6. The number of board feet in a 16 foot long log is approximated by the model $F(d) = 0.8d^2 - 1.4d + 9.6$ where "F" is the number of feet and "d" is the diameter of the log in inches.

function

$F*(d)$

(a) How many board feet are in a 48 foot long log with diameter 12 inches? [3]

$F(12) = 0.8(12)^2 + 1.4d + 9.6$

$= 141.6$ feet $\times 3$

$= 424.8$ feet

(b) What is the **minimum** board feet that could be produced from a 16 foot log? [4]

$F(d) = 0.8d^2 + 1.4d + 9.6$

0.76525

$= 0.8(d^2 + 1.75d + \_\_\_) + 9.6 - 0.6125$

$= 0.8(d + 0.875)^2 + 8.9875$

$V(-0.875, 8.9875)$

(d, F)

Minimum 8.9875 feet

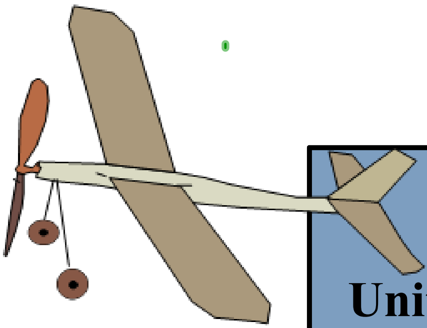
BONUS

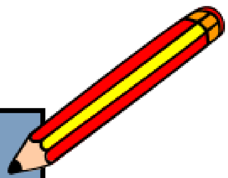
On a 42 km go-kart course Patrick goes 0.4 km/h faster than Joshua, but Patrick has trouble with his back wheels so he stops to fix them. This costs Patrick half an hour, so he arrives 15 minutes after Joshua. Determine how long it took each of them to complete the course. [4]

	d (km)	s (km/h)	t (h)
P	42	$x+0.4$	$\frac{42}{x+0.4} + 0.5$
J	42	x	$\frac{42}{x}$

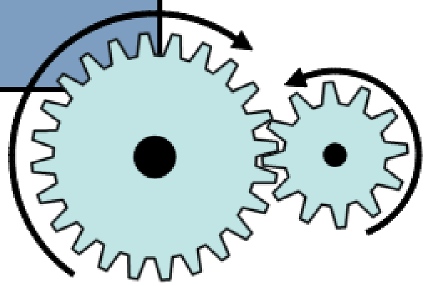
$\frac{15}{60}$

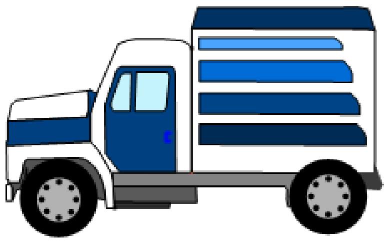
$$\left(\frac{42}{x+0.4} + 0.5\right) - \frac{42}{x} = \frac{1}{4}$$





Unit 2 - Rate of Change





Introduction…

Unit #2 - Rate of Change
An Introduction to Rates of Change

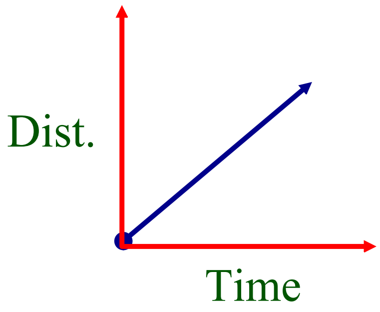
When you discuss with others the speed at which someone was driving, you mention kilometres **per** hour or miles **per** hour. When you discuss how much you are paid, you talk about earning so much **per** hour, **per** week, **per** lawn or **per** task. From discussions held in your Physical Education class, you know that an increase in physical activity will cause your heart to work harder and this will be measured in beats **per** minute.

These are all rates; rates in which we compare the change in the dependent variable of a function to the change in the independent variable. That is why you hear **"per"** when we talk about a rate. Dollars per week, metres per second, beats per minute, etc., tell you which two variables are being compared. The **"per"** tells you that the changes in the two variables are being compared. Since these rates compare how the two variables are changing in the relation of one to the other, we call this comparison a .

