Unit Test: Tuesday

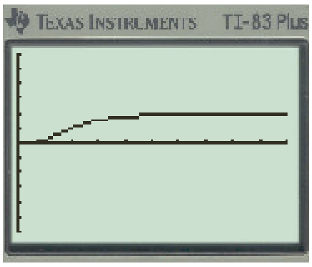
- Average Rate of Change
- Instantaneous Rate of Change
- Applications of Rate of Change

Review Work:

- Review Sheet
- Textbook:
  Page 90: # 15, 16
  Practice Questions: pgs. 105 - 106

Solutions...

page 90: #15. a) 44m; tangent is horizontal
b) ~ 6 s; -29.3 m/s, falling
c) 9.4 m/s

#16. 

b) 2 months, ~2.9 months
c) aproaches 2 m
d) 18 cm/month

Answers, Text pages 105–106

Question 1

(a) 73.75 km/h

(b) She was going fastest during the second and third hours, when she travelled 80 km in an hour and not just 75 km or 60 km.

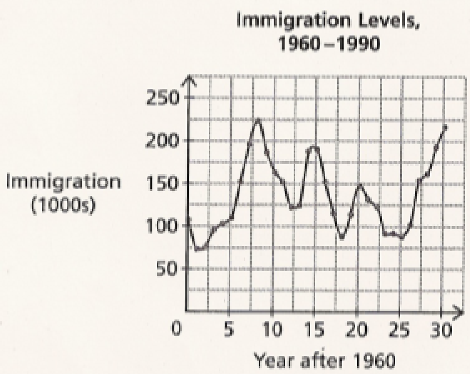
(c) She was going slowest during the fourth hour; you can't be sure it was so throughout.

Question 2

(a) from 1986 to 1991; you can tell by finding the amount of increase for each five-year period

(b) No; you don't know if there was the same number of students in each time period.

Question 3



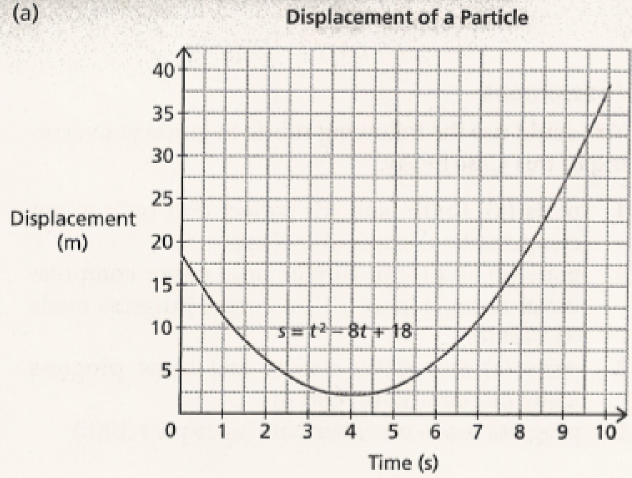
(a) The rate of change in immigration was positive during these time periods: 1961–1968, 1972–1975, 1978–1980, and 1985–1990.

It was negative during the other periods: 1960–1961, 1968–1972, 1975–1978, and 1980–1985.

(b) The rate of change was greatest during time periods for which the curve is steepest: from 1973 to 1974 and from 1986 to 1987.

Question 4

(a)

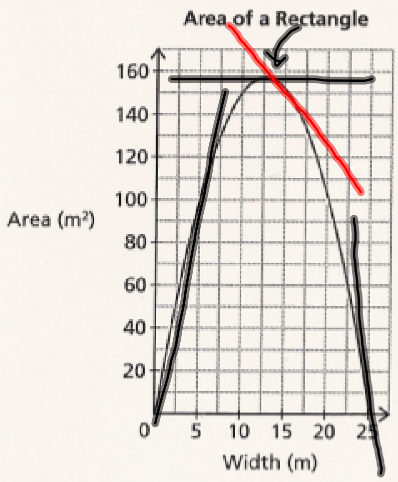


(b) The particle is moving fastest at the end of the 10 s. It is moving slowest at about 4 s after the start.

(c) The graph is steepest when $t = 10$ and is flattest when $t = 4$.

Question 5

The formula would be $A = w(25 - w)$.



$P = 2w + 2l$

$50 = 2(1) + 2(24)$

(a) No; the area increases at first and then decreases, as the length increases.

