

# WARM-UP...

State 4 coordinates that describe the point P in BOTH rads & deg...

- 1) (+, +)
- 2) (+, -)
- 3) (-, +)
- 4) (-, -)

$\pi \text{ Rad} = \frac{180^\circ}{\pi}$   
 $\rho = \frac{\pi}{180} \text{ Rad}$   
 $\frac{225\pi}{180}$

$(r, \theta)$

①  $(8, \frac{5\pi}{4})$

②  $(8, -\frac{3\pi}{4})$

③  $(-8, \frac{\pi}{4})$

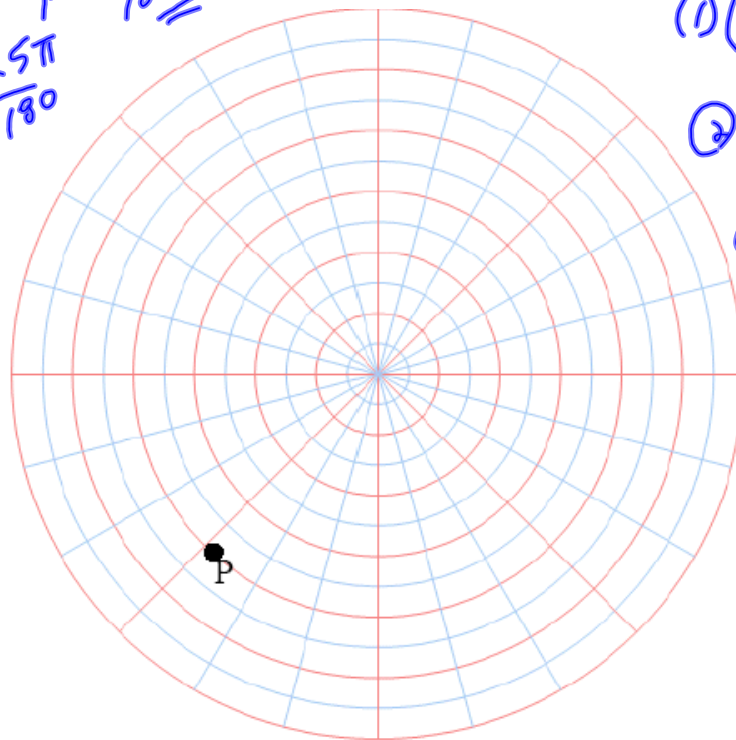
④  $(-8, -\frac{7\pi}{4})$

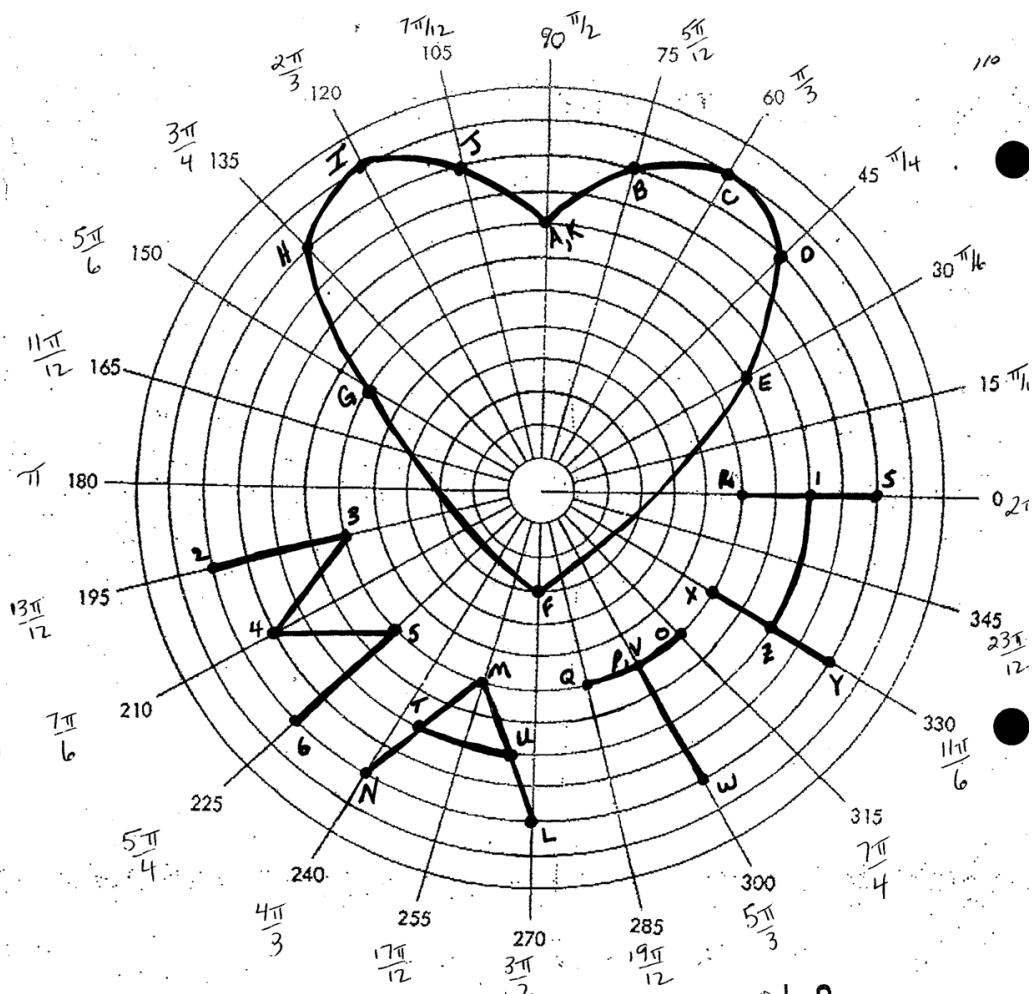
①  $(8, 225^\circ)$

②  $(8, -135^\circ)$

③  $(-8, 45^\circ)$

④  $(-8, -315^\circ)$





### Check Your Understanding

4. Create a picture by plotting and connecting the following points.

(a) Join these points with a smooth curve.

- $A \left( \frac{\pi}{2}, 8 \right)$ ,  $B \left( \frac{5\pi}{12}, 10 \right)$ ,  $C \left( \frac{\pi}{3}, 11 \right)$ ,  $D \left( \frac{\pi}{4}, 10 \right)$ ,  $E \left( \frac{\pi}{6}, 8 \right)$ ,  $F \left( \frac{\pi}{2}, 3 \right)$ ,  $G \left( \frac{2\pi}{3}, 7 \right)$ ,  $H \left( \frac{3\pi}{4}, -10 \right)$ ,  $I \left( \frac{5\pi}{6}, 10 \right)$ ,  $J \left( \frac{3\pi}{2}, -8 \right)$ ,  $K \left( \frac{7\pi}{6}, 10 \right)$ ,  $L \left( \frac{5\pi}{4}, -11 \right)$ ,  $M \left( \frac{5\pi}{3}, 10 \right)$ ,  $N \left( \frac{3\pi}{2}, 8 \right)$

