Solve the following Inequations:

① $3x^2-10x>8$

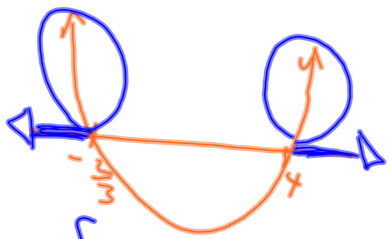
$3x^2-10x-8>0$

$3x^2-12x+2x-8>0$

$3x(x-4)+2(x-4)>0$

$(x-4)(3x+2)>0$ Above x-axis

Zeros: $x=4,\ -\frac{2}{3}$



$\{x \mid x<-\frac{2}{3} \text{ or } x>4,\ x\in\mathbb{R}\}$

Cases: $(+,+)$

$x-4>0$; $3x+2>0$

$x>4$; $3x>-2$

$x>-\frac{2}{3}$

$x>4$

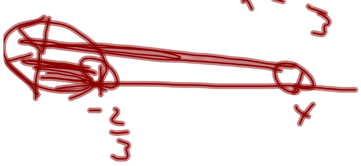
OR

II $(-,-)$

$x-4<0$; $3x+2<0$

$x<4$; $3x<-2$

$x<-\frac{2}{3}$



$x<-\frac{2}{3}$

② $-x^2+7\le 2x$

$-1\ (-x^2-2x+7)\le(0)^{-1}$

$x^2+2x-7\ge 0$

Find Zeros:

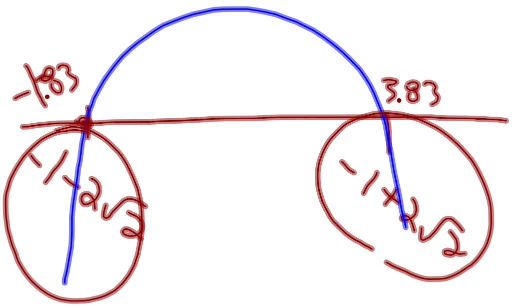
$$x=\frac{2\pm\sqrt{4-4(-1)(7)}}{2(-1)}$$

$$x=\frac{2\pm\sqrt{32}}{-2} \rightarrow 5.7$$

$$x=\frac{2\pm4\sqrt{2}}{-2}$$

$x=-1\pm2\sqrt{2}$

$x=-1+2\sqrt{2}$; $x=-1-2\sqrt{2}$



$\{x \mid x\le(-1-2\sqrt{2}) \text{ or } x\ge(-1+2\sqrt{2}),\ x\in\mathbb{R}\}$

1. e) $\sqrt{2}\,x^2 - 5x - \sqrt{8} = 0$

$$x = \frac{5 \pm \sqrt{25 - 4(\sqrt{2})(-\sqrt{8})}}{2\sqrt{2}}$$

$$x = \frac{5 \pm \sqrt{25 + 16}}{2\sqrt{2}}$$

$$x = \frac{5 \pm \sqrt{41}}{2\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$x = \frac{5\sqrt{2} \pm \sqrt{82}}{4}$$

The Nature of the Roots

- any quadratic equation can be solved using the quadratic formula...

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = \frac{-5 \pm (6)}{2}$

Discriminant

- enables you to determine the "Nature of the Roots" without actually finding the roots.

$D = b^2 - 4ac$

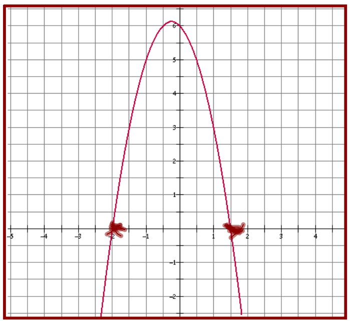
- there are THREE cases...

CASE #1: Real and Unequal Roots

- this happens when Discriminant > 0.
- the quadratic will have two real and unequal roots.

