Unit 4:

Linear Functions

6.1 Slope of a Line

LESSON FOCUS Determine the slope of a line segment and a line.

Make Connections

The town of Falher in Alberta is known as *la capitale du miel du Canada*, the Honey Capital of Canada. It has the 3-story slide in the photo.
How could you describe the steepness of the slide?

Some roofs are steeper than others. Steeper roofs are more expensive to shingle.
The steepness of a roof is measured by calculating its **slope.**

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

The **rise** is the vertical distance from the bottom of the edge of the roof to the top.
The **run** is the corresponding horizontal distance.
For each roof, we count units to determine the rise and the run.

Roof A

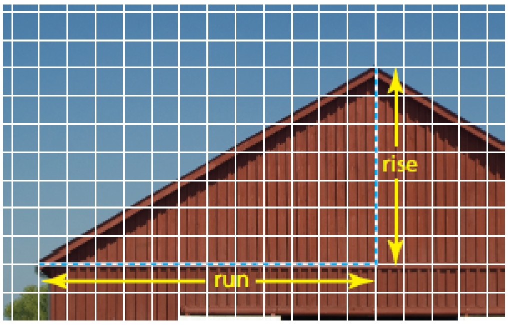
For Roof A

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{13}{13}$$

$$\text{Slope} = 1$$

Roof B



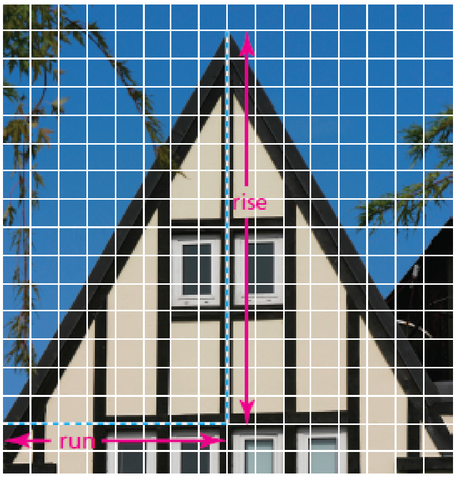
For Roof B

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{7}{12}$$

$$\text{Slope} = 0.58\overline{3}$$

Roof C



For Roof C

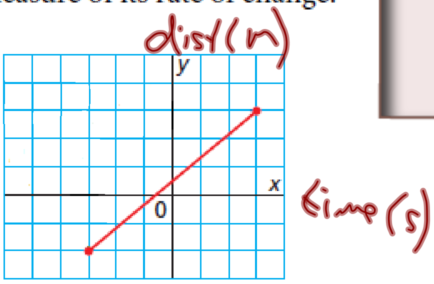
$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{14}{8}$$

$$\text{Slope} = 1.75$$

Roof C is the steepest because its slope is the greatest.
Roof B is the least steep because its slope is the least.

The slope of a line segment on a coordinate grid is the measure of its rate of change. From Chapter 5, recall that:



$$\text{Rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

$$\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

The change in y is the rise.
The change in x is the run.

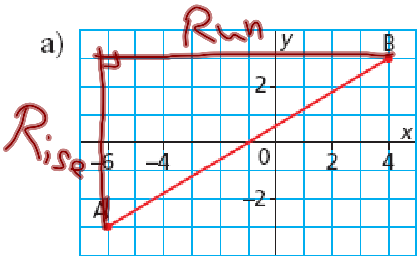
$$\text{So, slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{5\ \text{m}}{6\ \text{s}}$$

Example 1 Determining the Slope of a Line Segment

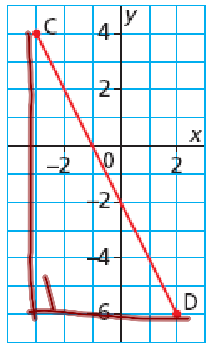
Determine the slope of each line segment.

a)



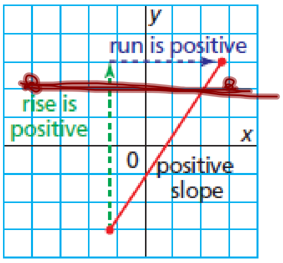
Slope $= \frac{6}{10} = \frac{3}{5}$

b)

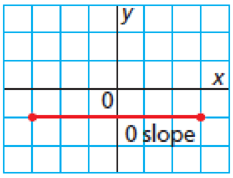


Slope $= \frac{-10}{5} = -2$

When a line segment goes up to the right, both y and x increase; both the rise and run are positive, so the slope of the segment is positive.



