

II. Factoring Trinomials:

Type 2: Polynomials of the form $ax^2 + bx + c$ $a \neq 1$

- Most efficient technique to factor most trinomials of this form is a process known as "**DECOMPOSITION**".

Consider the binomial product: $(3h + 4)(2h + 1)$

We can use the distributive property to expand:

$$\begin{aligned}(3h + 4)(2h + 1) &= 3h(2h + 1) + 4(2h + 1) \\&= 6h^2 + 3h + 8h + 4 \\&= 6h^2 + 11h + 4\end{aligned}$$

To factor $6h^2 + 11h + 4$ by decomposition, we reverse the steps above.

How well do we understand??

Fully factor the following as quickly as possible!

Factor

$$1. \quad 2x^2 + 3x + 1 \quad \text{+1}$$

$$\underline{2x^2 + 2x} + 1x + 1$$

$$2x(x+1) + 1(x+1)$$

$$(x+1)(2x+1)$$

$$3. \quad 2b^2 - 11b + 5 \quad \frac{-10}{\cancel{10}} \times \frac{-1}{\cancel{-1}} = 10$$

$$2b^2 - 10b - 1b + 5$$

$$2b(b-5) - 1(b-5)$$

$$2. \quad 2y^2 + 7y + 3 \quad \text{+6}$$

$$\underline{2y^2 + 4y} + 6y + 3$$

$$2y(\underline{2y+1}) + 3(\underline{2y+1})$$

$$4. \quad 3b^2 - 13b + 4 \quad \frac{-12}{\cancel{12}} \times \frac{-1}{\cancel{-1}} = 12$$

$$3b^2 - 12b - 1b + 4$$

$$3b(\underline{b-4}) - 1(\underline{b-4})$$

$$(b-4)(3b-1)$$

* 5. $2t^2 - t - 10$

$$2t^2 - 5t + 4t - 10$$

$$\begin{aligned} &t(\underline{2t-5}) + 2(\underline{2t-5}) \\ &(2t-5)(t+2) \end{aligned}$$

7. $12y^2 - 7y + 1$

$$\begin{array}{r} -5 \times 4 = -20 \\ -5 + 4 = -1 \end{array}$$

6. $3p^2 - 16p + 5$

* 8. $2t^2 + 5t - 12$

$$\begin{aligned} &2t^2 + 8t - 3t - 12 \\ &2t(\underline{t+4}) - 3(\underline{t+4}) \\ &(t+4)(2t-3) \end{aligned}$$

Factoring Special Polynomials

III. Perfect Square Trinomials:

Square each of the following binomials:

$$\begin{aligned}
 & x(x+3)^2 = x^2 + 6x + 9 \\
 & (5x+4)^2 = 25x^2 + 40x + 16 \\
 & (3x-1)^2 = 9x^2 - 6x + 1
 \end{aligned}$$

→ 1st & last perfect squares
 → Sum of 1st & last, root of last, doubled 3rd

Factor the following trinomial: $9w^2 + 48w + 64$

Plan A:

$$\begin{aligned}
 & 9w^2 + 24w + 16 \\
 & 3w(3w+8) + 8(3w+8) \\
 & (3w+8)(3w+8) \\
 & (3w+8)^2
 \end{aligned}$$

Plan B:

$$(3w+8)^2$$

→ Square Root of 1st & last terms
 → Roots of middle term

We say that $a^2 + 2ab + b^2$ is a **perfect square trinomial**.

The square of the 1st term in the binomial

↓ Twice the product of the first and second terms in the binomial

↓ The square of the 2nd term in the binomial

$$(a + b)^2 = a^2 + 2ab + b^2$$

How will we reverse this process and **FACTOR** a perfect square trinomial?

Factor the following trinomial: $25w^2 + 40w + 16$

$$= (5w + 4)^2$$

Factor each of the following if possible:

$$1. \ x^2 + 8x + 16$$

$$(x+4)^2$$

$$2. \ y^2 - 18y + 81$$

$$(y-9)^2$$

$$3. \ 100b^2 + 20b + 1$$

$$(10b+1)^2$$

$$4. \ 9x^2 + 42x + 49$$

$$(3x+7)^2$$

$$5. \ x^2 + 22xk + 121k^2$$

$$(x+11k)^2$$

$$6. \ x^{10} + 8x^5 + 16$$

$$\begin{array}{c} \cancel{q}^{\cancel{q}^2} \\ \sqrt{x^0} \\ \cancel{x}^{\cancel{x}^5} \\ (x^{\cancel{x}^5})^2 \end{array}$$

$$7. \ x^2 y^{10} - 14xy^5 + 49$$

$$(xy^5 - 7)^2$$

$$8. \ 3x^2 + 30x + 75$$

$$3(x^2 + 10x + 25)$$

$$9. \ 3a^2 + 18ab + 9b^2$$

$$\begin{array}{c} 3(x+5)^2 \\ 3(x+5)(x+5) \\ 3(x+5)(x+5) \end{array}$$