22.

| Month | 0 | 3 |
|---|---|---|
| Value | 1000 | 1030.30 |

$r = \frac{1030.30}{1000}$

$V = 1000(1.0303)^{\frac{M}{3}}$

$V = 1000(1.0303)^{\frac{36}{3}}$

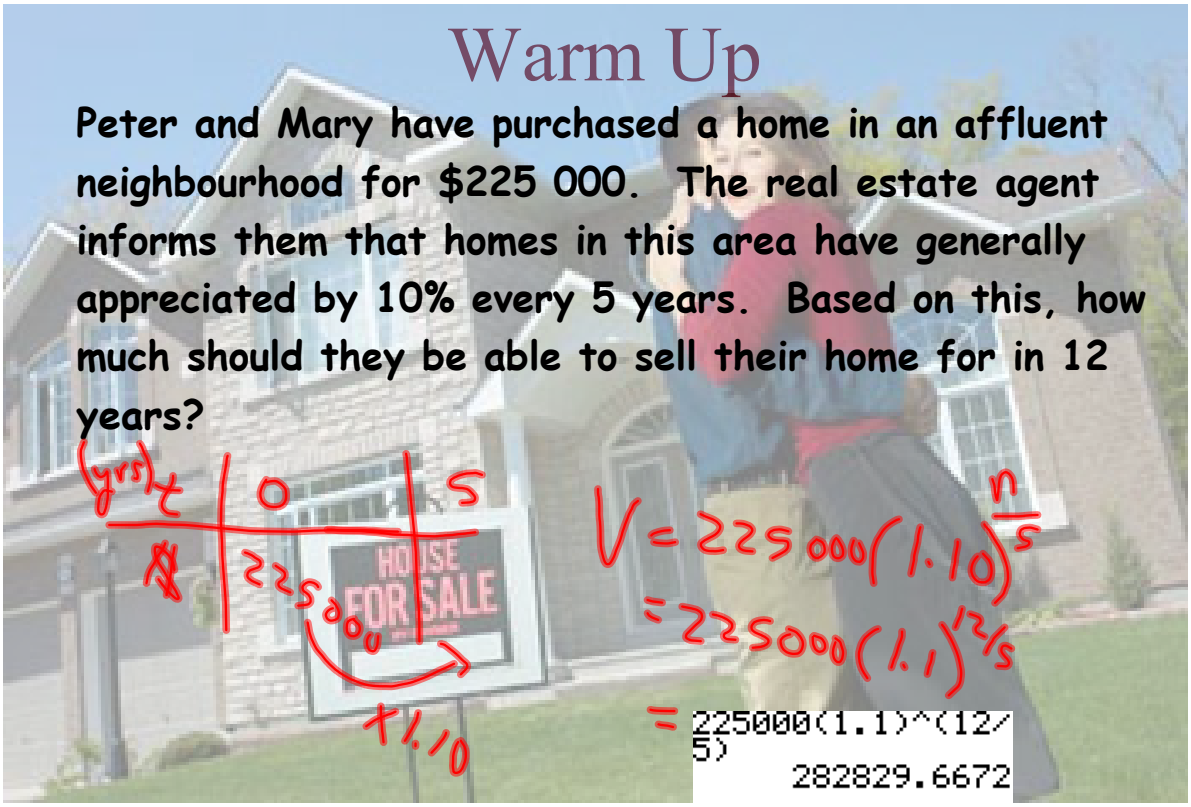
$V = \$1430.75$

$V = 1000(1.10462)^{\frac{M}{10}}$

$V = 1430.75$

# Warm Up

Peter and Mary have purchased a home in an affluent neighbourhood for $225 000. The real estate agent informs them that homes in this area have generally appreciated by 10% every 5 years. Based on this, how much should they be able to sell their home for in 12 years?




| (yrs) $t$ | 0 | 5 |
|---|---|---|
| $V$ | 225 000 | $\times 1.10$ |

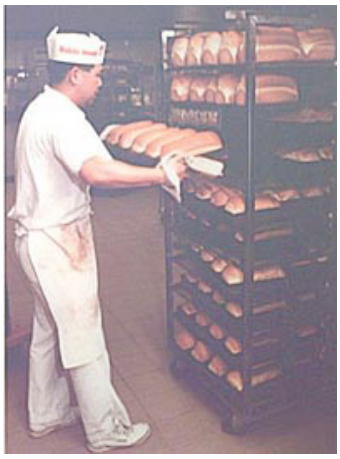
$$V = 225000(1.10)^{\frac{n}{5}}$$

$$= 225000(1.1)^{12/5}$$

$$= 225000(1.1)^{(12/5)}$$

$$282829.6672$$

# Applications with Exponential Growth



Yeast cells increase their numbers exponentially by a process called budding. They duplicate themselves about every half hour. The **doubling period** is thus said to be 0.5 hours.

**Example:**

A bacterial strain doubles every 3 minutes.
If there are 1000 bacteria initially, how many will there be after 0.25h?

