Test Answers

Avg. = 86%

1. a) Max = 253
Min = −3

b) Inc
$(-1,0)$ & $(1,\infty)$

Dec
$(-\infty,-1)$ & $(0,1)$

Local Max
$(0,1)$

Local Min
$(-1,-3)$ & $(1,-3)$

2. 7m x 7m x 3.5m

3. cost = $6600

4. 12 days

5. $\sqrt{3}$ km up the shore

6. r = 5 cm
h = 10 cm

$$(\theta)^2 = \left(\frac{10x}{\sqrt{x^2+90000}}\right)^2 - (8)^2$$

$$6^2 + 4^2 = 10^2$$

$$(8)^2 = \left(\frac{10x}{\sqrt{x^2+90000}}\right)^2$$

Example:

Using the function: $f(x) = \dfrac{x^2}{x-7}$

Determine each of the following...

- Intercepts — $f(x)$
- Intervals of increase/decrease — $f'(x)$
- Concavity — $f''(x)$
- Points of inflection ← $f''(x)$ & $f(x)$
- Local maximum and minimum values ← $f'(x)$ & $f(x)$

$$f'(x) = \frac{2x(x-7) - x^2(1)}{(x-7)^2}$$

$$= \frac{2x^2 - 14x - x^2}{(x-7)^2}$$

$$f'(x) = \frac{x^2 - 14x}{(x-7)^2}$$

$$f'(x) = \frac{x(x-14)}{(x-7)^2}$$

$$f''(x) = \frac{(2x-14)(x-7)^2 - (x^2-14x)(2(x-7))}{(x-7)^4}$$

$$f''(x) = \frac{(x-7)\left[(2x-14)(x-7) - 2(x^2-14x)\right]}{(x-7)^3}$$

$$f''(x) = \frac{\left[2(x-7)(x-7) - 2x(x-14)\right]}{(x-7)^3}$$

$$f''(x) = \frac{2(x^2 - 14x + 49 - x^2 + 14x)}{(x-7)^3}$$

$$f''(x) = \frac{98}{(x-7)^3}$$

$f(x)=\frac{x^2}{x-7}$ $\quad f'(x)=\frac{x(x-14)}{(x-7)^2}$ $\quad f''(x)=\frac{98}{(x-7)^3}$

Intercepts:

(x=0)

$y=\frac{0^2}{0-7}$

$y=0$

(y=0)

$0=x^2 \leftarrow$ Numerator

$x=0$

(0,0)

Inc/Dec:

f' critical Values:

$x=0, 14, 7$

	x	$x-14$	$(x-7)^2$	f'	f
$(-\infty,0)$	−	−	+	+	Inc
$(0,7)$	+	−	+	−	Dec
$(7,14)$	+	−	+	−	Dec
$(14,\infty)$	+	+	+	+	Inc

LOCAL MAX
(0,0)

LOCAL MIN
(14,28)

Concavity:

f'' critical Values

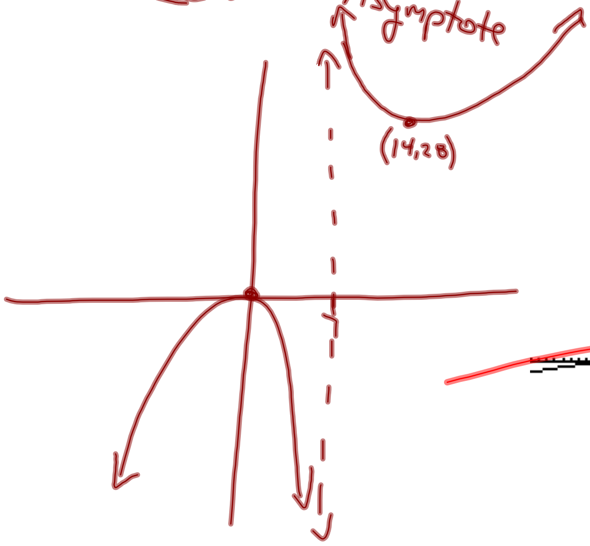
$x=7$

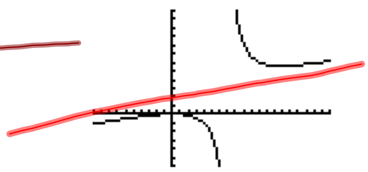
	98	$(x-7)^3$	f''	f
$(-\infty,7)$	+	−	−	Concave down
$(7,\infty)$	+	+	+	Concave up

Inflection Point(s): None

(7, ↑) ← undefined

→ x=7 is a Vertical Asymptote

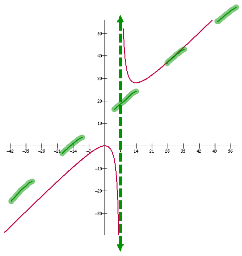




Let's look at homework question...

