Functions and Relations 112 Name: ______________

Optional Assignment – Review of Quadratics March 2011

1. Complete the chart shown for the quadratic: $f(x) = -2x^2 - 6x + 20$ [14]

• Complete the square (G → S) to find vertex...

$f(x) = -2(x^2+3x)+20$ (1)

$= -2(x^2+3x+\frac{9}{4}) - \frac{9}{4}(-2)+20$

$= -2(x+3/2)^2 + \frac{9}{2} + \frac{20 \cdot 2}{2}$ (1)

$= -2(x+3/2)^2 + \frac{49}{2}$ (1)

$f(x) = -2(x+1.5)^2 + 24.5$

• Solve quadratic equation to find x-int(s)

$0 = -2x^2 - 6x + 20$

$\div -2 \quad \div -2 \quad \div -2 \quad \div -2$

$0 = x^2 + 3x - 10$

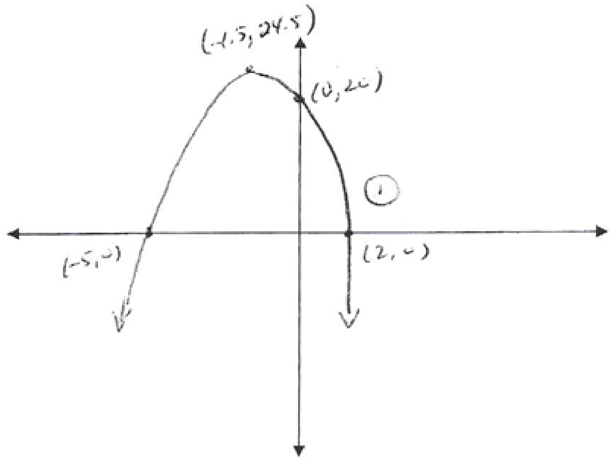
$\times \to -10 \; \{5, -2$
$+ \to 3$

$0 = (x+5)(x-2)$ (1)

$x = -5$ & $x = 2$

Vertex	$(-1.5, 24.5)$	(1)
Equation of axis of symmetry	$x = -1.5$	(1)
Domain	$x \in R$	(1)
Range	$y \leq 24.5$	(1)
x-intercept(s)	$(-5, 0)$ & $(2, 0)$	(1)
y-intercept	$(0, 20)$	(1)
Rule that maps $y = x^2$ to $f(x)$	$(x, y) \to (x - 1.5, -2y + 24.5)$ (1) (1)	

(1) Sketch
(label all key points found above)



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$-12 \div -\frac{2}{3}$

(6)

2. Express the function $y = -\frac{2}{3}x^2 - 12x + 1$ in transformational and standard form. [5]

$-12x \cdot -\frac{3}{2} = \frac{36}{2} = 18$

$-12 \div -\frac{2}{3}$
$-12x - \frac{3}{2}$
18

$y = -\frac{2}{3}(x^2+18x)+1$ (1)

$y = -\frac{2}{3}(x^2+18x+81) - 81(-\frac{2}{3}) + 1$ (1)

$y = -\frac{2}{3}(x+9)^2 + 54 + 1$

$y = -\frac{2}{3}(x+9)^2 + 55$ (S)

$y = -\frac{2}{3}(x+9)^2 + 55$

$y - 55 = -\frac{2}{3}(x+9)^2$ (1)

$-\frac{3}{2}(y-55) = (x+9)^2$ (T)

Standard Form: $y = -\frac{2}{3}(x+9)^2 + 55$ (1) Transformational Form: $-\frac{3}{2}(y-55) = (x+9)^2$ (1)

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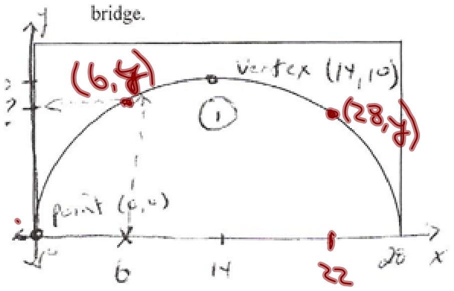
3. A parabolic arch supports a bridge that is spanning a stream. The bridge is 28 m wide and the furthest distance from arch to the stream is 10 m. Determine the distance from the stream up to the arch at a point 6 m from either end of the bridge. [4]



$y = a(x-14)^2 + 10$ (1)

$0 = a(0-14)^2 + 10$

$\frac{-10}{196} = \frac{196a}{196}$

$\frac{-5}{98} = a$

$-0.051 = a$ (1)

$y = \frac{-5}{98}(x-14)^2 + 10$

Let $x = 6$ to get y....