(b) Connect these points with straight lines in the order given.

- $O \left( \frac{\pi}{2}, 10 \right)$ ,  $P \left( \frac{3\pi}{4}, -6 \right)$ ,  $Q \left( \frac{5\pi}{6}, 10 \right)$ ,  $R \left( \frac{3\pi}{2}, -10 \right)$ ,  $S \left( \frac{7\pi}{6}, 10 \right)$ ,  $T \left( \frac{5\pi}{4}, -6 \right)$ ,  $U \left( \frac{5\pi}{3}, 10 \right)$ ,  $V \left( \frac{3\pi}{2}, -10 \right)$

(c) Join these points with a smooth curve.

- $W \left( \frac{\pi}{2}, 6 \right)$ ,  $X \left( \frac{5\pi}{12}, 10 \right)$ ,  $Y \left( \frac{\pi}{3}, 11 \right)$ ,  $Z \left( \frac{\pi}{4}, 10 \right)$ ,  $A \left( \frac{\pi}{6}, 8 \right)$ ,  $B \left( \frac{\pi}{2}, 3 \right)$ ,  $C \left( \frac{2\pi}{3}, 7 \right)$ ,  $D \left( \frac{3\pi}{4}, -10 \right)$ ,  $E \left( \frac{5\pi}{6}, 10 \right)$ ,  $F \left( \frac{3\pi}{2}, -8 \right)$ ,  $G \left( \frac{7\pi}{6}, 10 \right)$ ,  $H \left( \frac{5\pi}{4}, -11 \right)$ ,  $I \left( \frac{5\pi}{3}, 10 \right)$ ,  $J \left( \frac{3\pi}{2}, 8 \right)$

(d) Join these points with a straight line.