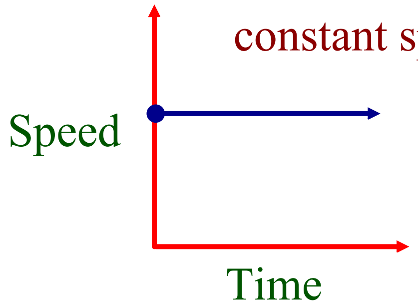
These are not the only rates we hear discussed every day. Revolutions per minute (rpm of a motor), pounds per square inch of pressure, birth rates per thousand people, rate of return or rate of interest measured in percent per day, month or year are all examples of rates. When we go to the sports pages, we hear more rates. Think of the many stats in baseball, the shooting percentages in basketball, save percentages or power play efficiency in hockey, yards per carry in football or serving percentages in volleyball or tennis; all of these are examples of rates and I am sure you can think of many more.

Interpreting distance/time graphs & speed/time graphs

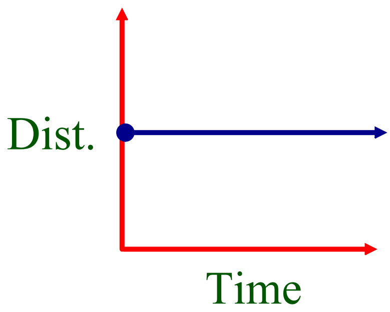
$$\text{speed} = \frac{\text{distance travelled}}{\text{time elapsed}}$$



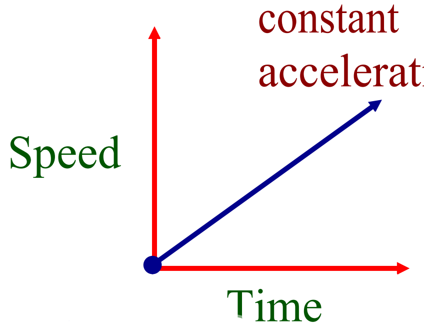
constant speed



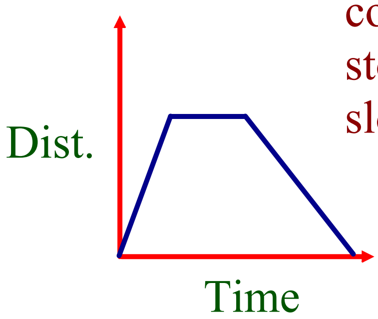
constant speed



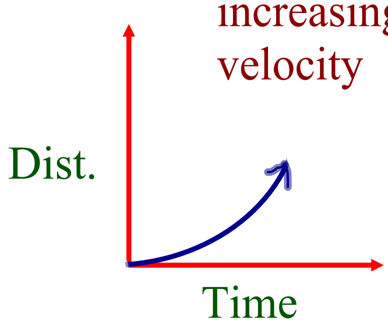
not moving



constant acceleration



constant speed,
stop,
slower speed back



increasing velocity

Interpreting data from a table...

$S = \frac{25\,km}{0.25\,h} = 100\,km/h$

Number of minutes after the start of trip	Total number of kilometers traveled
15	25
30	50
45	80
60	85
75	105
90	125
101.25	140

What is the average speed traveled during the first two 15 minute intervals of this trip?

$S = \frac{25\,km}{0.25\,h} = 100\,km/h$

What happened between 45 and 60 minutes during this trip?

Slowed down

What would go here if this person traveled at an average of 80 km/h between the 60th and 75th minutes of their trip?

$80\,km/h = \frac{d}{0.25\,h}$

$d = 20\,km$

What would go here if this person's average speed remained the same during the last 15 minutes of the trip as it was during the previous 15 minutes?

