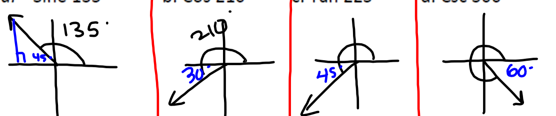


$\cos\frac{13\pi}{3}$ ← Break it apart

Nov 21-9:00 AM

Determine an exact value for

a. Sine 135° | b. Cos 210° | c. Tan 225° | d. Csc 300° (sin)



a. $+\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{\sqrt{2}}{2}$

b. $-\frac{\sqrt{3}}{2}$

c. $+\frac{1}{1}$ 1

d. $-\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{-2\sqrt{3}}{3}$

Oct 28-10:51 PM

$30°$ $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$45°$ $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

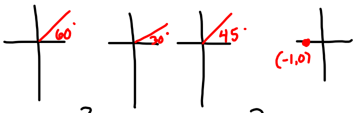
$60°$ $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Feb 26-11:13 AM

Example:

Without the use of a calculator evaluate each of the following expressions: (Sketch each angle on it's own axes)

$$\sin^2 60° - \csc 30° \sec^2 45° + \sec 180°$$



$$\left(+\frac{\sqrt{3}}{2}\right)^2 - (+2)\left(+\sqrt{2}\right)^2 + (-1)$$

$$\frac{3}{4} - (2)(2) - 1$$

$$\frac{3}{4} - 4 - 1$$

$$\frac{3}{4} - 5$$

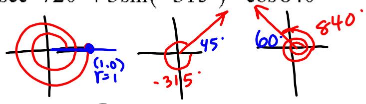
$$\frac{3}{4} - \frac{20}{4} \quad -\frac{17}{4}$$

Sep 7-11:13 PM

Extend the special angles into all FOUR quadrants

t a calculator determine the value of...

$$\sec^2 720° + 3\sin(-315°) - \cos 840°$$



$$\left(\frac{1}{1}\right)^2 + 3\left(+\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{2}\right)$$

$$1 + \frac{3}{\sqrt{2}} + \frac{1}{2}$$

$$\frac{3}{2} + \frac{3}{\sqrt{2}}$$

OR

$$\frac{3}{2} + \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{3}{2} + \frac{3\sqrt{2}}{2}$$

$$\frac{3\sqrt{2}}{2\sqrt{2}} + \frac{3(2)}{2\sqrt{2}}$$

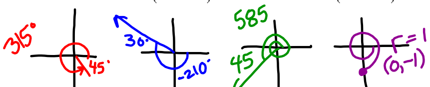
$$\frac{3\sqrt{2}+6}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{3(\cancel{2}) + \cancel{6}^{3}\sqrt{2}}{\cancel{2}(2)} = \frac{3+3\sqrt{2}}{2}$$

Sep 7-11:22 PM

Without a calculator determine the value of ...

$$\sec^2 315° - \sin(-210°) + 2\cot^2 585° \sin(-450°)$$

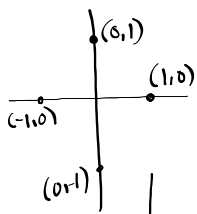


$$\left(+\sqrt{2}\right)^2 - \left(+\frac{1}{2}\right) + 2(+1)^2(-1)$$

$$2 - \frac{1}{2} - 2$$

$$-\frac{1}{2}$$

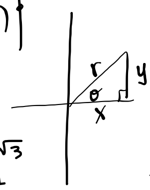
Sep 7-11:36 PM

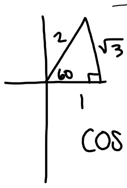


$\sin\theta = y/r$

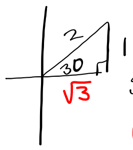
$\cos\theta = x/r$

$\tan\theta = y/x$





$\cos 60° = 1/2$



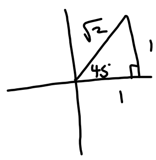
$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 45 = 1/\sqrt{2}$

$\cos 45 = 1/\sqrt{2}$



$30° \leftrightarrow \pi/6$

$60° \leftrightarrow \pi/3$

Apr 5-2:32 PM

Warm Up

Evaluate without the use of a calculator:

$\cot(780°)\sin(-390°) - \cos(-630°) - \sec(870°)\csc^2(-135°)$

$\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{2}\right) - (0) - \left(-\frac{2}{\sqrt{3}}\right)\left(-\frac{\sqrt{2}}{1}\right)^2$

$-\frac{1}{2\sqrt{3}} - \left(-\frac{2}{\sqrt{3}}\right)\left(\frac{2}{1}\right)$

$\frac{-1}{2\sqrt{3}} + \frac{4}{\sqrt{3}}$

$\frac{-1}{2\sqrt{3}} + \frac{8}{2\sqrt{3}}$

$\frac{+7}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{7\sqrt{3}}{2(3)} = \frac{7\sqrt{3}}{6}$

$\frac{3\sec(480°) - 4\sin^2(-315°)}{2\cot 210° \csc^2 135°} = \frac{3(\ )-4(\ )^2}{2(\ )(\ )}$

$\frac{3\left(-\frac{2}{1}\right) - 4\left(\frac{1}{\sqrt{2}}\right)^2}{2\left(\frac{\sqrt{3}}{1}\right)\left(\frac{\sqrt{2}}{1}\right)^2}$

$\frac{-6-2}{4\sqrt{3}} = \frac{-8}{4\sqrt{3}}$

$= \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

Oct 3-9:22 PM

12 r.p.m $\frac{216°}{360°} = 0.6$

12 times in one min.

Oct 27-3:11 PM

More Examples...

1. $2\sin 135° \cos 225° - 3\sec 300°$ (Solution is -7)
2. $\sin 300° \cot(-150°) + \sec 1980° \cos^2 855° - \sin(-450°)$ (Solution is -1)

*** Worksheet - Sketching Special Triangles ***

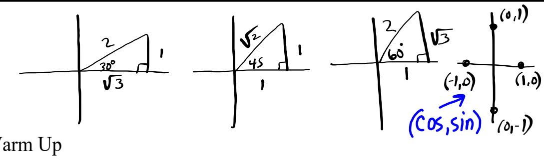
Worksheet - Special & Quadrantal Angles.doc

Feb 4-8:18 PM

Questions from the HOMEWORK???

Worksheet Solns - Special & Quadrantal Angles.doc

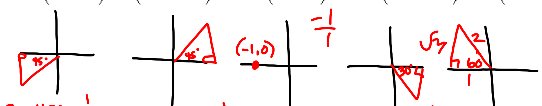
Feb 6-12:44 PM



Warm Up

Evaluate the following without using the trigonometric functions on a calculator:

$$3\csc(2745°)\cos(-2115°)-5\sec(1620°)+7\sin(-2190°)\cot^2(840°)$$



$\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\cos 180°$ $\sin 30° = \frac{1}{2}$ $\tan 60° =$

$$3(-\cancel{\sqrt{2}})\left(\frac{1}{\cancel{\sqrt{2}}}\right)-5(-1)+7\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)^2$$

$$-3+5+7\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right)$$

$$2+\frac{-7}{6}$$

$$\frac{12}{6}-\frac{7}{6}=\frac{5}{6}$$

$$\frac{5}{6}$$

Sep 21-9:42 AM

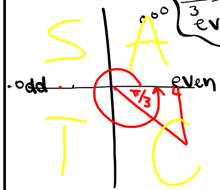
Hw #9c
page 202

$$\left(\cos\frac{5\pi}{3}\right)\left(\sec\frac{5\pi}{3}\right)$$

$\frac{6\pi}{3}=2\pi$ even

$$\left(\frac{1}{\cancel{2}}\right)\left(\frac{\cancel{2}}{1}\right)$$

$$(1)$$



$\cos\frac{\pi}{3}$

$\cos 60° = \frac{1}{2}$

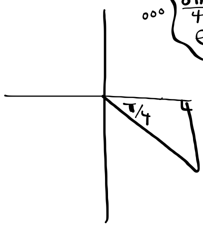
9e) $\left(\cos\frac{7\pi}{4}\right)^2+\left(\sin\frac{7\pi}{4}\right)^2$