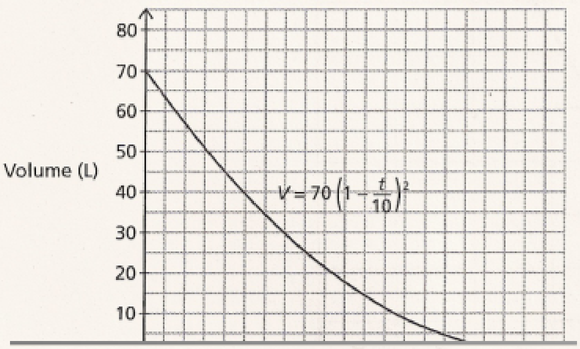
(b) The area changes "fastest" when the length and width are very different. You can tell because the graph is steepest there (where w is close to 0 or w is close to 25).

Question 6

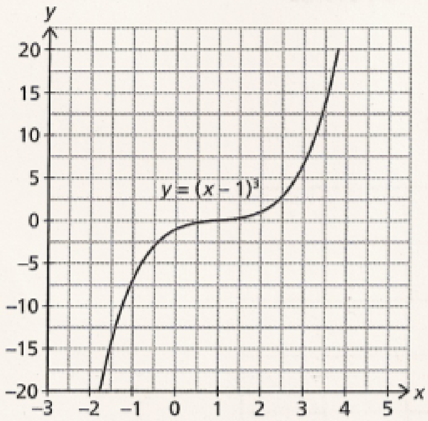
(a) -35.7 L during the first three minutes, an average of -11.9 L/min

(b) The rate of change in volume is greater at the beginning of the draining. You can tell because the curve is steeper there.

Volume of Water in a Pail



Question 7

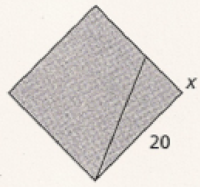


Answers will vary slightly, since students are estimating the tangent slopes. The tangents to the graph of $y = (x - 1)^3$ have these slopes:

- At $x = 0$, the slope is approximately 3.
- At $x = 1$, the slope is approximately 0.
- At $x = 5$, the slope is approximately 48.
- At $x = 10$, the slope is approximately 243.

The values make sense since the curve is flat at $x = 1$ and gets very steep as x gets very large. The rate of change is increasing at $x = 5$ and $x = 10$.

Question 8



The distance, d (in metres), to home plate when the runner is x metres down the path is given by $d = \sqrt{400 + x^2}$. Halfway down the path, $x = 10$. The slope of the tangent to the graph of the function at that value is about 0.45, so the change in distance to home plate is 0.45 m for 1 m along the path. That is the same as 0.45 m for $\frac{1}{7}$ s, or about 3.13 m/s.

Question 9

The volume of an ice cube is the cube of its side length. After 1.5 min, the ice cube has lost 12 mm on each side, so its side length is 28 mm. The volume has changed from 64 cm^3 to 21.952 cm^3, which is a decrease of 42.048 cm^3 in 1.5 min. This is equivalent to an average rate of change in volume of about -28.0 cm^3/min.

After 1.49 min, the volume is about 22.141 cm^3. The instantaneous rate of change in volume after 1.5 min is approximately $\frac{21.952 - 22.141}{1.5 - 1.49} = -18.9$ cm^3/min.

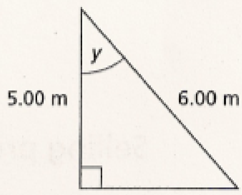
Question 10

(a) The height increases for about 2.5 s after the golf ball is hit.

(b) The height decreases from 2.5 s until the ball hits the ground, at a bit more than 5 s.

(c) When $t = 3$ s, the height is changing at about -4.4 m/s.

Question 11



Initially the ladder's base is about 3.32 m from the wall; the Pythagorean theorem can be used for this calculation. The base moves away at 25 cm/min, so after t min its distance from the wall is $3.32 + 0.25t$. This means that the sine of angle y is $\frac{3.32 + 0.25t}{6.00}$. For example, the graph of angle versus time is the graph of $y = \sin^{-1} \frac{3.32 + 0.25t}{6.00}$. The instantaneous rate of change in angle y after 10 min is about $\frac{76.9 - 75.9}{10.1 - 10}$, or 10°/min.

Attachments

Review - Rate of Change.doc