- $K \left( 0, 9 \right)$ ,  $L \left( -10, -9 \right)$ ,  $M \left( 9, 0 \right)$ ,  $N \left( -9, -9 \right)$

(e) Join these points with a smooth curve.

- $O \left( 8, 245^\circ \right)$ ,  $P \left( 8, 265^\circ \right)$ ,  $Q \left( -6, -240^\circ \right)$ ,  $R \left( 10, -60^\circ \right)$

(f) Join these points with a straight line.

- $S \left( 6, \frac{2\pi}{3} \right)$ ,  $T \left( 10, -\frac{\pi}{6} \right)$ ,  $U \left( 6, \frac{2\pi}{3} \right)$ ,  $V \left( 10, -\frac{\pi}{6} \right)$

(g) Join these points with a smooth curve.

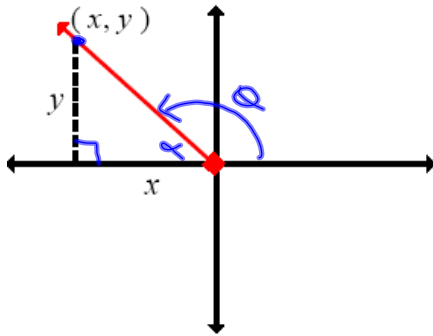
- $W \left( 8, 690^\circ \right)$ ,  $X \left( 8, 67^\circ \right)$ ,  $Y \left( 10, \frac{13\pi}{12} \right)$ ,  $Z \left( -6, \frac{\pi}{12} \right)$ ,  $A \left( 9, -\frac{5\pi}{6} \right)$ ,  $B \left( 6, \frac{3\pi}{4} \right)$ ,  $C \left( -10, -\frac{7\pi}{4} \right)$

(h) Connect these points with straight lines in the order given.

- $1 \left( 10, \frac{13\pi}{12} \right)$ ,  $2 \left( -6, \frac{\pi}{12} \right)$ ,  $3 \left( 9, -\frac{5\pi}{6} \right)$ ,  $4 \left( 6, \frac{3\pi}{4} \right)$ ,  $5 \left( -10, -\frac{7\pi}{4} \right)$ ,  $6 \left( 10, \frac{13\pi}{12} \right)$

## Converting Between Coordinate Systems

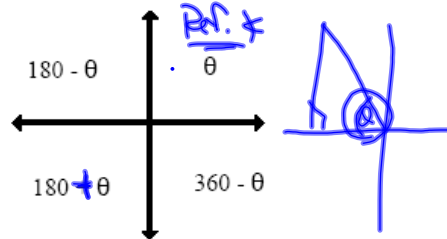
I. Rectangular  $\Rightarrow$  Polar  
 $(x, y)$   $(r, \theta)$



To get  $\theta$  we know that...

$$\tan \theta = \left| \frac{y}{x} \right|$$

to get the reference angle then we put it into the quadrant where the point lies.

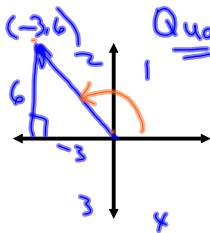


To get  $r$  we know that...

$$r = \sqrt{x^2 + y^2}$$

EXAMPLES...

#1.  $(-3, 6)$



$$r = \sqrt{9 + 36}$$

$$r = \sqrt{45}$$

$$r = 3\sqrt{5}$$

Always Positive!!

