1D - Types of Collisions

An elastic collision is a collision in which both momentum and kinetic energy are conserved.

Reminder: $E_k = \frac{1}{2} mv^2$

E_k -> kinetic energy (energy of motion) [J]
m -> mass [kg]
v -> velocity [m/s]

$kg\frac{m^2}{s^2} = J$

Conservation of Kinetic Energy:

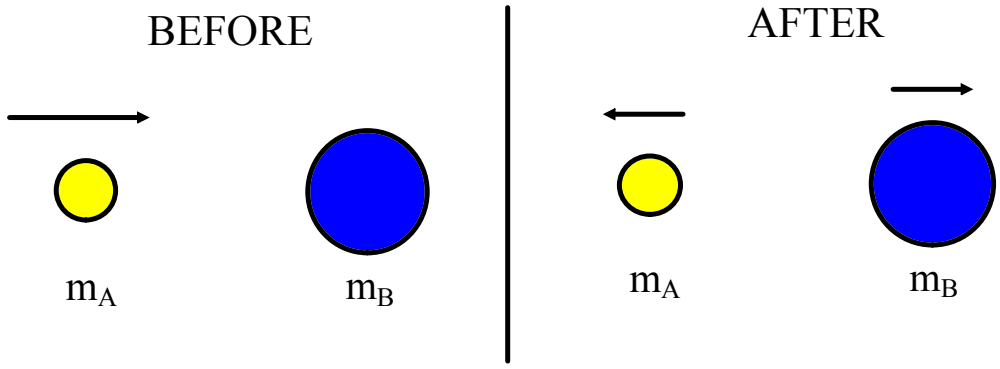
If you are told that a collision is elastic, you may need to use both a conservation of momentum equation and a conservation of energy equation.

Link

Link

Example - Elastic Collision:

Sphere A with mass 12 kg moving at 2.4 m/s east makes an elastic head-on collision with Sphere B, mass 36 kg, which is at rest. Find the velocities of the two spheres after the collision. Assume all motion is in one dimension. ($\vec{v_A}' = -1.2$ m/s, $\vec{v_B}' = 1.2$ m/s)



$m_A = 12$ kg
$m_B = 36$ kg
$\vec{v_A} = +2.4$ m/s
$\vec{v_B} = 0$ m/s
$\vec{v_A}' = ?$
$\vec{v_B}' = ?$
two unknowns

Conservation of Momentum

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$m_A v_A = m_A v_A' + m_B v_B'$$

Conservation of Kinetic Energy

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2$$

$$m_A v_A^2 = m_A (v_A')^2 + m_B (v_B')^2$$

Solve the momentum equation for v_B'

$$m_B v_B' = m_A v_A - m_A v_A'$$

$$v_B' = \frac{m_A (v_A - v_A')}{m_B}$$

Substitute the expression for v_B' above in the energy equation.

$$m_A v_A^2 = m_A (v_A')^2 + m_B \left[\frac{m_A (v_A - v_A')}{m_B}\right]^2$$

Expand to get a quadratic equation:

$$(v_A')^2 (m_A + m_B) + v_A' (-2 m_A v_A) + m_A v_A^2 - m_B v_A^2 = 0$$

(a = $(m_A + m_B)$, b = $(-2 m_A v_A)$, c = $m_A v_A^2 - m_B v_A^2$)

$$v_A' = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve for v_A' → two roots will result.

We want $v_A' = \dfrac{v_A (m_A - m_B)}{m_A + m_B}$

Substitute the expression for v_A' above into the expression for v_B' we got earlier.

$$v_B' = \frac{2 m_A v_A}{m_A + m_B}$$

So.

$$v_A' = \frac{v_A (m_A - m_B)}{m_A + m_B} \qquad v_B' = \frac{2 m_A v_A}{m_A + m_B}$$

$$v_A' = \frac{2.4\ m/s\ (12\ kg - 36\ kg)}{12\ kg + 36\ kg} \qquad v_B' = \frac{2(12\ kg)(2.4\ m/s)}{12\ kg + 36\ kg}$$

$$v_A' = -1.2\ m/s \qquad v_B' = +1.2\ m/s$$

Elastic collisions only occur at the atomic or subatomic levels.