15 min

| t (min) | 0 | 3 |
|---|---|---|
| N | 1000 | 2000 |

×2

$N = 1000(2)^{\frac{t}{3}}$

$N = 1000(2)^{\frac{15}{3}}$

$N = 32\,000$ Bacteria

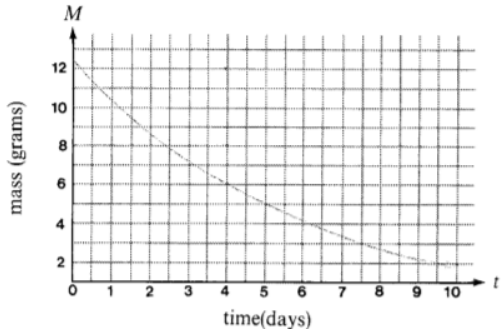
## Example - Half Life...

Radioactive elements decay over time. The data below were recorded for radon, a radioactive substance. The initial amount of radon was 12.6 g.

| amount of radon (grams) | 10.5 | 8.8 | 7.3 | 6.1 | 5.1 | 4.2 | 3.5 | 2.9 | 2.4 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| time (days) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

With the data displayed on a graph, you can see that the decay appears exponential.

The **half-life** of a radioactive element is the time taken for the element to decay by one half. In general, for a radioactive element the mass left after time, $t$, is given by




Example: 320 mg of Iodine 131 is stored in a laboratory for 40 d. At the end of this period only 10 mg of the element remains. What is the half-life of Iodine 131?

| $t$ (days) | 0 | 40 |
|---|---|---|
| Mass (mg) | 320 | 10 |

$\frac{10}{320} = \frac{1}{32}$

$M = 320(0.03125)^{\frac{t}{40}}$

$\frac{1}{2}$

$\frac{160}{320} = \frac{320\left(\frac{1}{32}\right)^{t/40}}{320}$

$\frac{1}{2} = \left(\frac{1}{32}\right)^{t/40}$

$2^{-1} = \left(2^{-5}\right)^{\frac{t}{40}}$

$2^{-1} = 2^{-\frac{5t}{40}}$

$-1 = -\frac{5t}{40}$

$-40 = -5t$

$8 = t$

$\frac{1}{2}$ life = 8 days


# Page 135: #26 - 32
# #33 (a - c)
# #34 - 36
# #39

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#26. $y = 5(1.2)^x$ *OR* $y = 5(3)^{\frac{x}{2}}$ *OR* $y = 5(9)^{\frac{x}{12}}$

#27. $78.11

#28. Initial - 4.22
Base - 2 (double)
Increment - every 16 years (2006 - 1990)

#29. Both are correct

#30. *a)* $y = 12(3)^{\frac{x}{2}}$

*b)* $y = 48\left(\frac{1}{2}\right)^{\frac{x}{3}}$

*c)* $y = 3(2)^{\frac{x}{4}}$

*d)* $y = 60(2)^{10x}$

*e)* $y = 6\left(\frac{1}{3}\right)^{5x}$

#31. a) 300 bacteria/cm$^2$
b) 20 min
c) 1697 bacteria/cm$^2$

#32. missing *x*-values: 12 & 18
missing *y*-values: 86.05 & 120.47

#33. *a)* $y = 0.87(0.82)^x$

*b)* $y = 0.87(0.76)^x$

*c)* $0.41\ candela/cm^2$

#34. *a)* $y = 2.8\left(\frac{1}{2}\right)^{\frac{x}{5750}}$

*b)* $0.66\ mg$

#35. $1414. 21 (wrong assumption)

#36. *a)* $y = 3500(0.629)^{\frac{x}{2}}$

b) Day 1 - 3500
Day 2 - 2200
Day 3 - 1384

c) 691 frogs

#39. $283 000

## Review of Applications of Growth and Decay...

Assume you invest \$5,000 in an account paying 8% interest compounded monthly. How much money will be in the account after 5 years?

8% /a

(months)

| t | 0 | 1 |
|---|---|---|
| \$ | 5000 | |

$1 + \frac{0.08}{12}$

$A = 5000(1.007)^{60}$

$= \$7598.68$

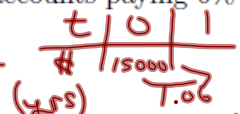
$A = 5000\left(1 + \frac{0.08}{12}\right)^{t}$

$A = \$7749.23$

Find the amount of money you will have after 10 years if \$15,000 is invested in accounts paying 6% interest compounded:

a. Annually

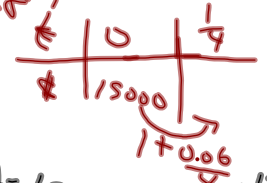
\$26 862.72




$\Rightarrow A = 15000(1.06)^{10}$

b. Quarterly

\$27 210.28




$A = 15000\left(1+\frac{0.06}{4}\right)^{4t}$

$= 15000\left(1+\frac{0.06}{4}\right)^{40}$

c. Monthly

\$27 290.95

$A = 15000\left(1+\frac{0.06}{12}\right)^{12\times 10}$

d. Daily

\$27 330.43

$A = 15000\left(1+\frac{0.06}{365}\right)^{365\times 10}$

Compound Interest $\Rightarrow A = P(1+i)^n$

$i$ = Annual Rate / compounds/year

$n$ = Total compounding intervals

\$1 000 000

$\rightarrow$ Invest \$900 000 @ 7¼%/a for 2 years

(compounded semi-annually)

$A = 900000\left(1+\frac{0.0725}{2}\right)^{4}$

$A = \$1\,037\,768.98$

Interest??

\$137 768.98