$\frac{8\pi}{4}=2\pi$ even

$$\left(\frac{1}{\sqrt{2}}\right)^2+\left(-\frac{1}{\sqrt{2}}\right)^2$$

$$\frac{1}{2}+\frac{1}{2}$$

$$1$$



$\sin\pi/4=-\frac{1}{\sqrt{2}}$

$\cos\left(\frac{\pi}{4}\right)$

$\cos(45°)=\frac{1}{\sqrt{2}}$

Mar 22-1:06 PM

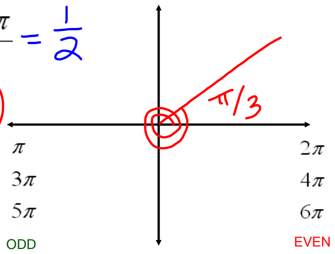
Oct 15-1:03 PM

p202 # 6ef
9 c,e

Mar 18-1:39 PM

Ex. $\cos\frac{13\pi}{3} = \frac{1}{2}$

ooo $\frac{12\pi}{3}$ 4π



Mar 19-1:24 PM

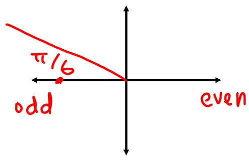
0
2π
4π
6π
even

$30° \to \pi/6$, $150° \to 5\pi/6$
$60° \to \pi/3$, $120° \to \frac{2\pi}{3}$
$45° \to \pi/4 \to 225° \to \frac{5\pi}{4}$

Oct 19-2:20 PM

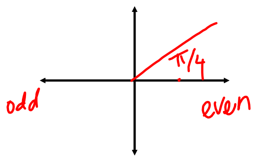
Ex. $\tan\frac{17\pi}{6} = -\frac{1}{\sqrt{3}}$

oo $\frac{18\pi}{6}$ 3π odd



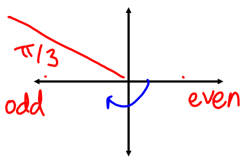
Ex. $\csc\frac{17\pi}{4} = +\sqrt{2}$

oo $\frac{16\pi}{4}$ 4π even



Ex. $\cos\left(-\frac{154\pi}{3}\right) = -\frac{1}{2}$

oo $-\frac{153\pi}{3}$ -51π

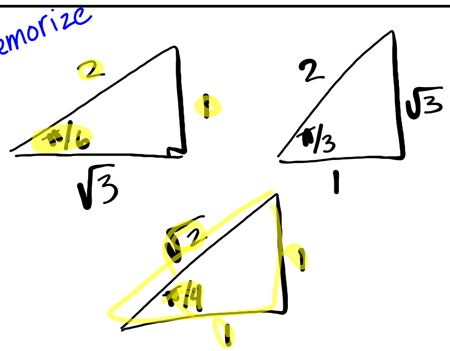


Nov 21-9:27 AM

$0, 2\pi, 4\pi, 6\pi \to 0°$
$\pi/2 \to 90°$ or $270°$
$\pi, 3\pi, 5\pi, \ldots \to 180°$
$\pi/3 \to 60°$ $\pi/4 \to 45°$
$\pi/6 \to 30°$

Mar 27-10:26 AM

Memorize



Mar 27-10:18 AM

$\sec \frac{14}{3}\pi$ $\quad \frac{15}{3}$

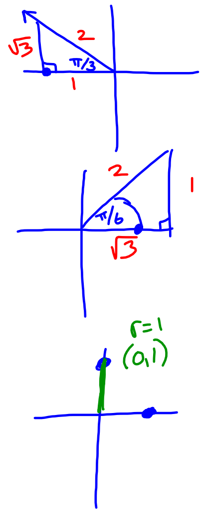
$\left(\frac{2}{-1}\right)$

$\tan^2 \frac{37\pi}{6}$

$\left(\frac{1}{\sqrt{3}}\right)^2$

$\csc \frac{17\pi}{2}$

$\left(\frac{1}{1}\right)$



Oct 19-2:51 PM

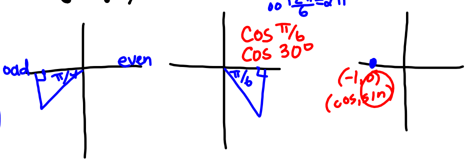
$\sin\left(-\frac{27}{4}\pi\right) \sec \frac{11\pi}{6} + \sin 15\pi$

$\frac{12\pi}{6} = 2\pi$

$\frac{28\pi}{4} = 7\pi$

$\sin \pi/4$

$\sin 45^\circ$



$\left(-\frac{1}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{3}}\right) + (0)$

$\frac{-2}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

$-\frac{2\sqrt{6}}{6}$

$= -\frac{\sqrt{6}}{3}$

Oct 19-2:56 PM

$\frac{15}{3} = 5$ $\quad \frac{24}{6} = 4$ $\quad \frac{12}{2} = 6$ $\quad \frac{28}{4}$

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right) + \csc\left(\frac{11\pi}{2}\right) + \sin^2\left(\frac{27\pi}{4}\right)$$

$\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{-1}\right) + \left(+\frac{1}{\sqrt{2}}\right)^2$

$\cos 113\pi$

Nov 22-10:11 AM

$\sin\left(-\frac{27\pi}{4}\right)\sec\left(\frac{11\pi}{6}\right)+\sin 15\pi$

$\pi/4$ $\pi/6$ $(-1,0)$ $r=1$

$\left(-\frac{1}{\sqrt{2}}\right)\left(+\frac{2}{\sqrt{3}}\right)+(0)$

$-\frac{2}{\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \frac{-2\sqrt{6}}{6} = \frac{-\sqrt{6}}{3}$

Mar 29-2:32 PM

$\sec\frac{14\pi}{3}\tan^2\frac{37\pi}{6}-\csc\frac{17\pi}{2}$

$\frac{15}{3}=5$ $\pi/3$ $\pi/6$ $(0,1)$ $r=1$

$(-2)\left(+\frac{1}{\sqrt{3}}\right)^2-(1)$

$(-2)\left(\frac{1}{3}\right)-1$

$-\frac{2}{3}-1$

$-\frac{2}{3}-\frac{3}{3} = -\frac{5}{3}$

Mar 29-2:12 PM

$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$

$(0,1)$ $r=1$

$r=1$ $(-1,0)$

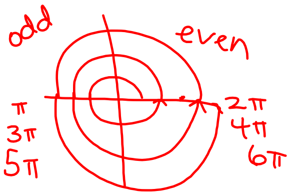
$(1,0)$ $r=1$

$(0,-1)$ $r=1$

Mar 29-2:20 PM

Mar 28-3:36 PM

Mar 28-3:36 PM



Mar 28-2:17 PM

Homework: Worksheet - Sketching Angles in Radians.doc

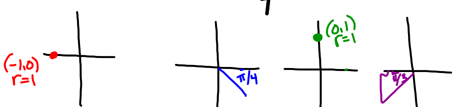
Solutions...