Example:

Using the function: $f(x)=\dfrac{x^2}{x-7}$ $= \dfrac{14^2}{7}$



Determine each of the following...

- Intercepts
- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values

$f'(x)=\dfrac{x(x-14)}{(x-7)^2}$ $f''(x)=\dfrac{98}{(x-7)^3}$

Intercepts:

x-Int. (y=0)

$0=\dfrac{x^2}{(x-7)}$

$x^2=0$

$x=0$

(0,0)

y-Int. (x=0)

$y=\dfrac{0^2}{0-7}$

$y=0$

Max/Min.

$f'(x)=\dfrac{x(x-14)}{(x-7)^2}$

Critical Values

$x=0, 14, 7$

	x	x-14	$(x-7)^2$	f'	f
$(-\infty,0)$	−	−	+	+	Inc
(0,7)	+	−	+	−	Dec
(7,14)	+	−	+	−	Dec
$(14,\infty)$	+	+	+	+	Inc

Local Max.
(0,0)

Local Min.
(14,28)

Concavity

$f''(x)=\dfrac{98}{(x-7)^3}$

C. Value ⇒ x=7

	98	$(x-7)^3$	f''	f
$(-\infty,7)$	+	−	−	Down
$(7,\infty)$	+	+	+	Up

Inflection Point: None

(7, undefined)

Asymptotes:

Horizontal:

$F(x)=\dfrac{x^2}{x-7}$

$\lim_{x\to\pm\infty}\dfrac{\frac{x^2}{x^2}}{\frac{x}{x^2}-\frac{7}{x^2}}$

$=\dfrac{1}{0-0}$

None

Vertical: (Set Den. = 0)

$x-7=0$

$x=7$

$\lim_{x\to7^-}\dfrac{x^2}{x-7}$

$=\dfrac{49}{\text{small}(-)}$

$\to-\infty$

$\lim_{x\to7^+}\dfrac{x^2}{x-7}$

$\dfrac{49}{\text{small}(+)}$

$\to\infty$

Example:

Sketch the function $f(x) = x^{\frac{2}{3}}(6-x)^{\frac{1}{3}}$. Use the following to help with the sketch:

- Intercepts
- Intervals of increase/decrease
- Concavity
- Points of inflection
- Local maximum and minimum values

$$f(x) = x^{2/3}(6-x)^{1/3}$$

$$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}(6-x)^{1/3} + x^{2/3}\left[\frac{1}{3}(6-x)^{-2/3}(-1)\right]$$

$$f'(x) = \frac{1}{3}x^{-1/3}(6-x)^{-2/3}\left[2(6-x) - x\right]$$

$$f'(x) = \frac{1}{3}x^{-1/3}(6-x)^{-2/3}(12-2x-x)$$

$$f'(x) = \frac{1}{\cancel{3}}x^{-1/3}(6-x)^{-2/3}\cancel{3}(4-x)$$

$$\boxed{f'(x) = x^{-1/3}(6-x)^{-2/3}(4-x)}$$

$$\frac{x^{-\frac{1}{3}}}{x^{-\frac{4}{3}}}$$

$$f''(x) = -\frac{1}{3}x^{-\frac{4}{3}}(6-x)^{-2/3}(4-x) + -\frac{2}{3}(6-x)^{-5/3}(-1)(x^{-\frac{1}{3}})(4-x) + (-1)(x^{-\frac{1}{3}})(6-x)^{-2/3}$$

$$f''(x) = -\frac{1}{3}x^{-4/3}(6-x)^{-5/3}\left[(6-x)(4-x) - 2x(4-x) + 3x(6-x)\right]$$

$$f''(x) = -\frac{1}{\cancel{3}}x^{-\frac{4}{3}}(6-x)^{-5/3}\left[24 \cancel{-10x} \cancel{+x^2} \cancel{-8x} \cancel{+2x^2} \cancel{+18x} \cancel{-3x^2}\right] \quad 8(24)$$

$$\boxed{f''(x) = -8x^{-4/3}(6-x)^{-5/3}}$$