For a horizontal line segment, the change in y is 0 and x increases.
The rise is 0 and the run is positive. So, any horizontal line segment has slope 0.

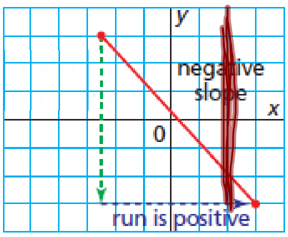


$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

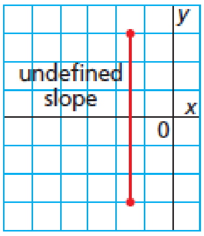
$$\text{Slope} = \frac{0}{\text{run}}$$

$$\text{Slope} = 0$$

When a line segment goes down to the right, y decreases and x increases; the rise is negative and the run is positive, so the slope of the segment is negative.



For a vertical line segment, y increases and the change in x is 0.
The rise is positive and the run is 0.

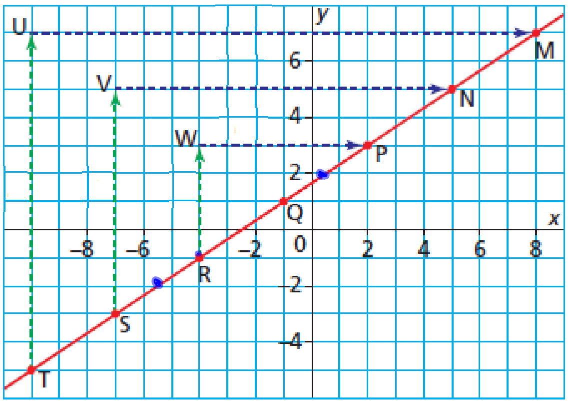


$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{\text{rise}}{0}$$

A fraction with denominator 0 is not defined.
So, any vertical line segment has a slope that is undefined.

We can show that the slopes of all segments of a line are equal.
On line MT, vertical and horizontal segments are drawn for the rise and run.
These segments form right triangles.
Consider the lengths of the legs
of these right triangles.



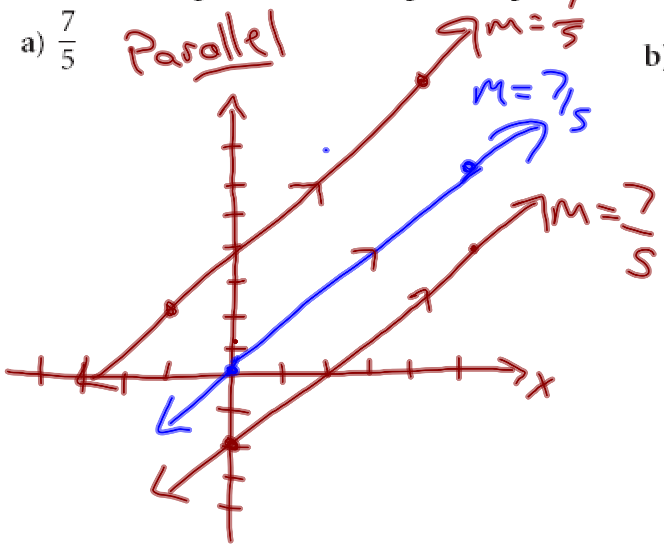
$\frac{TU}{UM} = ?$ $\frac{SV}{VN} = ?$ $\frac{RW}{WP} = ?$

?

