FUNCTIONS AND RELATIONS 111

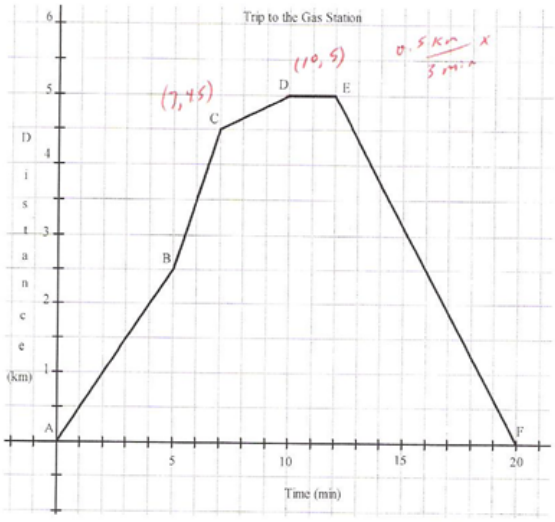
UNIT TEST: RATE OF CHANGE

NAME: Answer Key

NOVEMBER 2011

Part A – Multiple Choice (12 marks)
Shade the letter corresponding to the correct solution on the test. (TI-84 permitted for these questions)

Use the graph to the right to answer questions #1 - #4.



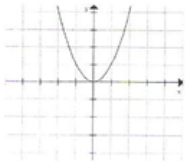
1. Which section of the trip has a **negative** rate of change?
[A] A to B [B] B to E
[C] A to F [D] C to F

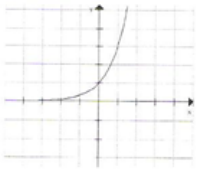
2. What is happening in section D to E?
[A] person is slowing down
[B] person is speeding up
[C] person is stopped
[D] not enough information

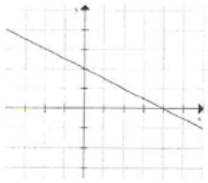
3. What is the average rate of change from C to D?
[A] 0.2 km/h [B] 0.3 km/h
[C] 10 km/h [C] 20 km/h

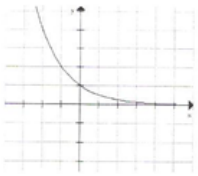
4. What is the **total distance** traveled?
[A] 4 km [B] 5 km
[C] 10 km [D] 17 km

5. Which of the following represent a **constant** rate of change?

[A] 

[B] 

[C] 

[D] 

6. Ethan wears a heart rate monitor when he jogs in the morning. He recorded his heart rates last Tuesday. What was the average rate of change of Ethan's heart rate (in beats per minute) over the interval from 4 minutes to 8 minutes?
[A] 5 beats/min [B] 11 beats/min
[C] 19 beats/min [D] 24 beats/min

min	beats
0	64
2	106
4	132
6	149
8	152
10	155

$\frac{132-152}{4-8} = \frac{-20}{-4}$

7. A company projects that its total savings S (in dollars) by converting to solar heating system with a solar collector area A (in m^2) will be $S = 360A - 0.10A^3$. Find the average rate of savings for the interval $A = 5\ m^2$ to $A = 20\ m^2$
[A] – \$4886.67/m^2 [B] \$0.10/m^2 [C] \$307.50/m^2 [D] \$359.48/m^2

$\frac{6400 - 1787.5}{20 - 5}$

8. A golf ball is hit into the air and follows the path modeled by the function $h = -2t^2 + 24t$, where h is the height of the ball in metres and t is the time in seconds. What time interval is the rate of change **decreasing**?
[A] 0 to 6 seconds [B] 0 to 12 seconds [C] 6 to 12 seconds [D] 12 to 24 seconds

$h = -2(t^2 - 12t + 36) + 72$
$-2(t-6) + 72$ (6, 72)

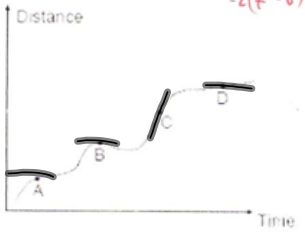
9. At what point is the instantaneous rate of change greatest?