1. $-\frac{5}{3}$
2. $\frac{-\sqrt{6}}{3}$
3. $-2-\sqrt{3}$
4. $\frac{-5}{3}$
5. $\frac{4+3\sqrt{3}}{6}$
6. $\frac{-10}{3}$
7. 0
8. $\frac{3+3\sqrt{3}}{-2}$

$(a+b)(a-b)$
a^2-b^2

$(a+b)(a-b)$
a^2-b^2

6. $\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$



$(-1) + \cancel{\sqrt{2}}\left(-\frac{1}{\cancel{\sqrt{2}}}\right)(1) - \left(-\frac{2}{\sqrt{3}}\right)^2$

$-1 - 1 - \frac{4}{3}$

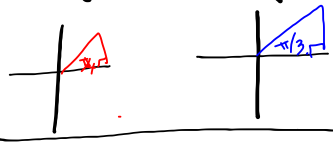
$-2 - \frac{4}{3}$

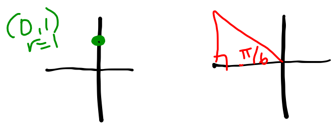
$-\frac{6}{3} - \frac{4}{3}$

$-10/3$

Nov 21-4:36 PM

8. $\dfrac{\tan\left(\frac{-15\pi}{4}\right) + \sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(\frac{-31\pi}{6}\right)}$



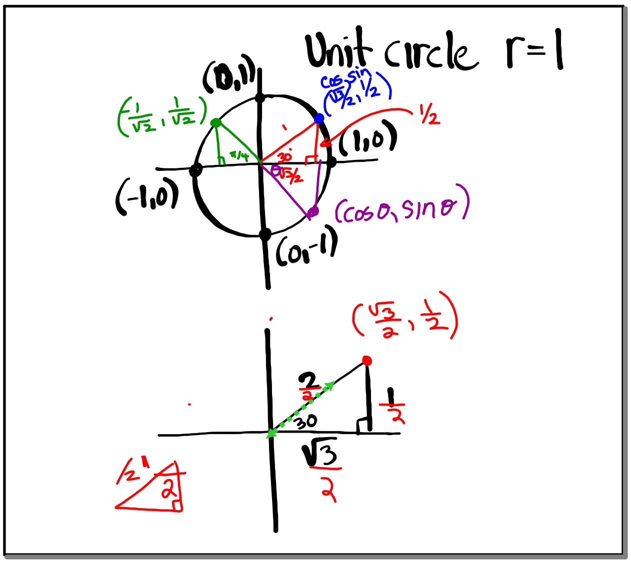


$\dfrac{(1)+(2)}{(1)+(-\sqrt{3})}$

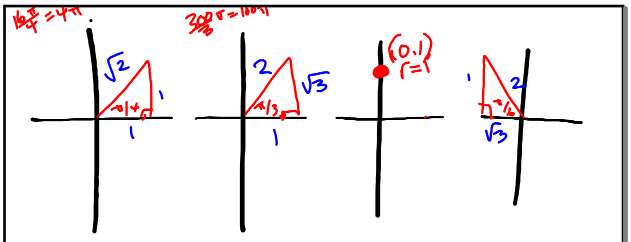
$\dfrac{3}{1-\sqrt{3}} \cdot \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$

$\dfrac{3+3\sqrt{3}}{-2} = \dfrac{-3-3\sqrt{3}}{2}$

Apr 3-2:30 PM



Apr 3-2:38 PM



$$\frac{\left(+\frac{1}{1}\right)+\left(+2\right)}{\left(\frac{1}{1}\right)+\left(-\sqrt{3}\right)}$$

$$\frac{3}{1-\sqrt{3}}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$\frac{3+3\sqrt{3}}{1-(3)}=\frac{3+3\sqrt{3}}{-2}$$

$$\frac{-3-3\sqrt{3}}{2}$$

Mar 29-9:55 AM

$$\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{6}}$$

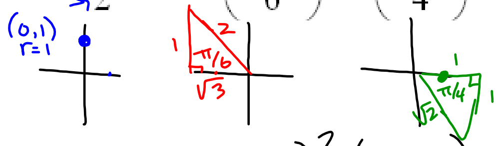
$$\frac{1}{2}\cdot\frac{1}{\sqrt{3}}=\frac{1}{2\sqrt{3}}$$

Mar 29-2:15 PM

Evaluate without the use of a calculator:

$$\sin\frac{9\pi}{2}-\cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

$\frac{16}{4}=4$



$$\left(\frac{1}{1}\right)-\left(-\frac{\sqrt{3}}{2}\right)^2\left(-\frac{1}{1}\right)$$