NOTE: If the discriminant is a perfect square, then the roots will be RATIONAL.
Otherwise, the roots will be IRRATIONAL. $\bar{Q}$

EXAMPLE: $2x^2 - x - 6 = 0$



Calculate the discriminant value

$D = b^2 - 4ac$

$D = 1 - 4(2)(-6)$

$D = 49$

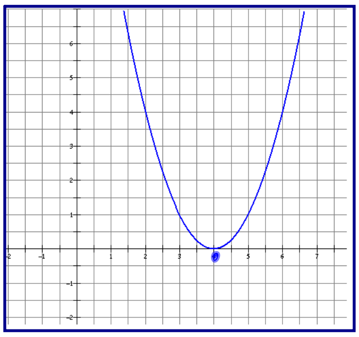
→ Real, Rational, Unequal

R Q

CASE #2: Real and Equal Roots

- this happens when Discriminant = 0. $x = \frac{3 \pm 0}{5}$
- the quadratic will have two real and equal roots (one real root).

EXAMPLE: $x^2 - 8x + 16 = 0$



Calculate the discriminant value

$D = b^2 - 4ac$

$D = 64 - 4(1)(16)$

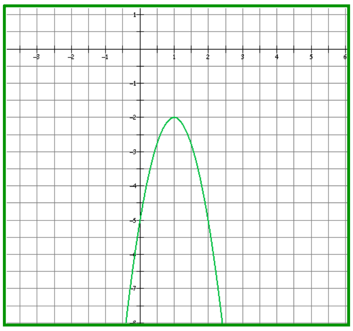
$D = 0$

Real, Equal (One Root)

CASE #3: Non-Real and Unequal Roots

- this happens when Discriminant < 0.
- the quadratic will have two non-real and unequal roots (imaginary/complex roots)

EXAMPLE: $-3x^2 + 6x - 5 = 0$



Calculate the discriminant value

$D = b^2 - 4ac$

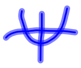
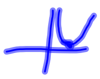
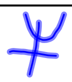
$D = 36 - 4(-3)(-5)$

$D = 36 - 60$

$D = (-\#)$

Complex Roots, Distinct

SUMMARY: Nature of the Roots

Value of the Discriminant			
$D = b^2 - 4ac$	**Real or Non-real**	**Equal or Unequal**	**Rational or Irrational**
1. $D > 0$ but not a perfect square 	Real	Unequal	Irrational
2. $D>0$ and is a perfect square 	Real	Unequal	Rational
3. $D = 0$ 	Real	Equal	Rational
4. $D < 0$ 	Non-real	Unequal	n/a

Sample Problem 1:

Describe the nature of the roots, if

a) $3x^2 + 6x + 1 = 0$

$D = 36 - 4(3)(1)$
$D = 24$

Real, Unequal, Irrational

b) $x^2 - 5x + 7 = 0$

$D = 25 - 4(1)(7)$
$D = -3$

Imaginary, Unequal

Another Example: $5x^2 - 5x - 1 = 0$

Sample Problem #2:

$ax^2+bx+c=0$

Find the value(s) of k so that $1x^2 + \underbrace{(k - 1)}_{b}x - k = 0$ has equal roots. $\rightarrow D=0$ $a=1$

$(k-1)^2-4(1)(-k)=0$

$k^2-2k+1+4k=0$

$k^2+2k+1=0$

$(k+1)^2=0$

Check: $k=-1$

$x^2-2x+1=0$

$(x-1)^2=0$

$x=1$

HOMEWORK...

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#58 c, e 61, 62, 63, 64

Solutions:

58 (c) $x^2+4x+1=0$

(e) $x^2-2x+3=0$

61. a) $t=-2$ or 6

b) $\{t>6$ or $t<-2\}$

c) $\{-2<t<6\}$

62. $p=-\frac{1}{4}$

63. Discriminant can not equal $\underline{\underline{0}}$.

64. $k=\frac{1}{4}$