$y = \frac{-5}{98}(6-14)^2 + 10$ (1)

$y = \frac{-5}{98}(64) + 10$

$y = 6.73$ m

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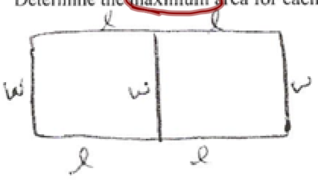
Distance = $\boxed{6.73\text{m}}$

4. Two equal rectangular playing fields, having one side in common, are to be enclosed and divided using 1200m of fencing. Determine the maximum area for each of the two fields. [5]



$3w + 4l = 1200$ (1)

$\frac{4l}{4} = \frac{1200-3w}{4}$

$l = \frac{1200-3w}{4}$

$A = 2lw$

$= 2\left(\frac{1200-3w}{4}\right)w$

$= 600w - \frac{3}{2}w^2$ (1)

$= -\frac{3}{2}w^2 + 600w$

$= -\frac{3}{2}(w^2 - 400w)$ (1)

$= -\frac{3}{2}(w^2 - 400w + 40000) - 40000\left(-\frac{3}{2}\right)$

$= -\frac{3}{2}(w-200)^2 + 60000$ (1)

vertex $(200, 60000)$

w A_{total}

$l = \frac{1200 - 3(200)}{4}$

$l = 150$

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Dimensions: 150m x 200m (1)
(one field)

Max Area: $\boxed{30000\text{m}^2}$
(one field)

5. Solve each of the following quadratic equations… [12]
note: - *if roots are non-real, express them as complex roots*
- *if roots are irrational, simplify the radical completely*

a) $4x^2 + 15 = 17x$

$x \to 60, + \to -17$ } $-12, -5$

$4x^2 - 17x + 15 = 0$

$4x^2 - 12x - 5x + 15 = 0$ (1)

$4x(x-3) - 5(x-3) = 0$

$(x-3)(4x-5) = 0$ (1)

$x - 3 = 0$ $4x - 5 = 0$

$\boxed{x = 3}$ (1) $4x = 5$

$\boxed{x = \frac{5}{4}}$

b) $x = 3 - \frac{8}{x+6}$

$x(x+6) = 3(x+6) - 8$

$x^2 + 6x = 3x + 18 - 8$

$x^2 + 6x - 3x - 10 = 0$

(1) $x^2 + 3x - 10 = 0$ $x \to -10, + \to 3$ } $5, -2$

(1) $(x+5)(x-2) = 0$

(1) $\boxed{x = -5}$ or $\boxed{x = 2}$

c) $4x^2 - 12x - 11 = 0$ $x \to -44, + \to -12$ } DNF

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-11)}}{2(4)}$ (1)

$= \frac{12 \pm \sqrt{144 + 176}}{8}$

$= \frac{12 \pm \sqrt{320}}{8}$ (1)

* $\sqrt{320}$
$\sqrt{64(5)}$
$8\sqrt{5}$

$= \frac{12 \pm 8\sqrt{5}}{8}$

$\therefore \boxed{x = \frac{3 \pm 2\sqrt{5}}{2}}$ (1)

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d) $x^2 + 14x + 74 = 0$ $x \to 74, + \to 14$ } DNF

$x = \frac{-14 \pm \sqrt{14^2 - 4(1)(74)}}{2(1)}$

$\sqrt{100i^2}$

$= \frac{-14 \pm \sqrt{196 - 296}}{2}$ (1)

* $\sqrt{-100}$
$10i$

$= \frac{-14 \pm \sqrt{-100}}{2}$ (1)

$= \frac{-14 \pm 10i}{2}$

$\therefore \boxed{x = -7 \pm 5i}$ (1)

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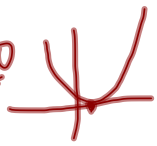
Ex. Find "K" such that the graph of

