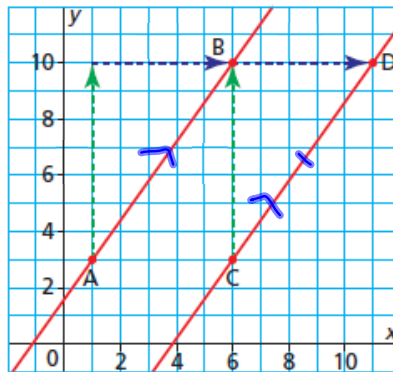When two lines have the same slope, congruent triangles can be drawn to show the rise and the run.
Lines that have the some slope are parallel.

Slope of AB = ?

Slope of CD = ?

?

Parallel lines MUST have equal slopes

Example 1   Identifying Parallel Lines

Line GH passes through $G(-4, 2)$ and $H(2, -1)$. Line JK passes through $J(-1, 7)$ and $K(7, 3)$. Line MN passes through $M(-4, 5)$ and $N(5, 1)$. Sketch the lines. Are they parallel? Justify the answer.

$$\underline{GH}$$

$$m = \frac{2-(-1)}{-4-2}$$

$$= \frac{3}{-6}$$

$$= -\frac{1}{2}$$

$$\underline{JK}$$

$$m = \frac{7-3}{-1-7}$$

$$= \frac{4}{-8}$$

$$= -\frac{1}{2}$$

$$\underline{MN}$$

$$m = \frac{5-1}{-4-5}$$

$$m = \frac{4}{-9}$$

$$\boxed{GH \parallel \overline{JK}}$$

The relationship between the slopes of AB and CD is true for any two oblique perpendicular lines. Horizontal and vertical lines are an exception.

The slope of a horizontal line is 0. The slope of a vertical line is $\frac{1}{0}$, which is not defined. So, the slopes of horizontal and vertical lines are not negative reciprocals.

Slopes of Perpendicular Lines

The slopes of two oblique perpendicular lines are negative reciprocals; that is, a line with slope a, $a \neq 0$, is perpendicular to a line with slope $-\frac{1}{a}$.
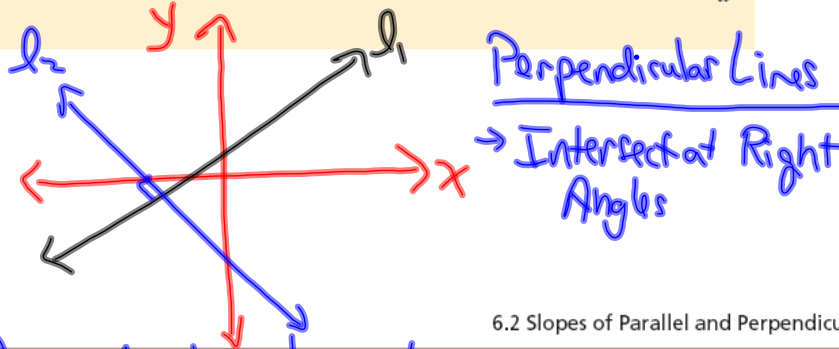
Perpendicular Lines
→ Intersect at Right Angles

Perpendicular lines have negative reciprocal Slopes

$EF \perp AB$

→ * Product of the slopes of $\perp$ lines must equal -1

$m_{EF} = -\frac{4}{5} \implies m_{AB} = \frac{5}{4}$

$\left(\frac{a}{b}\right)\left(-\frac{b}{a}\right) = -\frac{ab}{ab}$

$= -1$

Horizontal/Vertical line dilemma??

$m = \frac{0}{1}$

$m = -\frac{7}{0}$

m is undefined

3

Example 2 **Examining Slopes to Compare Lines**

Line PQ passes through P(-7, 2) and Q(-2, 10).
Line RS passes through R(-3, -4) and S(5, 1).

a) Are these two lines parallel, perpendicular, or neither? Justify the answer.

b) Sketch the lines to verify the answer to part a

$$M_{PQ} = \frac{8}{5} \qquad M_{RS} = \frac{-5}{-8} = \frac{5}{8}$$

$$\underline{Neither}$$

a) Determine the slope of a line that is perpendicular to the line through $E(2, 3)$ and $F(-4, -1)$.

b) Determine the coordinates of G so that line EG is perpendicular to line EF.

$$m_{EF} = \frac{3+1}{2+4}$$

$$m_{EF} = \frac{4}{6}$$

$$m_{EF} = \frac{2}{3}$$

$\perp$ to EF: $m = -\frac{3}{2} \leftarrow \Delta y$
$\qquad \qquad \qquad 2 \leftarrow \Delta x$

G

(b) $(6, -3)$ $\qquad (4, 0)$

$G(x, y)$ $\qquad m = -\frac{3}{2}$

$E(2, 3)$

$x - 2 = 2 \qquad y - 3 = -3$

$x = 4 \qquad \quad y = 0$

Example 4   **Using Slope to Identify a Polygon**

ABCD is a parallelogram. Is it
a rectangle? Justify the answer.

$$M_{AD} = \frac{4-2}{4+4} = \frac{2}{8} = \frac{1}{4}$$

$$m_{AB} = \frac{2-(-2)}{-4+3} = \frac{4}{-1} = -4$$
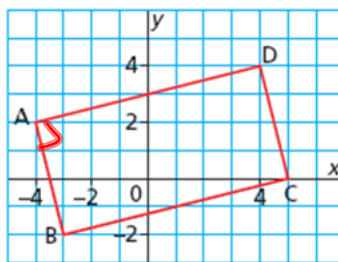
$$\therefore AD \text{ is } \perp \text{ to } AB$$

and thus all angles are 90°

Show that the triangle whose vertices have the coordinates **A** (3, 3), **B** (8, 17) & **C** (11, 5) is a right triangle.

$$M_{AB} = \frac{14}{5}$$

$$M_{BC} = \frac{12}{-3} = -4$$

$$M_{AC} = \frac{2}{8} = \frac{1}{4}$$

$$\therefore BC \perp \text{ to } AC$$

Try these...

The slopes of two lines are given as $3k - 5$ and $2(k - 3)$.
  (a) Determine the value of k if the lines are <mark>parallel</mark>

$$3k - 5 = 2(k - 3)$$
$$3k - 5 = 2k - 6$$
$$3k - 2k = -6 + 5$$
$$k = -1$$

  (b) Determine the value of k if the lines are <mark>perpendicular</mark>

$$\frac{(3k-5)^{-1}}{3k-5} = (2k-6)(3k-5)$$

Quadratic Equations

$$-1 = 6k^2 - 28k + 30$$
$$\longrightarrow 0 = 6k^2 - 28k + 31$$

$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{28 \pm \sqrt{(-28)^2 - 4(6)(31)}}{2(6)}$$

$$k = \frac{28 \pm \sqrt{40}}{12}$$

$$k = \frac{28 + \sqrt{40}}{12} \quad \text{OR} \quad k = \frac{28 - \sqrt{40}}{12}$$

Practice Problems...

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