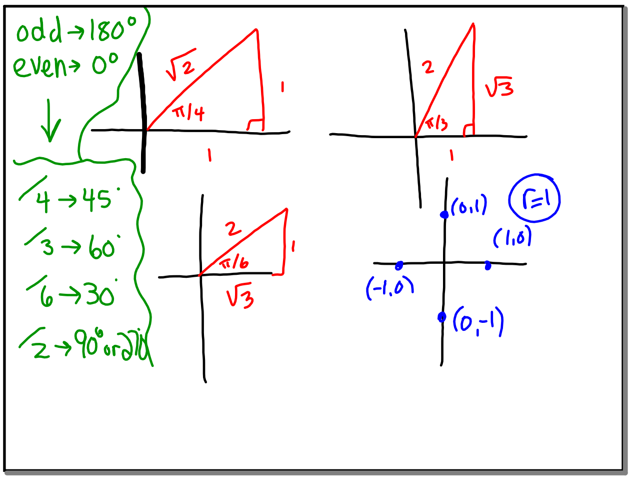
$$1-\left(\frac{3}{4}\right)(-1)$$

$$1+\frac{3}{4}$$

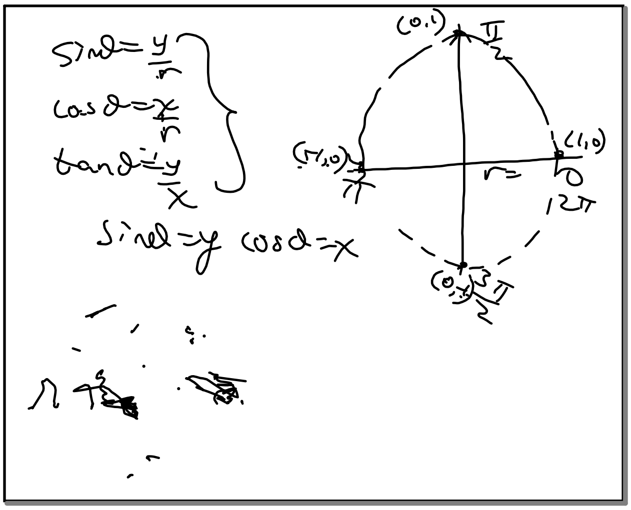
$$\frac{4}{4}+\frac{3}{4}$$

$$\frac{7}{4}$$

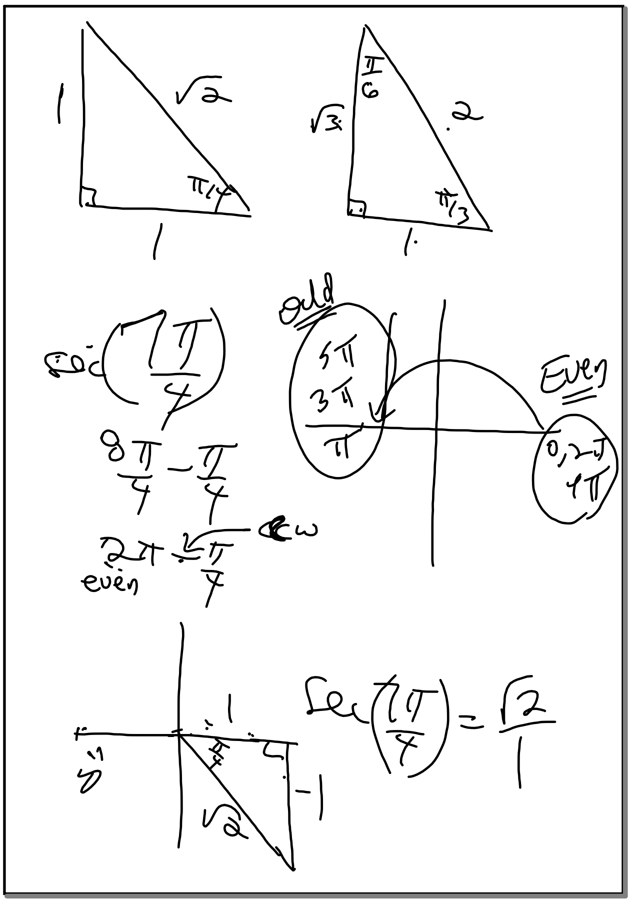
Nov 21-3:56 PM



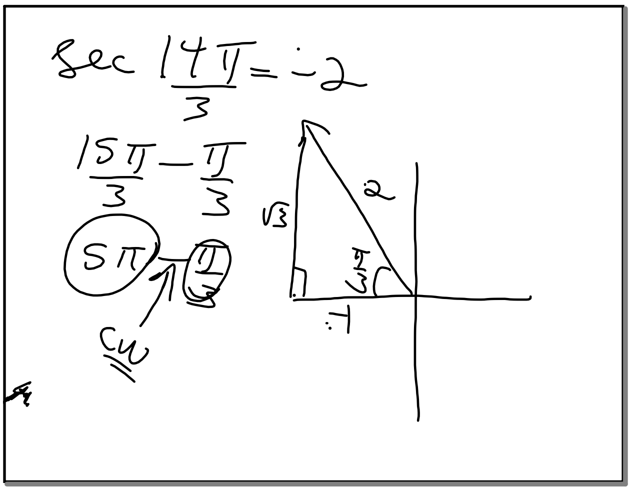
Feb 28-11:09 AM



Mar 27-2:28 PM



Mar 27-2:31 PM

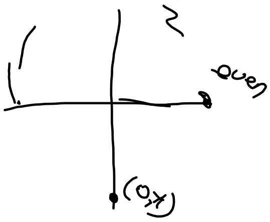


Mar 27-2:37 PM

$\sin\frac{39\pi}{2} = -1$

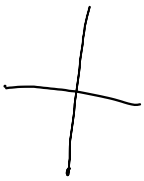
$\frac{40\pi}{2} - \frac{\pi}{2}$

$20\pi - \frac{\pi}{2}$



$\frac{38\pi}{2} + \frac{\pi}{2}$

$19\pi + \frac{\pi}{2}$



Mar 27-2:39 PM

$\sec\frac{14\pi}{3}\tan^2\frac{37\pi}{6} - \csc\frac{17\pi}{2}$

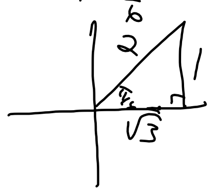
$\sec\frac{14\pi}{3}$

$= -2$

$\frac{37\pi}{6}$

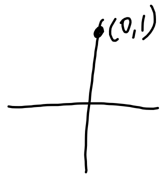
$\frac{36\pi}{6} + \frac{\pi}{6}$

$6\pi + \frac{\pi}{6}$



$\frac{18\pi}{2} - \frac{\pi}{2}$

$9\pi - \frac{\pi}{2}$



$= (-2)\left(\frac{1}{\sqrt{3}}\right)^2 - (1)$

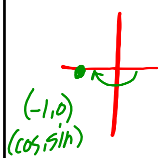
$= -2\left(\frac{1}{3}\right) - 1$

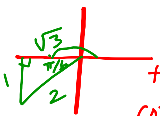
$= -\frac{2}{3} - \frac{3}{3} = -\frac{5}{3}$

Mar 27-2:43 PM

$\cos(-11\pi)$

$\cot\left(\frac{43\pi}{6}\right)$ $\frac{42\pi}{6} = 7\pi$





$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\cot\frac{\pi}{6} = \sqrt{3}$

$\frac{(-1)}{2-(\sqrt{3})}$ -1

$\frac{-1}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$\frac{-2-\sqrt{3}}{4-(3)}$ $\frac{-2\sqrt{3}}{1} = -2-\sqrt{3}$

Mar 23-9:37 AM

Introduction to Trigonometric Equations

trigonometric equation
- an equation involving trigonometric ratios

Focus on…
- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2\theta$. This means the same as $(\cos\theta)^2$.

Feb 26-1:34 PM

$(a-b)(a+b)$
a^2-b^2

$$\frac{2}{(3+\sqrt{5})}\cdot\frac{(3-\sqrt{5})}{(3-\sqrt{5})}$$

$$\frac{6-2\sqrt{5}}{9-(5)} = \frac{6-2\sqrt{5}}{4}$$

$$\frac{3-\sqrt{5}}{2}$$

Oct 21-2:22 PM

Let's start with basic LINEAR trigonometric equations...

...Pre-Calculus 110

Solve: $\sin\theta = \pm 0.9659$, $-360° < x < 720°$

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

$\theta = \sin^{-1}(0.9659)$
$= 75°$ reference angle

S | A
T | C

Quad I: $75°, -285°, 435°$

Quad II: $105°, -255°, 465°$

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\cos\theta = -0.6691$ $\quad -2\pi < \theta < 4\pi$

ref angle
$\cos\theta = 0.6691$
$\theta = 0.8378$

Quad II: $\pi - 0.8378$

$\boxed{2.3038}$
-3.9794
8.5870

Quad III: $\pi + 0.8378$

$\boxed{3.9794}$
-2.3038
10.2526

Mar 13-2:02 PM

i) $\tan x = -0.3124$
$x = -0.3028$

$x = 5.9804 \pm 2\pi n$

$x = 2.8388 \pm 2\pi n$

S | A
T | C

-0.3028

$\pi - 0.3028$

0.3028

k) $\sec x = -1.9105$ $\quad \boxed{x^{-1}}$ $\boxed{1/x}$

$\cos x = -0.5234$

$x = 2.1216 \pm 2\pi n$

$x = 4.1616 + 2\pi n$

$\pi + 1.0200$

4.1616 $\quad 2.1216$ $\quad 1.0200$

Mar 26-2:17 PM

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = -\pi/3$

$\pi + \frac{\pi}{3} = \frac{3\pi}{3} + \frac{\pi}{3} = \frac{4\pi}{3}$

$2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

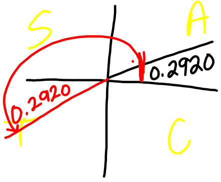
$\pi/3$ $\quad \pi/3$

Mar 26-2:58 PM

ex. $\tan\theta = 0.3006$ 1) $0 < \theta < 2\pi$ 2) general form

$\theta = \tan^{-1}(0.3006)$

$\theta = 0.2920 \pm 2\pi n$



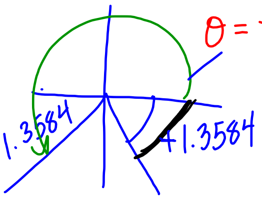
$\pi + 0.2920$

$3.434 \pm 2\pi n$

$\csc\theta = -1.0229$ $\boxed{x^{-1}}$

$\sin\theta = -0.9775$ $\boxed{y/x}$

$\theta = -1.3584 + 2\pi n$



$\pi + 1.3584 = 4.500$

$2\pi - 1.3584 = 4.9247$

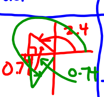
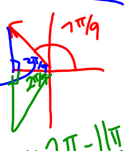
Mar 23-2:10 PM

