WARM UP

Solve the following quadratic equation by... (a) factoring
(b) completing the square
(c) using the quadratic formula

$$6x^2 - 13x = 5$$

$\to x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

a) $6x^2 - 13x - 5 = 0$ $\quad -30$

$6x^2 - 15x + 2x - 5 = 0$

$3x(2x-5) + 1(2x-5) = 0$

$(2x-5)(3x+1) = 0$

$2x - 5 = 0 \qquad 3x + 1 = 0$

$2x = 5 \qquad 3x = -1$

$x = \frac{5}{2} \qquad x = -\frac{1}{3}$

(b) $6x^2 - 13x = 5$

$6\left(x^2 - \frac{13}{6}x + \frac{169}{144}\right) = \frac{5}{1} + \frac{169}{24}$ $\quad +\frac{169}{24}$

$\left(\frac{1}{6}\right)\cancel{6}\left(x - \frac{13}{12}\right)^2 = \frac{289}{24}\left(\frac{1}{6}\right)$

$\sqrt{\left(x - \frac{13}{12}\right)^2} = \sqrt{\frac{289}{144}}$

$x - \frac{13}{12} = \pm\frac{17}{12}$

$x = \frac{13}{12} \pm \frac{17}{12}$

$x = \frac{30}{12}$ or $x = -\frac{4}{12}$

$x = \frac{5}{2} \qquad x = -\frac{1}{3}$

Solve:

$\frac{3(x+2)^2}{3} = \frac{21}{3}$

$\sqrt{(x+2)^2} = \sqrt{7}$

$x + 2 = \pm\sqrt{7}$

$x = -2 \pm \sqrt{7}$

(c) $6x^2 - 13x - 5 = 0$

$x = \frac{13 \pm \sqrt{169 - 4(6)(-5)}}{2(6)}$

$x = \frac{13 \pm \sqrt{289}}{12}$

$x = \frac{13 \pm 17}{12}$

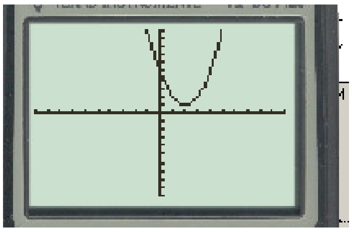
$x = \frac{5}{2}$ or $x = -\frac{1}{3}$

Non-Real Roots

- **What if it is not possible to factor a quadratic equation and you cannot use completing the square or quadratic formula because there is a negative under the radical sign????**

There are no *x*-intercepts!!!!!!!

EXAMPLE: $y = x^2 - 4x + 5$, $x \in \mathbb{C}$



Look, it doesn't cross the *x*-axis

What happens if I try to use the quadratic formula?

$$x = \frac{-(-4) \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

a =
b =
c =

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

Impossible??

$\sqrt{9} = \sqrt{3 \times 3} = 3$

$$x = \frac{4 \pm \sqrt{4i^2}}{2}$$

$$x = \frac{4 \pm 2i}{2}$$

$x = 2 \pm i$

$x = 2 + i$ OR $x = 2 - i$

Complex Numbers

- If a quadratic does not have any x-intercepts, we say that there are "no real roots".
- There are two roots from the imaginary number system.
- Imaginary Number System.....Huh????

Foundation of Complex Number System →

If $i^2 = -1$

then, $i = \sqrt{-1}$

A number system developed by Leonard Euler and popularized by Carl Friedrich Gauss

