# Warm Up

Evaluate the following limits, if they exist:

1. $\lim_{x\to 8} \frac{x-8}{\sqrt[3]{x}-2}$

$\lim_{x\to 8} \frac{\cancel{(\sqrt[3]{x}-2)}(x^{2/3}+2\sqrt[3]{x}+4)}{\cancel{\sqrt[3]{x}-2}}$

$= 8^{2/3}+2\sqrt[3]{8}+4 = 12$

2. $\lim_{x\to 3} \frac{x^{-2}-3^{-2}}{x-3}$

$\lim_{x\to 3} \frac{\left(\frac{1}{x^2}-\frac{1}{9}\right)}{x-3}$

$\lim_{x\to 3} \left(\frac{9-x^2}{9x^2}\right)\cdot\frac{1}{(x-3)}$

$\lim_{x\to 3} \frac{\overset{-1}{\cancel{(3-x)}}(3+x)}{9x^2\cancel{(x-3)}}$

$= \frac{-1(3+3)}{9(3)^2} = \frac{-6}{81} = \frac{-2}{27}$

3. $\lim_{x\to 1} \frac{x^3-1}{x^3-x^2-4x+4}$ $\quad \frac{0}{0}$

Denominator:

$x^3-x^2-4x+4$

$x^3-4x-x^2+4$

$x(x^2-4)-1(x^2-4)$

$(x^2-4)(x-1)$

$(x-2)(x+2)(x-1)$

OR

Factor Theorem

$(x-1)$ is a factor

Synthetic Division:

Remainder

$$\begin{array}{r|rrr|r} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$

$(x-1)(x^2-4)$

$(x-1)(x-2)(x+2)$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x-2)(x+2)}$

$= \frac{1+1+1}{(1-2)(1+2)} = \frac{3}{-3} = -1$

4. $\lim\limits_{h \to 0} \dfrac{(a+h)^2 - a^2}{h}$

$$\lim_{h \to 0} \frac{[(a+h)-a][(a+h)+a]}{h}$$

$$\lim_{h \to 0} \frac{\cancel{(h)}(2a+h)}{\cancel{h}} = 2a+0$$

$$= 2a$$

Pg. 19

5.e) $\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h}\left(\frac{\sqrt{9+h}+3}{\sqrt{9+h}+3}\right)$

$\lim_{h\to 0} \frac{\cancel{(9+h)-9}\ \cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}$

$= \frac{1}{\sqrt{9+0}+3} = \frac{1}{6}$

#9b) $\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}\left(\frac{\sqrt{6-x}+2}{\sqrt{6-x}+2}\right)\left(\frac{\sqrt{3-x}+1}{\sqrt{3-x}+1}\right)$

$$\lim_{x\to 2} \frac{[(6-x)-4]\left(\sqrt{3-x}+1\right)}{[(3-x)-1]\sqrt{6-x}+2}$$

$$\lim_{x\to 2} \frac{\cancel{(2-x)}\left(\sqrt{3-x}+1\right)}{\cancel{(2-x)}\left(\sqrt{6-x}+2\right)}$$

$$= \frac{\sqrt{3-2}+1}{\sqrt{6-2}+2} = \frac{2}{4} = \frac{1}{2}$$

# Recall from our prior discussions that ...

**1 Theorem** $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$

Let's look at a couple of unique functions:

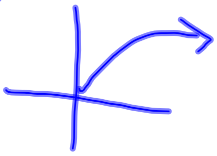
[3]

$y = \sqrt{x-1}$

1) $\lim_{x \to 1} \sqrt{x-1}$

$y = \sqrt{x}$

$\lim_{x \to 1^-} \sqrt{0.999... - 1}$



$\sqrt{-\#}$

$\therefore$ DNE

$y = \sqrt{x-1}$

$\therefore \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\therefore \lim_{x \to 1} f(x)$ D.N.E

2) $\lim\limits_{x \to -2} \frac{|x+2|}{x+2}$


$$\lim_{x \to -2^-} \frac{|-2.000..1 + 2|}{-2.000..1 + 2} = \frac{|-0.00...1|}{-0.000...1} = \frac{0.0000...1}{-0.000...1} = -1$$

$$\lim_{x \to -2^+} \frac{|-1.999 + 2|}{-1.999.. + 2} = \frac{0.000..1}{0.000..1} = 1$$

$$\therefore \lim_{x \to -2} \frac{|x+2|}{x+2}$$

Does Not Exist

# Piecewise Defined Functions

**Definition:**

- **Functions defined by different formulas in different parts of their domains**

Example:

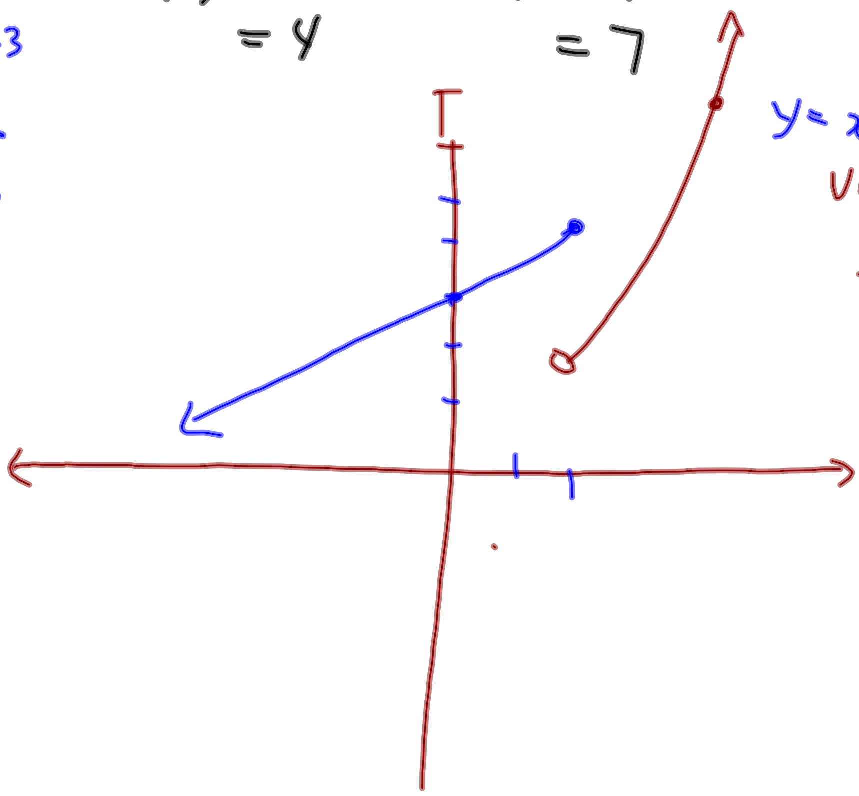
$$f(x) = \begin{cases} x+3 & if\ x \le 2 \\ x^2 - 2 & if\ x > 2 \end{cases}$$

1) Determine $f(1)$ , $f(3)$, and $f(2)$.
2) Sketch $f(x)$.

① $f(1) = 1+3 = 4$ $\quad$ $f(3) = (3)^2 - 2 = 7$ $\quad$ $f(2) = 2+3 = 5$

$y = x+3$

| x | y |
|---|---|
| 2 | 5 |
| 0 | 3 |



$y = x^2 - 2$

Vertex: $(0,-2)$

| x | y |
|---|---|
| 2 | 2 |
| 3 | 7 |