Solve: $\sec\theta = -1.3054,\ -2\pi \le x \le 2\pi$

Mar 13-2:08 PM

Solve: $\sec\theta = (-1.3054)^{-1},\ -2\pi \le x \le 2\pi$

$\cos\theta = -0.7660$

Could work with degrees	Could work with ≈ rad	Could work with exact radians
$\theta = 140°$  $= 220°$	$\theta = 2.4$  $\theta = 3.88$	$\theta = 140°$ $\theta = \frac{2\pi}{9}$ 
$140 \cdot \frac{\pi}{180} = \boxed{\frac{7\pi}{9}} - 2\pi$	$2.4 - 2\pi = -3.88$	$\pi - 7\pi/9$
$220 \cdot \frac{\pi}{180} = \boxed{\frac{11\pi}{9}}$	$3.88 - 2\pi = -2.4$	$\frac{9}{9}\pi - \frac{7\pi}{9}$
$-\frac{11\pi}{9}$, $-\frac{7\pi}{9}$		$\frac{2\pi}{9}$
		$\pi + \frac{2\pi}{9} = \frac{11\pi}{9}$

Mar 13-2:08 PM

Mar 24-10:09 AM

Ex. $\sqrt{2}\cos\theta + 1 = 0, \; -360° \le \theta \le 720°$

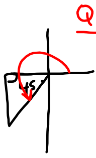
S | A
T | C

$\sqrt{2}\cos\theta = -1$

$\cos\theta = -\frac{1}{\sqrt{2}}$

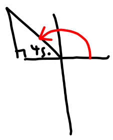
$\boxed{\begin{array}{l} \text{ref } \theta \\ \cos 45° = \frac{1}{\sqrt{2}} \\ \theta = 45° \end{array}}$

Quad III



225°
-135°
585°

Quad II



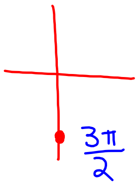
135°
-225°
495°

Dec 15-7:05 PM

Ex. $\sin x + 1 = 0, \; -2\pi \le x \le 4\pi$

$\sin x = -1$

$x = -90°$



$\frac{3\pi}{2}$

$-\frac{\pi}{2} + 2\pi$
$-\frac{\pi}{2} + \frac{4\pi}{2}$
$\frac{3\pi}{2}$

$\frac{3\pi}{2} + 2\pi$
$\frac{3\pi}{2} + \frac{4\pi}{2}$
$\frac{7\pi}{2}$

$-90°$ $\;-\frac{\pi}{2}$
$270°$ $\;\frac{3\pi}{2}$
$630°$ $\;\frac{7\pi}{2}$

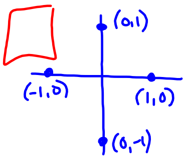
Dec 15-7:11 PM

P232 #3, 5ac, 6ac, 8ac, 9ac, 10ac

8 a) $\sin\frac{\pi}{6} = \sin \boxed{}$

c) $\sin\frac{11\pi}{6} = \sin\boxed{}$

9 a) $\cos\theta = \pi/6$

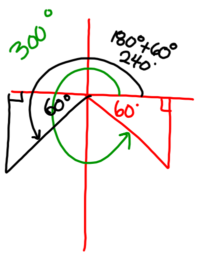


Mar 29-10:39 AM

5c) $\csc\theta = -\frac{2}{\sqrt{3}}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = -60°$

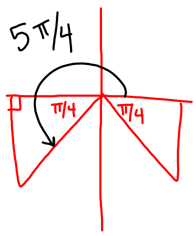


6a) $\sin x = -\frac{\sqrt{3}}{2}$

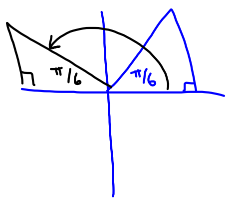
c) $\csc x = -\sqrt{2}$

$\sin x = -\frac{1}{\sqrt{2}}$

$\pi + \frac{\pi}{4} = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$



8 a) $\sin\frac{\pi}{6} = \sin\boxed{\frac{5\pi}{6}}$



$\pi - \frac{\pi}{6}$

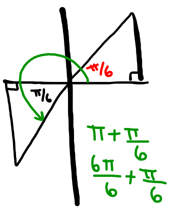
$\frac{6\pi}{6} - \frac{\pi}{6}$

$\frac{5\pi}{6}$

$\boxed{8c)}$ $\boxed{10a}$

Mar 30-9:56 AM

$10a)\ \tan\frac{\pi}{6} = \tan\boxed{\frac{7\pi}{6}}$

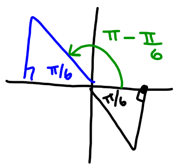


$\pi + \frac{\pi}{6}$

$\frac{6\pi}{6} + \frac{\pi}{6}$

$\frac{7\pi}{6}$

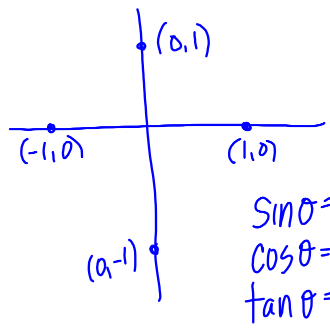
$10c)\ \tan\frac{11\pi}{6} = \tan\boxed{\frac{5\pi}{6}}$



$\pi - \frac{\pi}{6}$

$\frac{6\pi}{6} - \frac{\pi}{6}$

Mar 30-2:22 PM



$\sin\theta = y/r$

$\cos\theta = x/r$

$\tan\theta = y/x$

Mar 30-9:50 AM

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1,\ -2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0,\ 0° \le x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$

$3\cos\theta - \cos\theta = 2$

$2\cos\theta = 2$

$\cos\theta = 1$

$\theta = 0$

$\boxed{-2\pi, 0, 2\pi}$

$\cos\theta = x/r$, $\cos\theta = \frac{1}{1}$, (1,0)

b) $4\sec x + 8 = 0$

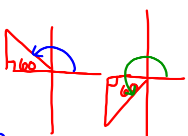
$4\sec x = -8$

$\sec x = -2$

$\boxed{120°, 240°}$

$\cos 60° = \frac{1}{2}$

$\sec 60° = 2$



120°

240°

Mar 13-2:20 PM

Check-Up:

Solve:

$-\pi < \theta < -\frac{\pi}{2}$

① $\cot\theta = 0.7834,\ -\frac{\pi}{2}$

② $3\cos x + 5 = 6,\ -360° \le x \le 720°$

③ $2\csc x(1 - \csc x) = 0,\ -4\pi < x < 4\pi$

Mar 18-8:45 AM

① $\cot\theta = 0.7834$, $-\pi < \theta < -\frac{\pi}{2}$ ~~$-\frac{\pi}{2} < \theta < \pi$~~