complex number: $a + bi$, where a & $b \in R$ and $i \in C$

Real Part Imaginary Part

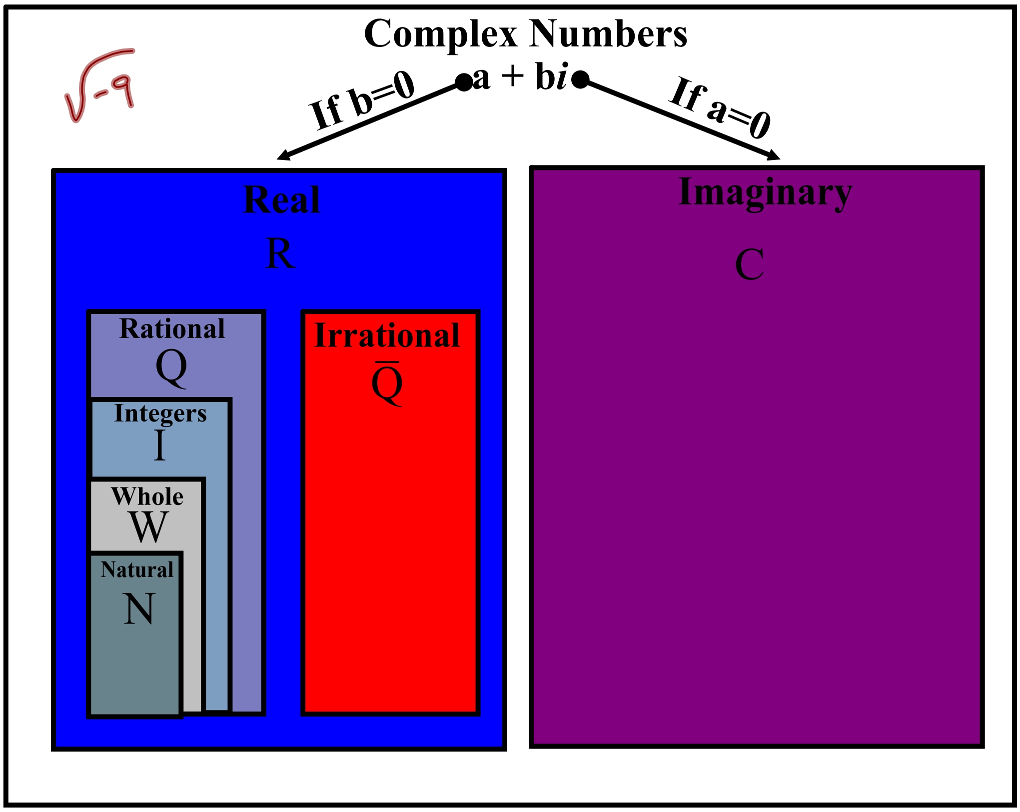
ex. 1) $7 - 2i$

2) $-2 + i$

3) $7i$

4) π

"The Complete Number System"



$\sqrt{-9}$

→ Move into complex # system

$i^2 = -1$

$\sqrt{9(-1)}$

$\sqrt{9i^2}$

$\pm 3i$

$\sqrt{-25}$

$\sqrt{25i^2}$

$\pm 5i$

Example #2: **Solve the following equation. If roots are non-real, express them as complex roots.**

$$2x^2 -6x + 17 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(17)}}{2(2)}$$

a =

b =

c =

$$x = \frac{6 \pm \sqrt{36-136}}{4}$$

$$x = \frac{6 \pm \sqrt{-100}}{4}$$

$$x = \frac{6 \pm \sqrt{100i^2}}{4}$$

$i^2 = -1$

$$x = \frac{6 \pm 10i}{4}$$

$$\cancel{x = \frac{2 \pm 3i}{2}}$$

$$\cancel{x = 1 \pm 3i}$$

$$x = \frac{3}{2} \pm \frac{5}{2}i$$

$$x = \frac{3}{2} + \frac{5}{2}i \text{ or } x = \frac{3}{2} - \frac{5}{2}i$$

Example #3: **Solve the following equation. If roots are non-real, express them as complex roots.**

$$\cancel{(3-x)}(x+2)\frac{3}{\cancel{3-x}}+\frac{4}{\cancel{x+2}}(3-x)\cancel{(x+2)}=4(3-x)(x+2)$$

$3(x+2)+4(3-x)=4(3-x)(x+2)$

$3x+6+12-4x=4(3x+6-x^2-2x)$

$18-x=-4x^2+4x+24$

-24

$4x^2-5x-6=0$

$4x^2-8x+3x-6=0$

$4x(x-2)+3(x-2)=0$

$(x-2)(4x+3)=0$

$x=2,-\frac{3}{4}$

Example #4: Solve the following equation. If roots are non-real, express them as complex roots.

$$\frac{3\sqrt{2}-x}{\sqrt{x}} = \frac{\sqrt{2x}}{\sqrt{2}-x}$$

Bonus

HOMEWORK...

page 47: #17 (2nd column)

page 49: #30 (2nd column)
#31 (b & c)
#32 (1st column)

Page 47...

SOLUTIONS...

#17. $a)\ -\frac{3}{4}\ \&\ 2$ $\quad b)\ \frac{-9\pm\sqrt{26}}{5}$

$c)\ \frac{-5\pm\sqrt{70}}{3}$ $\quad d)\ \frac{9\pm\sqrt{129}}{4}$

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#30. $a)\ \frac{-3\pm\sqrt{89}}{8}$ $\quad b)\ \frac{3\pm\sqrt{41}}{8}$

$c)\ 4\pm\sqrt{22}$ $\quad d)\ \frac{-3\pm\sqrt{137}}{8}$

$e)\ 3\pm2\sqrt{3}$ $\quad f)\ \frac{1\pm\sqrt{5}}{2}$

$g)\ \frac{-1\pm i\sqrt{11}}{2}$ $\quad h)\ \frac{2\pm i\sqrt{14}}{2}$

#31. $a)\ -0.986\ \&\ 2.510$ $\quad b)\ -1.750\ \&\ 0.667$

$c)\ -\frac{\sqrt{2}}{4}\ \&\ -\sqrt{2}$ $\quad d)\ \frac{\sqrt{3}\pm i\sqrt{33}}{6}$

#32. $a)\ \frac{-5\pm\sqrt{145}}{12}$ $\quad b)\ \frac{-3\pm i\sqrt{39}}{4}$

$c)\ \frac{2\pm\sqrt{13}}{9}$ $\quad d)\ -2\ \&\ -\frac{3}{4}$

$e)\ \frac{-\sqrt{6}\pm\sqrt{14}}{2}$ $\quad f)\ \frac{5\sqrt{2}\pm\sqrt{26}}{2}$

$g)\ \frac{-3\pm\sqrt{29}}{2}$