5 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example:

Find the critical values of $f(x) = x^{\frac{3}{5}}(4 - x)$ and determine all intervals of increase and decrease as well as any local extrema.

$f'(x) = \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1)$

$f'(x) = \frac{12}{5}x^{-2/5} - \frac{3}{5}x^{3/5} - x^{3/5}$

$f'(x) = \frac{12}{5}x^{-2/5} - \frac{8}{5}x^{3/5}$

$f'(x) = \frac{4}{5}x^{-2/5}(3 - 2x) \quad \frac{x^{3/5}}{x^{-2/5}}$

$m^6 + 3m^4$

$m^4(m^2+3)$

smallest

Critical Values:

undefined if $x = 0$; $3 - 2x = 0$

$-2x = -3$

$x = \frac{3}{2}$

	$\frac{4}{5x^{2/5}}$	$3-2x$	f'	f
$(-\infty, 0)$	+	+	+	Inc
$(0, \frac{3}{2})$	+	+	+	Inc
$(\frac{3}{2}, \infty)$	+	−	−	Dec

```
(3/2)^(3/5)(4-3/2
        3.188561252
```

Local Max.

$(\frac{3}{2}, 3.2)$

Local Min.

None

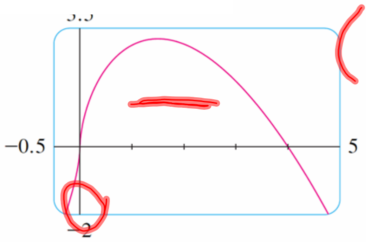


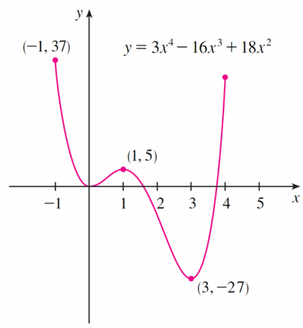
FIGURE 11

How do we determine absolute maximum and minmum values?

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example:



Given the function...

$$f(x) = 3x^4 - 16x^3 + 18x^2 \;,\; -1 \le x \le 4$$

Determine the absolute maximum and minimum values of the function.

$f'(x) = 12x^3 - 48x^2 + 36x$

Critical Values:

$12x^3 - 48x^2 + 36x = 0$

$12x(x^2 - 4x + 3) = 0$

$12x(x-3)(x-1) = 0$

$x = 0, 3, 1$

Table of Values

Sub into f(x)

x	y
-1	
0	
1	
3	
4	

X	Y1
-1	37
0	0
1	5
3	-27
4	32

X=

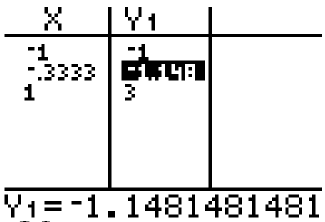
Absolute Maximum = 37

Absolute Minimum = -27

Example 2:

Using Calculus methods determine the absolute maximum and minimum values of the function given below:

$f(x) = x^3 + 2x^2 + x - 1$ over the interval [-1 , 1]



X	Y1
-1	-1
-.3333	-1.148
1	3

Y1=-1.1481481481

$f'(x) = 3x^2 + 4x + 1$

$3x^2 + 4x + 1 = 0$

$3x^2 + 3x + x + 1 = 0$

$3x(x+1) + 1(x+1) = 0$

$(x+1)(3x+1) = 0$

$x = -1, -\frac{1}{3}$

x	y	
-1	-1	
$-\frac{1}{3}$	$-\frac{31}{27}$	Abs. Min.
1	3	Abs Max.

Given the function $f(x) = 4x^4 - 8x^2 + 1$ determine ...

(a) the absolute maximum and minimum values on the interval [0 , 3].

(b) the intervals of increase/decrease and local extreme values on the interval (-∞ , ∞).

$f'(x) = 16x^3 - 16x$

$0 = 16x(x^2 - 1)$

$0 = 16x(x-1)(x+1)$

Critical Values: $x = 0, 1, -1$

a)

x	y
0	1
1	-3 ← Abs Min.
3	253 ← Abs. Max.

b)

	$16x$	$x-1$	$x+1$	f'	f
$(-\infty, -1)$	−	−	−	−	Dec
$(-1, 0)$	−	−	+	+	Inc
$(0, 1)$	+	−	+	−	Dec
$(1, \infty)$	+	+	+	+	Inc

Local Max.

$(0, 1)$

Local Min.

$(-1, -3)$ & $(1, -3)$

Homework

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#3 d, f, l
#4 d, f, h, i
#6
#7