$\boxed{x^{-1}}$

$\tan\theta = 1.2765$

$\theta = 0.9063$

Quad I

Quad III

$-\pi + 0.9063$

$\boxed{-2.2353}$

Mar 18-8:46 AM

② $3\cos x + 5 = 6$, $-360° \leq x \leq 720°$

$3\cos x = 1$

$\cos x = \frac{1}{3}$

$x = 70.5°$

$x = 71°$

Quad I	Quad IV
71°	289°
−289°	−71°
431°	649°

Mar 18-8:46 AM

③ $2\csc x(1 - \csc x) = 0$, $-4\pi < x < 4\pi$

~~$2\csc x = 0$~~

~~$\csc x = 0$~~

$\csc x = 0$, $\sin x = \text{DNE}$, not possible

$1 - \csc x = 0$

$1 = \csc x$

$\csc x = 1$, $\sin x = 1$

$2a(1-a) = 0$

$a = 0$ $a = 1$

$\pi/2$

$-\frac{3\pi}{2} - 2\pi = -\frac{3\pi}{2} - \frac{4\pi}{2} = -\frac{7\pi}{2}$

$\frac{\pi}{2} - 2\pi = \frac{\pi}{2} - \frac{4\pi}{2} = -\frac{3\pi}{2}$

$\frac{\pi}{2}$

$\frac{\pi}{2} + 2\pi = \frac{\pi}{2} + \frac{4\pi}{2} = \frac{5\pi}{2}$

Mar 18-8:46 AM

$x^2 + 4 = 0$

$x^2 = -4$

no value of x would make $x^2+4=0$

no solution

Oct 22-2:45 PM

Review Factoring

GCF

$3b^2-12b=0$

$3b(b-4)=0$

$3b=0 \quad b-4=0$

$b=0 \quad b=4$

Simple Trinomial

$x^2-13x-30=0$

$(x-15)(x+2)=0$

$x=15 \quad x=-2$

D.O.S

$4x^2-25=0$

one way

$(2x-5)(2x+5)=0$

$2x-5=0$, $2x=5$, $x=5/2$ $\quad x=-\frac{5}{2}$

another way

$\frac{4x^2}{4}=\frac{25}{4}$

$x^2=\frac{25}{4}$

$x=\pm\sqrt{\frac{25}{4}}$

$x=\pm\frac{5}{2}$

$x^2-7=0$

$(x-\sqrt{7})(x+\sqrt{7})=0$

$x=\sqrt{7} \quad x=-\sqrt{7}$

$x^2=7$

$x=\pm\sqrt{7}$

Oct 22-2:35 PM

$(m-2)(m+5)=0$

$m-2=0$, $m=2$

$m+5=0$, $m=-5$

$2x(x+7)=0$

$x=0$

$x=-7$

Oct 22-2:37 PM

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2x^2+x=1$ $\xrightarrow{\text{Treat the same}}$ Solve: $2\sin^2 x+\sin x=1,\ 0\le x\le 4\pi$

$2x^2+x-1=0$

$(\cancel{2x+2})(2x-1)=0$

$(x+1)(2x-1)=0$

$x=-1 \quad 2x=1, x=1/2$

let $a=\sin x$

$2a^2+a=1$

$2a^2+a-1=0$

$(\cancel{2a+2})(2a-1)=0$

$(a+1)(2a-1)=0$

$a=-1 \quad 2a=1, a=1/2$

$\sin x=-1 \quad \sin x=\frac{1}{2}$

$\frac{3\pi}{2}$

$\frac{3\pi}{2}$

$\frac{3\pi}{2}+2\pi$

$\frac{3\pi}{2}+\frac{4\pi}{2}$

$7\pi/2$

$\pi/6$

$\frac{\pi}{6}$

$\frac{\pi}{6}+2\pi$

$\frac{\pi}{6}+\frac{12\pi}{6}$

$13\pi/6$

$\pi-\frac{\pi}{6}=\frac{6\pi}{6}-\frac{\pi}{6}$

$\frac{5\pi}{6}$

$\frac{5\pi}{6}+\frac{12\pi}{6}$

$17\pi/6$

Mar 13-2:29 PM

Review Factoring

GCF

$3b^2-12b=0$

$3b(b-4)=0$

$3b=0$, $b=0$ $\quad b-4=0$, $b=4$

ex $\cos^2\theta-\cos\theta=0$

let $a=\cos\theta$

$a^2-a=0$

$a(a-1)=0$

$a=0 \quad a=1$

$\cos\theta=0 \quad \cos\theta=1$

OR

$\cos\theta(\cos\theta-1)=0$

$\cos\theta=0 \quad \cos\theta=1$

D.O.S

$x^2-49=0$

$(x-7)(x+7)=0$

$x=7 \quad x=-7$

Alternative

$x^2-49=0$

$x^2=49$

$x=\pm\sqrt{49}$

$x=\pm 7$

ex $x^2-13=0$

$(x-\sqrt{13})(x+\sqrt{13})$

$x=\sqrt{13} \quad x=-\sqrt{13}$

OR

$x^2=13$

$x=\pm\sqrt{13}$

ex. $x^2+25=0$

$x^2=-25$

~~$x=\pm\sqrt{-25}$~~

error

Mar 30-2:39 PM

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0,\ -2\pi \le \theta \le 4\pi$

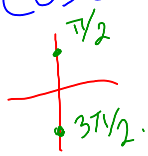
$a = \cos\theta \qquad a^2 - \frac{1}{2}a = 0$

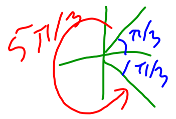
$a\left(a - \frac{1}{2}\right) = 0$

$a = 0 \qquad | \qquad a - \frac{1}{2} = 0$

$a = \frac{1}{2}$

$\cos\theta = 0 \qquad | \qquad \cos\theta = \frac{1}{2}$





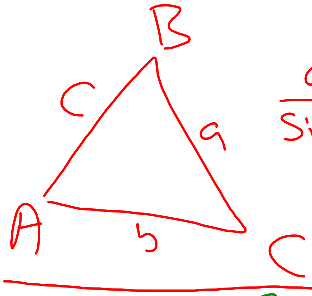
$\frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{\pi}{3} \quad \frac{5\pi}{3}$

$-\frac{3\pi}{2},\quad -\frac{\pi}{3} \quad -\frac{5\pi}{3} \quad -\frac{\pi}{3}$

$\frac{5\pi}{2} \quad \frac{7\pi}{2} \quad \frac{7\pi}{3} \quad \frac{11\pi}{3}$

Nov 28-8:50 AM

Sine Law



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Cosine Law

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

211 #3,4

Oct 22-3:13 PM

$\boxed{\text{let } a = \sin x}$

Ex. $6\sin^2 x - \sin x = 2,\ \boxed{-2\pi \le \theta \le 4\pi}$

$6a^2 - a = 2$

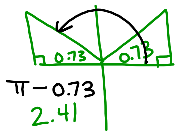
$6a^2 - a - 2 = 0$

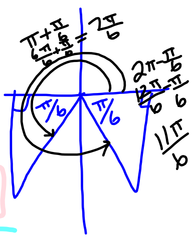
$\cancel{(6a-4)(6a+3)} = 0$

$(3a-2)(2a+1) = 0$

$a = \frac{2}{3} \qquad a = -\frac{1}{2}$

$\sin x = \frac{2}{3} \qquad \sin x = -\frac{1}{2}$





$x = \sin^{-1}\left(\frac{2}{3}\right)$

$x = 0.73, 2.41, \frac{7\pi}{6}, \frac{11\pi}{6}$

$x = 7.01, 8.69, \frac{19\pi}{6}, \frac{23\pi}{6}$

$x = -5.55, -3.87, -\frac{5\pi}{6}, -\frac{\pi}{6}$

$+2\pi$

$+\frac{12\pi}{6}$

Dec 15-7:11 PM

Your Turn

Solve for θ.