This is because collisions at the macroscopic level involve energy transformations in which kinetic energy is converted into thermal energy, sound energy, etc.

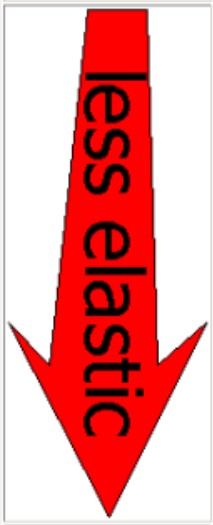
An inelastic collision results when kinetic energy is not conserved. Momentum is still conserved in an inelastic collision.

A perfectly inelastic or completely inelastic collision results when objects lock or stick together upon collision.

Simulation

Collision Types

Most text books describe three different types of collisions. In reality, it is more true to say there are two ends of a spectrum (range) of collision types. Regardless of what type of collision occurs, the total momentum of the system will always be conserved if there are no external forces present.

Collision Type	properties or characteristics	examples of collisions
Totally Elastic	● No kinetic energy is lost (KE is conserved). Since KE is not lost, no energy can go into heat or sound. ● No damage caused to either colliding object.	➤ Atomic or nuclear particles with similar charges. ➤ Magnets with similar poles facing each other.
	●Less damage or heat created. ●Less sound created. ● More damage or heat created. ● More sound created.	➤ A well inflated basketball, or any ball that bounces some. ➤ An under-inflated basketball, or any ball that doesn't bounce much.
Totally Inelastic	● The colliding objects stick together and become one. ● Involves the greatest loss of Kinetic Energy. (keep in mind that if it loses KE then the energy will need to appear as heat and or sound) ● The colliding objects usually suffer some kind of permanent damage	➤ A block of soft clay dropped onto a hard floor. ➤ A bad car accident where the cars stick together. ➤ Two train cars collide and stick together.

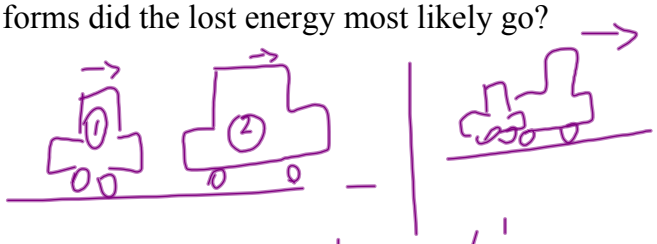
Example - Inelastic Collision:
In an accident due to slippery roads, a compact car, mass 575 kg moving at +15 m/s, smashes into the rear of a car with a mass of 1575 kg originally moving at +5.0 m/s. They lock together and slide forward.

a) What is the final velocity of the wrecked cars? (7.7 m/s)
b) How much kinetic energy was lost? (2.1×10^4 J lost)
c) What fraction of the original energy was lost? (1/4)
d) Into what forms did the lost energy most likely go?

Example - Inelastic Collision:

In an accident due to slippery roads, a compact car, mass 575 kg moving at +15 m/s, smashes head on into the rear of a car with a mass of 1575 kg originally moving at +5.0 m/s. They lock together and slide forward. perfectly inelastic

a) What is the final velocity of the wrecked cars? (7.7 m/s)
b) How much kinetic energy was lost? (2.1×10^4 J lost)
c) What fraction of the original energy was lost? (1/4)
d) Into what forms did the lost energy most likely go?



a) $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$

Note: $v_1' = v_2' = v'$

$m_1v_1 + m_2v_2 = v'(m_1 + m_2)$

$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$v' = 7.7\ m/s$

b) $\Delta E_K = E_{Kf} - [E_{Ki}]$

$$= \left[\frac{1}{2}m_1(v_1')^2 + \frac{1}{2}m_2(v_2')^2\right] - \left[\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right]$$

E_{Kf} E_{Ki}

$\Delta E_K = -2.1 \times 10^4\ J$

c) $$\frac{\Delta E_K}{E_{Ki}} = \frac{-2.1 \times 10^4\ J}{8.4 \times 10^4\ J} = -\frac{1}{4}$$

d) → heat, sound, bending metal takes energy.

Handout - Elastic and Inelastic Collisions

Attachments

p12-lad 2d collision.jpg

p12-lad 2d collision 2.jpg

p12-lad 2d collision 3.jpg