EXAMPLE:

Given $f(x) = \sqrt[3]{2x-1}$, find $f'''(5)$

$$f(x) = (2x-1)^{1/3}$$

$$f'(x) = \frac{1}{3}(2x-1)^{-2/3}(2)$$

$$f''(x) = -\frac{2}{9}(2x-1)^{-5/3}(4)$$

$$f'''(x) = \frac{10}{27}(2x-1)^{-8/3}(8)$$

$$f'''(5) = \frac{10}{27}(10-1)^{-8/3}(8)$$

$$= \frac{10}{27}\left(\frac{1}{9^{8/3}}\right)(8)$$

$$= \frac{80}{27\left(\sqrt[3]{9}\right)^8} =$$

Rectilinear Motion and Derivatives

Any motion along a straight line is called rectilinear motion.

Displacement - Velocity - Acceleration

If s represents a function that measures displacement, then $\frac{ds}{dt}$ would represent ??? $\quad velocity = \frac{ds}{dt}$

The rate of change of the velocity...ie. $\frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

would represent?? $\quad acceleration = \frac{dv}{dt}$

So it follows that the second derivative of displacement will give us acceleration:

$$a = \frac{d^2s}{dt^2}$$ ←------- Notice the notation

Example

If the displacement (in metres) at time t (in seconds) of an object is given by

disp. → $s = 4t^3 + 7t^2 - 2t$,

find the acceleration at time $t = 10$.

$s' = 12t^2 + 14t - 2$ (velocity)

$s'' = 24t + 14$ (accel)

$s''(10) = 24(10) + 14$

$= 254 \ m/s^2$

Example:

- The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$, where t is measured in seconds and s in meters.
  a) Find the velocity at time t.
  b) What is the velocity after 2 s? After 4 s?
  c) When is the particle at rest?
  d) When is the particle moving forward (that is, in the positive direction)?
  e) Draw a diagram to represent the motion of the particle.
  f) Find the total distance traveled by the particle during the first five seconds.
  g) Find the acceleration at time t and after 4 s.
  h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.
  i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

a) Find the velocity at time t.

$s'(t) = 3t^2 - 12t + 9$

b) What is the velocity after 2 s? After 4 s?

$s'(2) = 3(2)^2 - 12(2) + 9$
$= 12 - 24 + 9$
$= -3 m/s$

$s'(4) = 3(4)^2 - 12(4) + 9$
$= 57 - 48$
$= \underline{9 m/s}$

$s = f(t) = t^3 - 6t^2 + 9t$

c) When is the particle at rest? $(v = 0)$

$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} = \frac{0}{3}$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3$ OR 1

1 second & 3 seconds

d) When is the particle moving forward (that is, in the positive direction)?

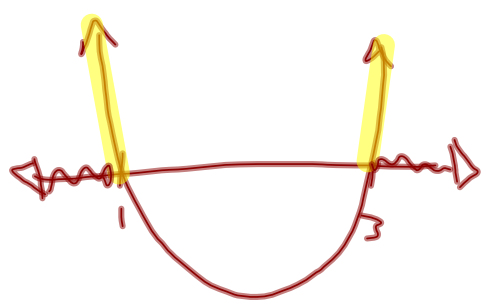
(Velocity > 0)

$+3t^2 - 12t + 9 > 0$

zeros $= 1$ & 3

$\{t \mid 0 \leq t < 1$ OR $t > 3, t \in R\}$

Graph



e) Draw a diagram to represent the motion of the particle.

$s = f(t) = t^3 - 6t^2 + 9t + 3$

t	s
0	0
1	4
3	0
5	20

← Always start

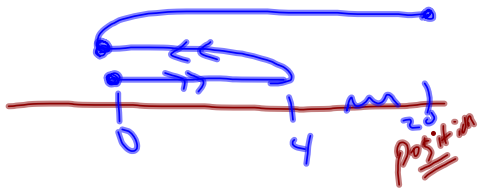
Any time velocity = 0

← End Time

$s(0) = 0$

$s(1) = 1 - 6 + 9$

$s(3) = 0$



f) Find the total distance traveled by the particle during the first five seconds.

distance = 4m + 4m + 20m

= 28m

g) Find the acceleration at time t and after 4 s.

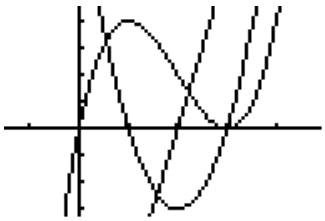
$s = f(t) = t^3 - 6t^2 + 9t$

$s''(t) = 6t - 12$

$s''(4) = 6(4) - 12$

$= 12 \text{ m/s}^2$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.



i) When is the particle speeding up? When is it slowing down?

$s = f(t) = t^3 - 6t^2 + 9t$

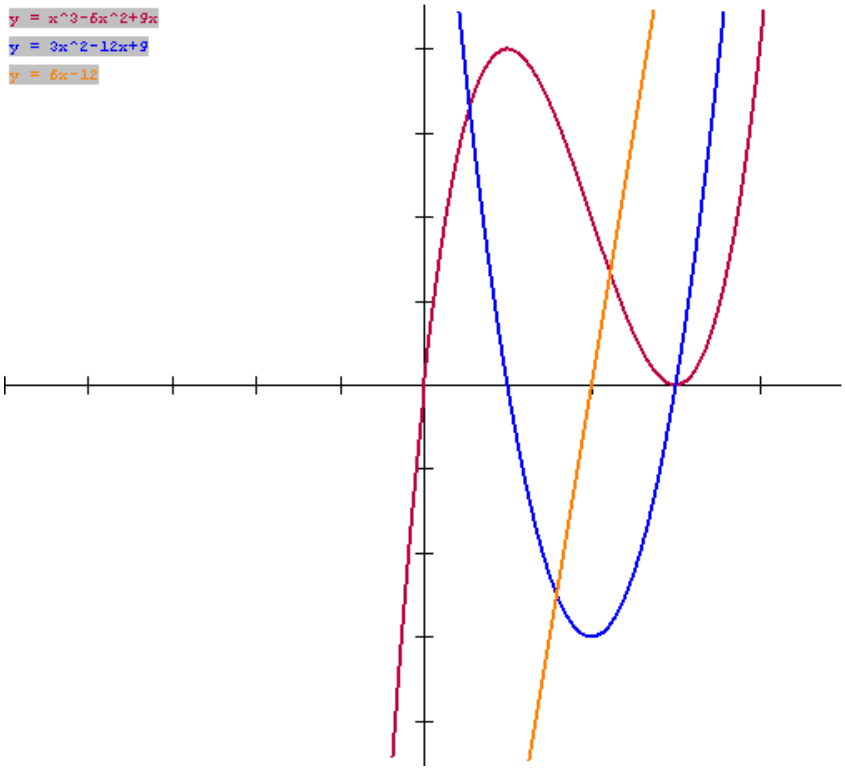
Speeding Up: ① $v > 0$ & $a > 0$

OR ② $v < 0$ & $a < 0$

Slowing Down: ① $v > 0$ & $a < 0$

OR ② $v < 0$ & $a > 0$

h) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.



i) When is the particle speeding up? When is it slowing down?

Time to check your understanding...

(seconds)

A particle moves according to a law of motion $s(t) = 2t^3 - 9t^2 + 12t + 1,\ t \geq 0$.

(a) Determine the velocity of the particle when it has acceleration 2 units/s^2.

(b) When is this particle moving in a positive direction?

(c) Sketch the path of this particle, and determine how far it has traveled during the first 8 seconds.

Attachments

Mr lams data.84state