$S = \frac{20\,km}{0.25\,h} = 80\,km/h$

$(t)\ 80\,km/h = \frac{15\,km}{t\,h}\ (t)$

$t = \frac{15}{80}\,h \times \frac{60\,min}{1\,h} = 11.25$

HOMEWORK

- Page 76: Investigation #1

  Do questions #1 - 6

  Note: Convert minutes to hours

Sample Problems – Slope and Function Notation

Slope: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

WARM-UP...

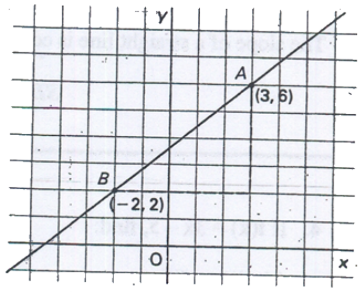
Lines that rise from left to right have a positive slope.
Lines that fall from left to right have a negative slope.

Function Notation: $f(x) = x + 3$ "f at x equals x plus three"
$h(t) = t^2 - 3t + 2$ "h at t equals t squared minus three times x plus two"

$f(2) = (2) + 3 = 5$ $h(2) = (2)^2 - 3(2) + 2 = 4 - 6 + 2 = 0$

1. a) Find the slope of the line.

$m = \dfrac{6-2}{3-(-2)} = \dfrac{4}{5}$



b) Find the slope of the line containing the given points.

(i) (3, 2), (10, 14) (ii) (-6, 10), (-11, 7) (iii) (-9, -13), (1, 1)

2. a) If f(x) = 2x + 5, find f(-1). b) If $h(t) = 3t^2 - 6t + 5$, find h(1.7).

3. If $g(x) = 3x^2 - 5x$, find g(4) – g(-1).

$g(4) = 3(4)^2 - 5(4) = 48 - 20 = 28$

$g(-1) = 3(-1)^2 - 5(-1) = 3 + 5 = 8$

$= 28 - 8 = 20$

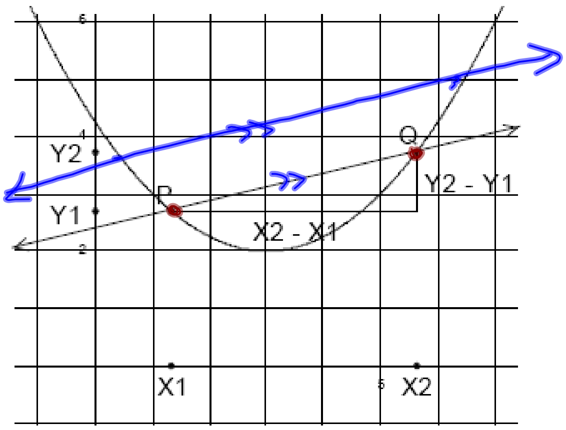
Average Rate of Change

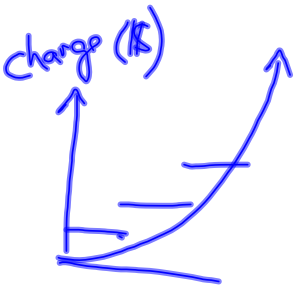
The average rate of change for a function y = f(x) is the ratio of the change in y to the change in x. Thus, if the change in x is from x_1 to x_2, the

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Slope of a Secant

line that cuts graph in 2 places





- The average rate of change of a linear function is constant.
- A variable rate of change implies some kind of curve.
- Rates of change tell how one thing is changing in relation to another.
- The average rate of change of y with respect to x over an interval from a to b is "Slope"
- A function with a positive slope indicates a positive rate of change, and hence is called an ***increasing function***. Ex: work longer hours, earn more money if being paid hourly.
- A negative slope indicates a negative rate of change and is called a ***decreasing function***. Ex: as height above sea level increases, air temperature decreases.
- The linear function with a zero rate of change is called a ***constant function***. Ex: On salary, pay remains the same despite number of hours worked.
- For nonlinear functions, the ***rate of change*** is not the same everywhere. It ***varies*** from place to place. Ex: speed of a baseball tossed into the air.
- The rate of change is closest to zero at the maximum or minimum point.

Attachments

Derivatives Worksheet.doc