$$(3\sqrt{5}, 117^\circ)$$

$$\tan^{-1}(2) = 63.43494882^\circ$$

Qual.: 2  
 $\tan \theta = \left| \frac{-3}{-5} \right| = 2$

(Ref:  $63^\circ$ ) Q2

$$\theta = 180 - 63 = 117^\circ$$

2.  $(-12, -5) \rightarrow$  Q3

$$r = \sqrt{144 + 25}$$

$$r = 13$$

$$\tan \theta = \frac{5}{12}$$

(Ref:  $23^\circ$ )

$$\theta = 180 + 23$$

$$\theta = 203^\circ$$

$$(13, 203^\circ)$$

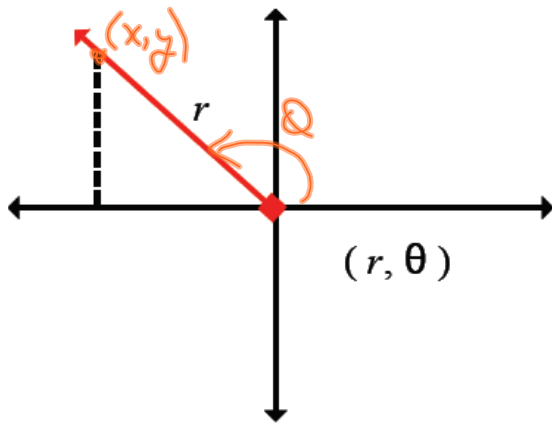
$$(13, -157^\circ)$$

$(0, 4) \leftarrow$  Quadrantal



## II. Polar $\Rightarrow$ Rectangular

$(r, \theta)$        $(x, y)$



## FORMULA!!!

$$\cos \theta = \frac{x}{r} \quad \text{so} \quad x = r \cos \theta$$

$$\sin \theta = \frac{y}{r} \quad \text{so} \quad y = r \sin \theta$$

## Examples

$$3 \cos(150^\circ) = -2.598076211$$

#1.  $(3, 150^\circ)$

$$3 \cos(150^\circ) = -2.598076211$$

$$3 \sin(150^\circ) = 1.5$$

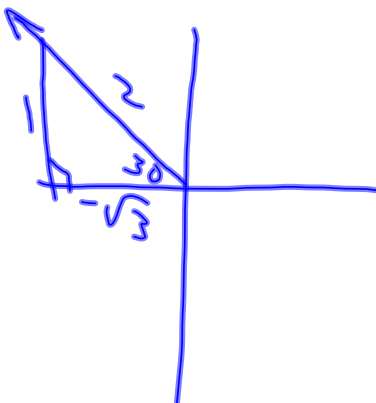
$$x = 3 \cos 150^\circ$$

$$x = \frac{-3\sqrt{3}}{2} = 3\left(-\frac{\sqrt{3}}{2}\right)$$

$$y = 3 \sin 150^\circ$$

$$y = \underline{1.5}$$

150°



#2.  $(-5, -440^\circ)$

$$x = -5 \cos(-440^\circ)$$

$$-5 \cos(-440^\circ) = -4.924038765$$

$$x =$$

$$y = -5 \sin(-440^\circ)$$

$$y = -5 \sin(-440^\circ) = 4.924038765$$

Bonus #1

$$|2x-1| - |x+5| = 3$$

Case 1: (+, +)  $\rightarrow$  If  $2x-1 > 0$  &  $x+5 > 0$

$$(2x-1) - (x+5) = 3$$

$$x-6=3$$

$$x=9$$

okay

$$2x > 1 \quad x > -5$$

$$x > \frac{1}{2}$$

$$\therefore x > \frac{1}{2}$$

Case 2: (+, -)  $\rightarrow$  If  $2x-1 > 0$  &  $x+5 < 0$

$$(2x-1) + (x+5) = 3$$

$$3x+4=3$$

$$3x=-1$$

$$x = -\frac{1}{3}$$

$$x > \frac{1}{2} \quad x < -5$$

$\emptyset$

Case 3: (-, +)  $\rightarrow$  If  $2x-1 < 0$  &  $x+5 > 0$

$$-(2x-1) - (x+5) = 3$$

$$-2x+1-x-5=3$$

$$-3x-4=3$$

$$-3x=7$$

$$x = -\frac{7}{3}$$

okay

$$2x < 1 \quad x > -5$$

$$x < \frac{1}{2}$$

$$\therefore \left\{ -5 < x < \frac{1}{2} \right\}$$

Case 4: (-, -)  $\rightarrow$  If  $2x-1 < 0$  &  $x+5 < 0$

$$-(2x-1) + (x+5) = 3$$

$$-x+4=3$$

$$-x=-1$$

$$x = -1$$

Not  $< -5$

$$x < \frac{1}{2} \quad x < -5$$

$$\therefore x < -5$$