[A] A

[B] B

[C] C

[D] D



10. The height of a pebble fired by a sling shot is given by $h(t) = 25t - 4.9t^2$, where $h(t)$ is height in metres and t is time in seconds after the pebble leaves the sling shot. Determine the instantaneous rate of change in the height of the pebble after 3 seconds.

[A] 30.9 m/s [B] –4.4 m/s [C] 44.1 m/s [D] –4.9 m/s

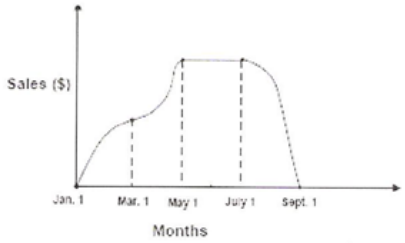
11. The graph below shows the sales of a company over an eight month period. Between what dates was there a negative average rate of change in sales?

[A] Jan. 1 → Mar. 1

[B] Mar. 1 → May 1

[C] May 1 → July 1

[D] July 1 → Sept. 1



12. The table below shows the height, in metres, of a dropped ball over time, in seconds. Based on this data, what is the best approximation for the instantaneous rate of change, in m/s, at 4 seconds?

t	3.98	3.99	4.00	4.01	4.02
$h(t)$	22.382	21.992	21.6	21.208	20.814

[A] -39.2 [B] -0.026

[C] 0 [D] 21.6

PART B – Open Response (36 marks)

Show **ALL** your work in the space that is provided. **Be sure to pay attention to units!!!**

1. The distance-versus-time data from the odometer in a police car during the 1 minute interval between 3:34 PM and 3:35 PM on Friday afternoon is recorded as follows:

Time (s)	0	5	10	15	20	25	30	35	40	45	50	55	60
Distance (m)	88	171	268	356	361	368	374	485	630	771	940	1116	1298

83 97 88 5 7 6 111 145 141 169 176 182

(a) What was the average speed of the police car, in **km/h**, between 3:34:15 PM and 3:34:45 PM? [3]

ARC = $\frac{356 - 771 \text{ m}}{15 - 45 \text{ sec}} = \frac{-415 \text{ m}}{-30 \text{ sec}} \times \frac{3600 \text{ sec}}{1 \text{ h}} \times \frac{1 \text{ Km}}{1000 \text{ m}} = 49.8 \text{ Km/h}$

(b) The police officer claims that he had to slow down for an oncoming ambulance for about 15 seconds. During which interval did this most likely occur? Provide a BRIEF reason to justify your choice. [2]

3:34:15 – 3:34:30

– only travelled 83 m

(c) The officer is being reprimanded for traveling in excess of the posted 110 km/h speed limit during this one minute interval. Provide data that clearly demonstrates that this officer was in fact speeding between 3:34 PM and 3:35 PM on Friday. [3]

3:34:55 – 3:35

IRC (or ARC) = $\frac{1298 - 1116 \text{ m}}{60 - 55 \text{ sec}} = \frac{182 \text{ m}}{5 \text{ sec}} \times \frac{3600 \text{ sec}}{1 \text{ h}} \times \frac{1 \text{ Km}}{1000 \text{ m}} = 131.04 \text{ Km/h}$

Speeding !!

2. A TI-84 calculator is carelessly thrown from the top of a University residence building by a frustrated Calculus student. The height of the calculator above the ground, in metres, at any time t seconds is given by the function $h(t) = -4.9t^2 + 19.6t + 6$.

(a) Determine the average rate of change in the height of the calculator between 1 second and 3 seconds. [3]

t	h
1	20.7
3	20.7

ARC $= \frac{20.7 - 20.7}{2}$ m/sec $= 0$ m/sec

(b) Determine the height reached by the calculator the instant that it has an instantaneous rate of change of 0 m/s. [4]
(Must be done algebraically, TI-84 not permitted)

$h(t) = -4.9(t^2 - 4t + 4) + 6 + 19.6$

$h(t) = -4.9(t-2)^2 + 25.6$

$(2, 25.6)$

Calculator would be at a height of 25.6m

(c) Approximate the instantaneous rate of change in the height of the calculator when it is 25.404m above the ground for the **second** time. *(Tangent feature of TI-84 not permitted)* [6]

