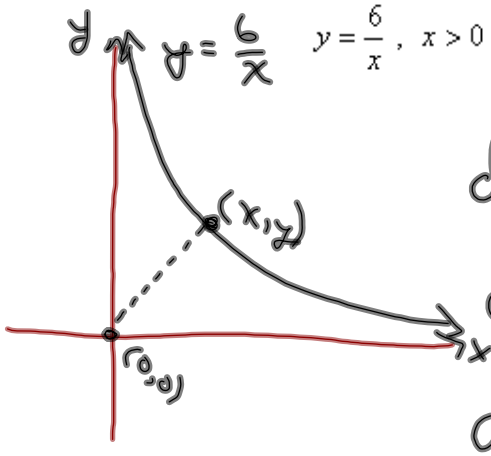
Determine the coordinates of the point closest to the origin on the graph of $y = \frac{6}{x}$, $x > 0$



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d = \sqrt{(x-0)^2+(y-0)^2}$$

$$d = \sqrt{x^2+y^2}$$

$$d = \sqrt{x^2+\left(\frac{6}{x}\right)^2}$$

$$d = \sqrt{x^2+36x^{-2}}$$

$$d' = \frac{1}{2}\left(x^2+36x^{-2}\right)^{-\frac{1}{2}}\left(2x-72x^{-3}\right)$$

$$0 = \frac{2x-72x^{-3}}{2\sqrt{x^2+36x^{-2}}}$$ Let Num = 0

$$2x-\frac{72}{x^3}=0$$

$$2x^4-72=0$$

$$\frac{2x^4}{2}=\frac{72}{2}$$

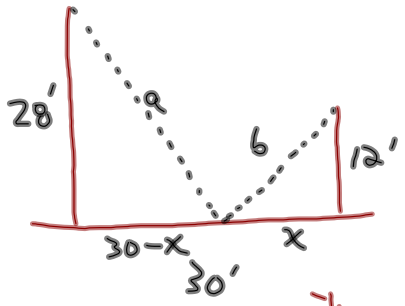
$$\sqrt[4]{x^4}=\sqrt[4]{36}$$

$$x=\sqrt{6} \Rightarrow y=\frac{6}{\sqrt{6}}\left(\frac{\sqrt{6}}{\sqrt{6}}\right)$$

$$y=\frac{6\sqrt{6}}{6}$$

$$(\sqrt{6},\sqrt{6})$$

Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached at a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?



Wire

$(a+b) = \sqrt{28^2+(30-x)^2} + \sqrt{12^2+x^2}$

l

$l' = \frac{1}{2}\left(28^2+(30-x)^2\right)^{-1/2}\left(2(30-x)^1(-1)\right) + \frac{1}{2}\left(12^2+x^2\right)^{-1/2}(2x)$

$0 = \frac{x-30}{\sqrt{28^2+(30-x)^2}} + \frac{x}{\sqrt{12^2+x^2}}$

$\left(-\frac{x+30}{\sqrt{784+(30-x)^2}}\right)^2 = \left(\frac{x}{\sqrt{144+x^2}}\right)^2$

$\frac{(-x+30)^2}{784+(30-x)^2} = \frac{x^2}{144+x^2}$

$(x^2-60x+900)(144+x^2) = x^2(784+900-60x+x^2)$

$144x^2+x^4-8640x-60x^3+129600+900x^2 = 784x^2+900x^2-60x^3+x^4$

$-640x^2-8640x+129600 = 0$

$2x^2+27x-405 = 0$

$x = \frac{-27 \pm \sqrt{27^2-4(2)(-405)}}{2(2)}$

$x = \frac{-27 \pm 63}{4}$

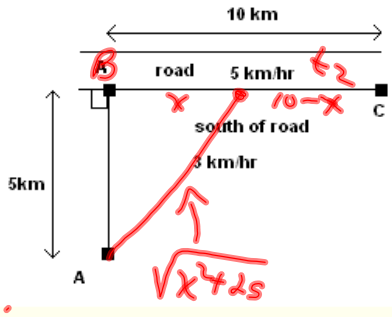
$x = \frac{-27 \pm 63}{4}$

$x = -\frac{90}{4}$ or $x = 9$

Example 5:

You decide to walk from point A (see figure below) to point C. To the south of the road through BC, the terrain is difficult and you can only walk at 3 km/hr. However, along the road BC you can walk at 5 km/hr. The distance from point A to the road is 5 km. The distance from B to C is 10 km. What path you have to follow in order to arrive at point C in the shortest (minimum) time possible?

$t = \frac{d}{s}$



$t = \frac{d}{s}$

Minimize Time: $T = \frac{\sqrt{x^2+25}}{3} + \frac{10-x}{5}$

$T = \frac{1}{3}(x^2+25)^{1/2} + 2 - \frac{1}{5}x$

$T' = \frac{1}{6}(x^2+25)^{-\frac{1}{2}}(2x) - \frac{1}{5}$

$0 = \frac{x}{3\sqrt{x^2+25}} - \frac{1}{5}$

$\left(\frac{1}{5}\right)^2 = \left(\frac{x}{3\sqrt{x^2+25}}\right)^2$

$\frac{1}{25} = \frac{x^2}{9(x^2+25)}$

$9(x^2+25) = 25x^2$

$9x^2 + 225 = 25x^2$

$\frac{225}{16} = \frac{16x^2}{16}$

$\sqrt{\frac{225}{16}} = \sqrt{x^2}$

$\frac{15}{4} = x$

$3.75 = x \Rightarrow$ 3.75 km east of B along the road

practice problems...

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#5, 6, 8, 10, 11, 12, 14, 16, 17, 19, 20