$\cos^2\theta - \cos\theta - 2 = 0,\ 0° \le \theta < 360°$

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

$a^2 - a - 2 = 0$

$(a-2)(a+1) = 0$

$a = 2$ $\quad a = -1$

$\cos\theta = 2$ $\quad \cos\theta = -1$

$\theta = \cos^{-1}(2)$

~~θ = error~~

$r = 1$

$(-1, 0)$

$\boxed{\theta = 180°}$

Mar 13-2:39 PM

$\cos\theta = -\frac{1}{\sqrt{2}}$

$\theta = 135°, 225°$

$135° + 360n,\ 225° + 360n$

Oct 28-2:31 PM

General Solution of a Trigonometric Equation

Solve: $3\cos^2\theta - \cos\theta = 2;\ \theta \in \mathbb{R}$

$\cos\theta = \frac{x}{r}$

$3a^2 - a - 2 = 0$

$\cancel{(3a-3)}(3a+2) = 0$

$(a-1)(3a+2) = 0$

$a = 1$ $\quad a = -\frac{2}{3}$

$\cos\theta = 1$ $\quad \cos\theta = -\frac{2}{3}$

$\theta = 0°$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta = 132°, 228°$

$132°$ $\quad 48°$ $\quad 48°$

$0° + 360n,\ 132° + 360n,\ 228° + 360$

where $n \in I$

Mar 13-2:42 PM

Determine the general solution for $\sin^2\theta - 1 = 0$ over the real numbers if x is measured in radians.

Did You Know?

$2n$, where $n \in I$, represents all even integers.

$2n + 1$, where $n \in I$, is an expression for all odd integers.

$a^2 - 1 = 0$

$(a-1)(a+1) = 0$

$a = 1$ $\quad a = -1$

$\sin\theta = 1$ $\quad \sin\theta = -1$

$(0, 1)$, $r = 1$ $\quad \theta = \frac{\pi}{2}$

$(0, -1)$, $r = 1$ $\quad \theta = \frac{3\pi}{2}$

$\pi + \frac{\pi}{2}$

$x = \frac{\pi}{2} + 2\pi n$, where $n \in I$

$x = \frac{3\pi}{2} + 2\pi n$, where $n \in I$

or

$x = \frac{\pi}{2} + \pi n$, where $n \in I$

or

$(2n + 1)\left(\frac{\pi}{2}\right)$, $n \in I$

Mar 13-2:45 PM

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Mar 13-2:47 PM

$b^2 + 7b + 12$

$\sin^2 x + 7\sin x + 12 = 0$

$(\sin x + 4)(\sin x + 3) = 0$

Oct 26-3:09 PM

p215-217

pick a few from each

Thursday Test

Oct 26-3:11 PM

Practice Problems:

~~Pages 212 - 214~~
~~#11 - 23~~

$\boxed{\text{p211 } \#5, 7}$

Mar 21-10:16 AM

Check-Up problem...

ve:

$\sin x \sec x + 2\sin x = 0\ ,\ x \in R$ (x is measured in radians)

$\sin x(\sec x + 2) = 0$

$\sin x = 0 \qquad \sec x = -2$

$x = \pi/3$

$x = 0 + 2\pi n$

$x = \pi + 2\pi n$

$x = \pi - \pi/3 = \frac{2\pi}{3} + 2\pi n$

$x = \pi + \pi/3 = \frac{4\pi}{3} + 2\pi n$

Mar 21-10:17 AM

homework page 211

#5 a,c,e

#7 b,d

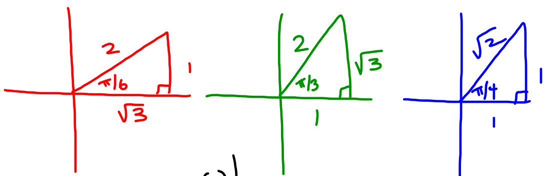
#22

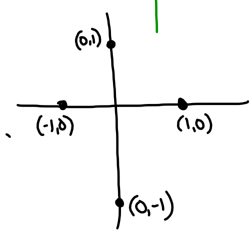
Test topics

- Radians ↔ Degrees
- Arc Length
- Sector Area
- Angular Velocity
- Special Angles
- "odd/even" → Evaluating Special Angles (Order of Ops) Long question
- Solving Trig Equations 10
  - linear
  - quadratic
  - with specific domain
  - with General Domain

Apr 3-10:31 AM

Apr 4-9:47 AM



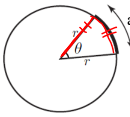


$\boxed{\sin 30° = \frac{1}{2}}$

Apr 3-3:03 PM

Unit Review...

What topics have we covered??



$a = \theta r$

angle in Radian

arc Length

radius

$1^\circ = \frac{\pi}{180}$ radians $1 \text{ rad} = \frac{180}{\pi}$

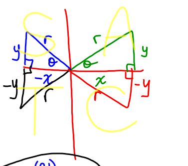
TRIG RATIOS on the CARTESIAN PLANE

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$

"Primary"   "Reciprocal"

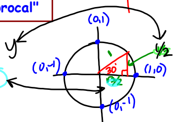


Special Angle

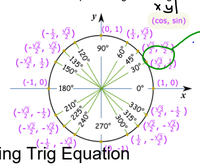
$\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$



Unit Circle of Special Angles in Degrees



$\frac{\sin 30^\circ}{\cos 30^\circ} = \tan 30$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

Solving Trig Equation

Mar 21-10:20 AM

Measurement of area of sector of a circle in Radian

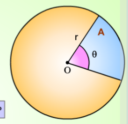
In general, if the angle of a sector, θ, is measured in degree,

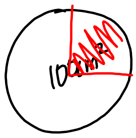
then the area of the sector, $A = \frac{\theta}{360} \times \pi r^2$

If θ is measured in radians,

then the area of the sector, $A = \frac{\theta}{2\pi} \times \pi r^2$ $2\pi \text{ rad} = 360^\circ$

$\therefore A = \frac{1}{2} r^2 \theta$





$\frac{90}{360} \cdot A\text{circle}$

$\frac{1}{4}(100)$

25 cm.

$\frac{47}{360} \cdot \pi(5)^2$

10.3

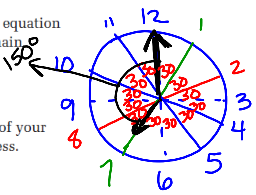
Oct 28-2:58 PM

Review...

C4 a) Determine all solutions for the equation $2\sin^2\theta = 1 - \sin\theta$ in the domain $0^\circ \le \theta < 360^\circ$.

b) Are your solutions exact or approximate? Why?

c) Show how you can check one of your solutions to verify its correctness.



A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

$150^\circ \cdot \frac{\pi}{180}$ $\frac{150\pi}{180}$ $\frac{5\pi}{6} = 2.618$

Determine the angular velocity of the minute hand on a clock.