$-4.9t^2 + 19.6t + 6 = 25.404$

$-4.9t^2 + 19.6t - 19.404 = 0$

$t = \frac{-19.6 \pm \sqrt{(19.6)^2 - 4(-4.9)(-19.404)}}{2(-4.9)}$

$t = \frac{-19.6 \pm 1.96}{-9.8}$

$t = 2$ sec or 1.8 sec

t	h
2.1	25.551
2.3	25.159

IRC $= \frac{25.551 - 25.159}{2.1 - 2.3}$ m/sec

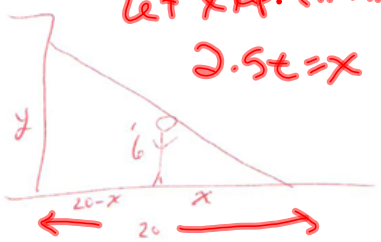
$= \frac{0.392\text{m}}{-0.2\text{sec}} = -1.96$ m/sec

You MUST COMPLETE at least 3 of the following 4 problems (Question 5 is considered 2 problems). Any extra problems completed will be considered as BONUS marks

3. A spotlight is located on the ground 20 feet away from a wall and a 6 foot tall person is walking toward the wall at a rate of 2.5 feet/sec. How fast is the height of the shadow changing when the person is 8 feet from the wall? Is the shadow increasing or decreasing in height at this time? *(Tangent feature of TI-84 permitted to determine final solution)* [5]

Let x rep. time in s

$2.5t = x$



$\frac{6}{y} = \frac{x}{20}$

$120 = xy$

$120 = (2.5t)y$

$y = \frac{48}{t}$

IRC @ 4.8sec $= -2.08$ m/s

$x = 2.5t$

$12 = 2.5t$

$t = 4.8$ sec

- Decreasing

4. A large spherical weather balloon is being inflated with hydrogen gas in such a manner that its surface area is increasing at a rate of 5.8 m^2/minute. Determine the rate at which the radius of the weather balloon is increasing the instant it has a volume of $\frac{16\pi}{3}\ m^3$.

(Tangent feature on TI-84 permitted to determine final solution) [5]

(*Volume of a sphere*: $V = \frac{4}{3}\pi r^3$, *Surface area of a sphere*: $SA = 4\pi r^2$)

"t" Rig time in minutes

$SA = 4\pi r^2$

$r = \sqrt{\frac{SA}{4\pi}}$

$r = \sqrt{\frac{5.8t}{4\pi}}$

$SA = 5.8t$

$31.65 = 5.8t$

$t = 5.46$ min

$\frac{16\pi}{3} = \frac{4}{3}\pi r^3$

$8 = r^3$ $r = 1.6m$

$r = 2m$

F

$SA = 4\pi(\sqrt[3]{4})^2$

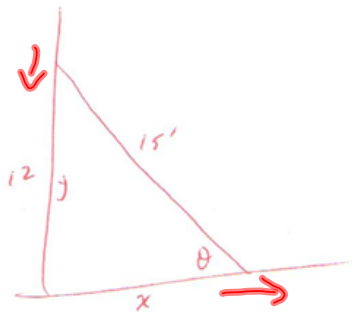
$= 31.65\ m^2$

IRC @ 5.46 sec = 0.145 m/minute

5. A ladder 15 feet tall leans against a vertical wall of a house in such a manner that it is resting against the ledge of a window located 12 feet above the ground. The bottom of the ladder begins to slide away horizontally from the house at constant speed of 4 ft/sec.

(a) Determine the rate at which the top of the ladder is sliding down the wall 8 seconds after the bottom begins to slide away.

(Tangent feature on TI-84 permitted to determine final solution) [5]



$x = 9 + 4t$

$y = \sqrt{15^2 - (9+4t)^2}$

@ $t = 8$ sec

IRC = -6.95 feet/sec

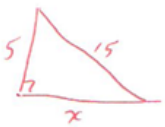
(b) Determine the rate at which the angle between the ground and the ladder is decreasing the instant the top of the ladder is situated 5 feet above the ground on the side of the house. [5]

(Tangent feature on TI-84 permitted to determine final solution)

$\cos\theta = \frac{9+4t}{15}$

$\theta = \cos^{-1}\left(\frac{9+4t}{15}\right)$

IRC @ 1.29 sec = -46.31°/sec



$x = \sqrt{15^2 - 25}$

$x = 14.14$ feet

$9 + 4t = 14.14$

$t = 1.29$ sec