Warm Up

If $h(x) = \dfrac{3}{\sqrt{1-3x}}$, find $h'(-1)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = \frac{3}{\sqrt{1-3(x+h)}} = \frac{3}{\sqrt{1-3x-3h}}$$

$$\lim_{h \to 0} \frac{\dfrac{3}{\sqrt{1-3x-3h}} - \dfrac{3}{\sqrt{1-3x}}}{h}$$

$$\lim_{h \to 0} \left[\frac{3\sqrt{1-3x} - 3\sqrt{1-3x-3h}}{(\sqrt{1-3x-3h})(\sqrt{1-3x})} \right] \cdot \frac{1}{h} \quad \left(\frac{3\sqrt{1-3x} + 3\sqrt{1-3x-3h}}{3\sqrt{1-3x} + 3\sqrt{1-3x-3h}} \right)$$

$$\lim_{h \to 0} \frac{9(1-3x) - 9(1-3x-3h)}{(\sqrt{1-3x-3h})(\sqrt{1-3x})(h)(3\sqrt{1-3x} + 3\sqrt{1-3x-3h})}$$

$$\lim_{h \to 0} \frac{\cancel{9} \cancel{-27x} \cancel{-9} \cancel{+27x} + 27\cancel{h}}{\sqrt{1-3x-3h}\sqrt{1-3x}\,(\cancel{h})(3\sqrt{1-3x} + 3\sqrt{1-3x-3h})}$$

$$-3h \to 0$$

$$= \frac{27}{(\sqrt{1-3x})^2 (6\sqrt{1-3x})}$$

$$= \frac{9}{2(1-3x)\sqrt{1-3x}} \rightarrow h'(-1) = \frac{9}{2(1-3(-1))\sqrt{1-3(-1)}}$$

$$= \frac{9}{2(4)(2)} = \frac{9}{16}$$

$$h(x) = \frac{3}{\sqrt{1-3x}}$$

$$h(x) = 3(1-3x)^{-1/2}$$

Chain Rule:

$$h'(x) = \frac{-3}{2}(1-3x)^{-3/2}(-3)$$

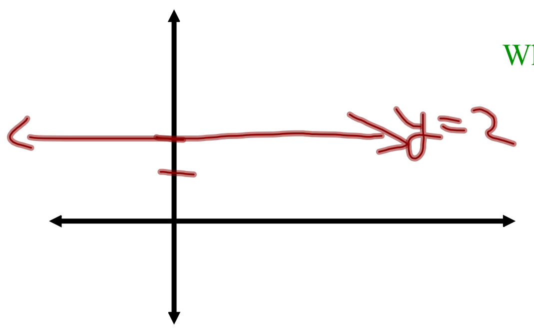
Unit Review: Limits

- Determining limits from a graph
- Evaluating indeterminate limits
- One-sided limits
- Continuity - Piecewise functions
- Limits at Infinity
- Limits of trigonometric functions
- Using limits to determine slope
- Definition of a derivative

Differentiation Rules

I. Constant Functions

- Sketch the function $y = 2$



What is the slope? = 0

The derivative of a constant will always be equal to "0".

Formal Proof:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{c - c}{h}$$

$$= \lim_{h \to 0} 0 = 0$$

II. Power Functions

We want to come up with a rule to differentiate functions of the form $f(x) = x^n$, $x \in R$

Here are a couple derivatives that we would have already looked at using limits:

$$\frac{d}{dx}(x^2) = 2x^1 \qquad \frac{d}{dx}(x^3) = 3x^2$$

$\frac{dy}{dx}$

Using the definition of a derivative to differentiate $f(x) = x^4$ would lead to ...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h\to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3) = 4x^3$$

$f(x) = x^{75}$
$f'(x) = 75x^{74}$

Do you see a pattern emerging?

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Let's practice using the power rule...

Differentiate each of the following functions:

1. $f(x) = x^{25}$

$f'(x) = 25x^{24}$
$f'(2) = ??$

2. $f(x) = x^{-5}$

$f'(x) = -5x^{-6}$

3. $f(x) = \frac{1}{x^{10}}$

$f(x) = x^{-10}$
$f'(x) = -10x^{-11}$

4. $f(x) = \sqrt[5]{x^7}$

$f(x) = x^{7/5}$
$f'(x) = \frac{7}{5}x^{2/5}$

Constant Multiples

- The following formula says that the derivative of a constant multiplied by a function is the constant multiplied by the derivative of the function:

The Constant Multiple Rule If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

$\cancel{f(x) = x^3\sqrt{x}}$

EXAMPLE 4

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}(x^4) = 3(4x^3) = 12x^3$

(b) $\frac{d}{dx}(-x) = \frac{d}{dx}[(-1)x] = (-1)\frac{d}{dx}(x) = -1(1) = -1$

$f(x) = 10x^7$

$f'(x) = 10(7x^6) = 70x^6$

Sums and Differences

- These next rules say that the derivative of a sum (difference) of functions is the sum (difference) of the derivatives:

The Sum Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

The Difference Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

1. $f(x) = 2x^5 - 3x^{-3} + 2\sqrt{x} - \frac{5}{x} + \pi^2$

$2x^{1/2}$

$5x^{-1}$

$f'(x) = 10x^4 + 9x^{-4} + x^{-1/2} + 5x^{-2} + 0$

2. $f(x) = \left(2x^3 - 5\right)^2$

$f(x) = 4x^6 - 20x^3 + 25$

$f'(x) = 24x^5 - 60x^2$

3. $f(x) = \frac{7\sqrt{x}}{4} - \frac{6}{x^5} + \frac{10x^3}{\sqrt[4]{x}} + 6x^{30} - ex + 9$

2.7...

$\frac{x^{3/1}}{x^{1/4}} = \frac{x^{12/4}}{x^{1/4}}$

$= x^{11/4}$

$f(x) = \frac{7}{4}x^{1/2} - 6x^{-5} + 10x^{11/4} + 6x^{30} - ex^1 + 9$

$\frac{110}{4}$

$f'(x) = \frac{7}{8}x^{-1/2} + 30x^{-6} + \frac{55}{2}x^{7/4} + 180x^{29} - e$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}\left[\overset{\text{1st}}{f(x)}\,\overset{\text{2nd}}{g(x)}\right] = \overset{\text{1st}}{f(x)}\,\overset{\text{der}}{\frac{d}{dx}[g(x)]} + \overset{\text{2nd}}{g(x)}\,\underbrace{\frac{d}{dx}[f(x)]}_{\text{der of 1st}}$$

Express the product rule verbally if you are considering a function of the form...

$f(x)$= (First) x (Second)

" The derivative of the product of two functions is the derivative of the first multiplied by the second, plus the first multiplied by the derivative of the second"

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

Examples:

$$f(x) = \underbrace{(7x^3 - x^2 + 5)}_{1^{\text{st}}}\underbrace{(x^9 + 3x - 5)}_{2^{\text{nd}}}$$

$$f'(x) = (21x^2 - 2x)(x^9 + 3x - 5) + (7x^3 - x^2 + 5)(9x^8 + 3)$$

$$h(t) = (t^3 - 5t)(6\sqrt{t} - t^{-5})$$

$$h'(t) = (3t^2 - 5)(6t^{1/2} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

$$g(x) = \underbrace{(7x^3 - 5)(4x^2 - 2x + 3)}_{1^{\text{st}}}\underbrace{(9 - x^6)}_{2^{\text{nd}}}$$

$$f(x) = (x^{12} - 5x^3)(7x^3 + x - 5)(4\sqrt{x} + 2)$$