Mar 21-10:21 AM

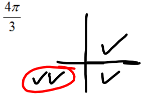
Solve: $6\sin^2\theta - 3\sin\theta = 0$, $0 \le \theta \le 360^\circ$

[A] $0^\circ, 30^\circ, 180^\circ, 330^\circ, 360^\circ$ [B] $0^\circ, 30^\circ, 180^\circ, 150^\circ, 360^\circ$

[C] $30^\circ, 90^\circ, 120^\circ, 270^\circ$ [D] $0^\circ, 180^\circ, 210^\circ, 330^\circ, 360^\circ$

If $\csc\theta < 0$ and $\tan\theta > 0$, then which of the following could be a possible measure of angle θ?

[A] $\frac{11\pi}{6}$ [B] $\frac{4\pi}{3}$ [C] $\frac{3\pi}{4}$ [D] $\frac{\pi}{2}$

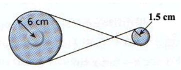


What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$ [B] $\frac{\pi}{4}$ [C] $-\frac{\pi}{4}$ [D] $\frac{7\pi}{4}$

Mar 22-9:21 AM

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20°

[C] 20 radians [D] 5°

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Solve: $2(1-\sin\theta)^2 + \sin\theta = 2(3-4\sin^2\theta)$, $-360° \le \theta \le 720°$

Mar 22-10:43 AM

Little Johnny has a rock tied to the end of a piece of rope 1.5 m long and he is swinging it around his head in a circular pattern. Mrs. Centripetal, his physics teacher, is watching Johnny out the window of her physics lab and notes that the rock is making 12 revolutions every 48 seconds.

(a) Determine the angular velocity with which little Johnny is twirling the rope above his head. [2]

(b) The rock comes flying from the rope 3 minutes after Mrs. Centripetal started to time little Johnny. How far did the rock travel during the 3 minutes? [2]

Mar 22-10:49 AM

Test Topics Radian Measure

Radians ↔ Degrees

Arc Length $a = \theta r$

Sector Area $A = \frac{\theta}{360} \cdot \pi r^2$

OR $A = \frac{1}{2}\theta r^2$

radians

Angular Velocity $\omega = V_a = \frac{\theta}{t}$

Linear Velocity $V = \frac{d}{t}$

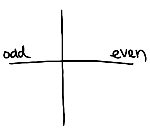
$V = V_a(r)$

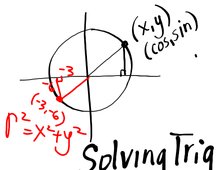
ex 5 rpm $= \frac{5 \text{ rev}}{1 \text{ min}}$ $V_a = \frac{5(2\pi)}{1 \text{ min}}$

Special Angles

30, 45, 60

$\pi/6$ $\pi/4$ $\pi/3$





$r^2 = x^2 + y^2$

Solving Trig Equations

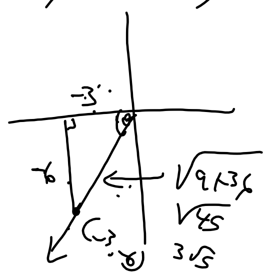
Solving 211 (omit 14-17, 19, 20, 23-)

Review p215

218 (omit 6, 13, 14, 16)

Apr 11-9:56 AM

#4) P(-3,-6)



$\sqrt{9+36}$

$\sqrt{45}$

$3\sqrt{5}$

$\csc\theta = \frac{\sqrt{5}}{-2}$ ✓

$\sin\theta = \frac{-6}{3\sqrt{5}} = \frac{-2}{\sqrt{5}} \cdot \left(\frac{\sqrt{5}}{\sqrt{5}}\right)$

$= \frac{-2\sqrt{5}}{5}$

$\cos\theta = \frac{-\cancel{3}}{\cancel{3}\sqrt{5}}$

$\tan\theta = 2$ $\cot\theta = \frac{1}{2}$

$\sec\theta = -\sqrt{5}$

$\cos\theta = \frac{-1}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$

$= \frac{-\sqrt{5}}{5}$

Apr 11-12:54 PM

M/choice

#6) 3.86

b) $\cos(7\pi/12) = (\ \ )^{-1}$

sec = ↓

Apr 11-12:58 PM

#6) $12\sin^2 x - \sin x - 6 = 0$, $-2\pi \le \theta \le 2\pi$

Let $m = \sin x$

$12m^2 - m - 6 = 0$

$\left(\frac{12m-9}{3}\right)\left(\frac{12m+8}{4}\right) = 0$

$(4m-3)(3m+2) = 0$

$m = \frac{3}{4}$ OR $m = -\frac{2}{3}$

$\sin x = \frac{+3}{4}$ $\sin x = -\frac{2}{3}$

$180-\theta$	θ
$180+\theta$	$360-\theta$

(Ref ∡ 49, Q1,2) (Ref ∡ 42°, Q 3,4)

$x = 49°, 131°$ $x = 222°, 318°$

$x = -311, -229°$ $x = -138°, -42°$

$x = \frac{49\pi}{180}, \frac{131\pi}{180}$

Apr 11-1:01 PM

Warm Up

A basketball rolling across the floor completes 75 revolutions per minute. The linear velocity of the basketball is 2.5 m/s. Find the radius of the basketball and its angular velocity.

$V_a = \frac{75\text{ rev}}{1\text{ min}} \xrightarrow{\times 2\pi} \frac{150\pi}{1\text{ min}} \cdot \frac{1\text{ min}}{60\text{ sec}} = \frac{5\pi}{2}\text{ rad/s}$

$\boxed{V = V_a \cdot r}$

$r = \frac{V}{V_a} = \frac{2.5\text{ m/s}}{\frac{5\pi}{2\text{ s}}} = 0.32\text{ m}$

Nov 21-4:24 PM

Example $\boxed{x^2+y^2=r^2}$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$. List all 6 trig ratios

$(-2\sqrt{3})^2 + (-4)^2 = r^2$

$4(3) + 16 = r^2$

$12 + 16 = r^2$

$28 = r^2$

$\sqrt{28} = r$

$\sqrt{4}\sqrt{7} = r$

$2\sqrt{7} = r$

exact

$\sin\theta = \frac{y}{r} = \frac{-4}{2\sqrt{7}} = \frac{-2}{\sqrt{7}} = \frac{-2\sqrt{7}}{7}$

$\csc\theta = \frac{r}{y} = \frac{2\sqrt{7}}{-4} = \frac{-\sqrt{7}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-2\sqrt{3}}{2\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{-\sqrt{21}}{7}$

$\sec\theta = \frac{r}{x} = \frac{2\sqrt{7}}{-2\sqrt{3}} = \frac{-\sqrt{21}}{3}$

$\tan\theta = \frac{y}{x} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\theta = \frac{x}{y} = \frac{-2\sqrt{3}}{-4} = \frac{\sqrt{3}}{2}$

To find θ (in radians)

$\tan\theta = \frac{2\sqrt{3}}{3}$

$\theta = 0.85707$

$+\ \pi$

$\theta = 3.99866$

$\theta = 4$ radians $(-2\sqrt{3}, -4)$

Feb 26-2:56 PM

Attachments

Worksheet - Sketching Angles in Radians.doc

Worksheet - Special & Quadrantal Angles.doc

Worksheet Solns - Special & Quadrantal Angles.doc

M4H00390.THM

M4H00376.MP4

M4H00376.THM

M4H00383.MP4

M4H00383.THM

M4H00385.MP4

M4H00385.THM

M4H00390.MP4