$y = 9x^2 + 3Kx + K$

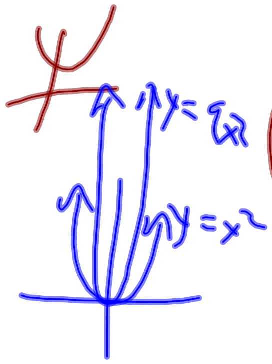
(a) is tangent to the x-axis $D = 0$



(b) has real & distinct Roots $D > 0$

(c) Does Not cross the x-axis

$D < 0$



Discriminant

$D = b^2 - 4ac$

$D = (3k)^2 - 4(9)(k)$

(a) $9k^2 - 36k = 0$

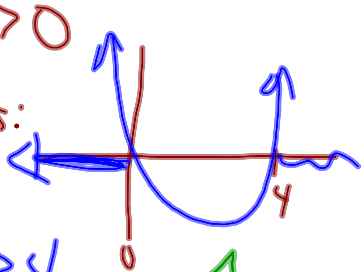
$9k(k-4) = 0$

$9k = 0$ or $k - 4 = 0$

$k = 0$ $\quad$ $k = 4$

(b) $9k^2 - 36k > 0$

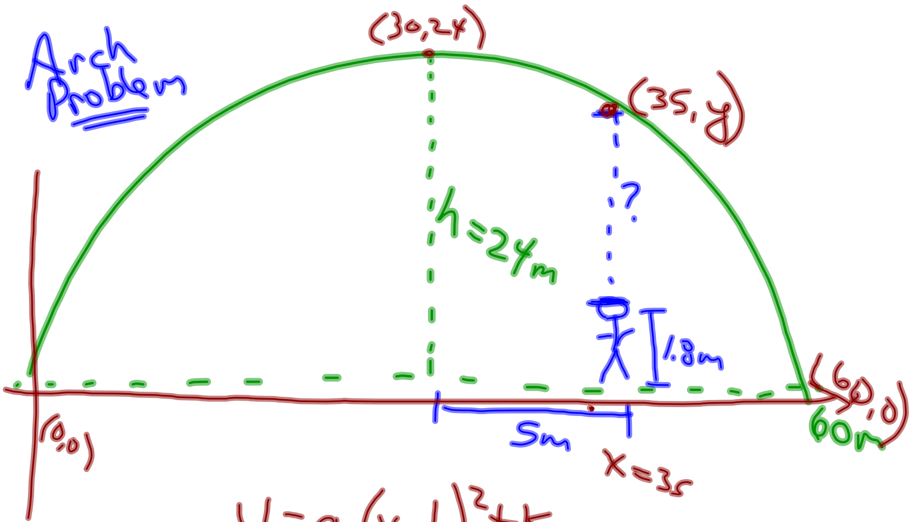
① Plot Zeros:



$k < 0$ or $k > 4$

(b) $9k^2 - 36k < 0$

$0 < k < 4$



$$y = a(x-h)^2 + k$$

$$0 = a(0-30)^2 + 24$$

$$\frac{-24}{900} = a \Longrightarrow y = \frac{-24}{900}(x-30)^2 + 24$$

at $x = 35$ $\quad y = -\frac{24}{900}(35-30)^2 + 24$

$$y = 23.\overline{3}$$

Clearance: $23.\overline{3}\text{m} - 1.8\text{m}$

$$= 21.5\text{m}$$

Passes through $(0,-1)$, $(3,-7)$ & $(5,-31)$
Determine function is general form.

$$y = ax^2 + bx + c$$

Sub. $(0,-1)$:
$$-1 = 0 + 0 + c$$
$$c = -1$$

Sub $(3,-7)$:
$$-7 = a(3)^2 + b(3) - 1$$
$$\boxed{-6 = 9a + 3b} \Longrightarrow 3b = -6 - 9a$$
$$b = -2 - 3a$$

Sub. $(5,-31)$:
$$-31 = a(5)^2 + b(5) - 1$$
$$\boxed{-30 = 25a + 5b}$$

$$-30 = 25a + 5(-2-3a)$$
$$-30 = 25a - 10 - 15a$$
$$-20 = 10a$$
$$-2 = a$$

$$b = -2 - 3(-2)$$
$$= 4$$

$$y = -2x^2 + 4x - 1$$