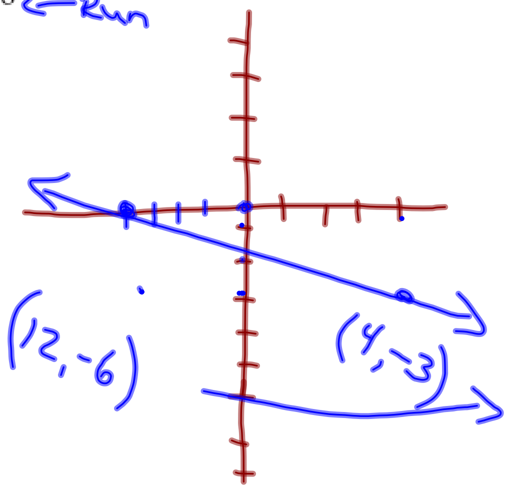
Example 2 Drawing a Line Segment with a Given Slope

Draw a line segment with each given slope.

a) $\frac{7}{5}$ Parallel



b) $-\frac{3}{8}$ ← Rise ← Run

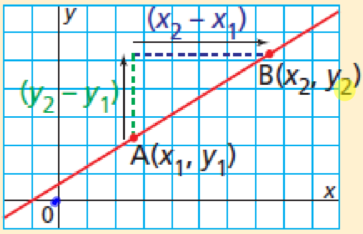


Example 3 leads to a formula we can use to determine the slope of any line.

Slope of a Line

A line passes through $A(x_1, y_1)$ and $B(x_2, y_2)$.

Slope of line $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$

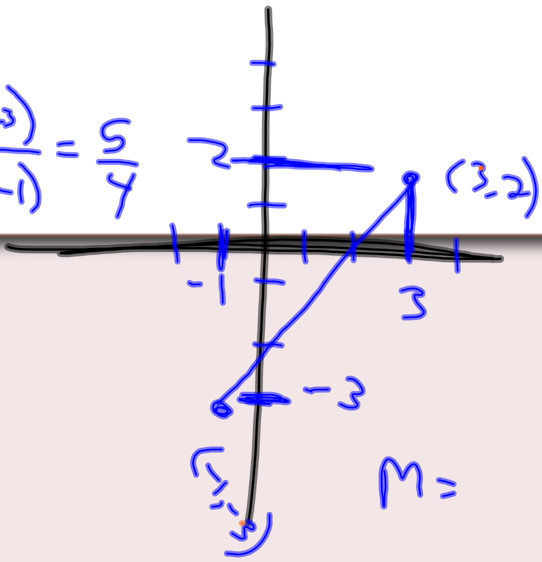


$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

A lowercase "m" is used to represent slope

$$m = \frac{2-(-3)}{3-(-1)} = \frac{5}{4}$$

$$= \frac{5}{4}$$



$m =$

$$m = \frac{-3-2}{-1-3} = \frac{-5}{-4} = \frac{5}{4}$$

Examples:

Determine the slope of a line passing through the following coordinate pairs:

$(-5,4)$ *and* $(3,-1)$

$m = \frac{4-(-1)}{-5-3} = \frac{5}{-8}$

$m = \frac{\Delta y}{\Delta x}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$(-4,-6)$ *and* $(-1,2)$

$m = \frac{2-(-6)}{-1-(-4)}$

$= \frac{8}{3}$

Example 4 Interpreting the Slope of a Line

Yvonne recorded the distances she had travelled at certain times since she began her cycling trip along the Trans Canada Trail in Manitoba, from North Winnipeg to Grand Beach. She plotted these data on a grid.

a) What is the slope of the line through these points? $m = \frac{4}{2} = 2$

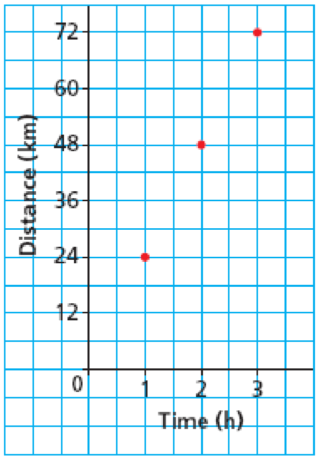
b) What does the slope represent? 2 km/h

c) How can the answer to part b be used to determine:

i) how far Yvonne travelled in $1\frac{3}{4}$ hours?

ii) the time it took Yvonne to travel 55 km?

Graph of a Bicycle Ride



CHECK YOUR UNDERSTANDING

SOLUTION