Warm Up...

1. Express $1-\sqrt{3}i$ in polar form.

2. Express $-3-4i$ in polar form.

3. Express the number 4 in polar form.

4. Express the polar form $\sqrt{8}(cos1395° + isin1395°)$ as a complex number in rectangular form.

1. Express $1-\sqrt{3}i$ in polar form. $\boxed{a+bi = r(\cos\theta + i\sin\theta)}$

$(1, -\sqrt{3}) \Rightarrow$ Quad 4

$r = \sqrt{1+3}$

$r = 2$

$\tan\theta = \left|\frac{\sqrt{3}}{1}\right|$

(Ref ∡: $60°$) $\Rightarrow$ Q4

$\theta = 300°$

$= 2(\cos 300° + i\sin 300°)$

$= 2\,\text{cis}(300°)$

2. Express $-3-4i$ in polar form.

$(-3, -4) \rightarrow$ Quad. 3

$r = \sqrt{9+16}$

$r = 5$

$\tan\theta = \frac{4}{3}$

(Ref ∡: $53°$) Q3

$180-\theta$	θ
$180+\theta$	$360-\theta$

$\theta = 180° + 53°$

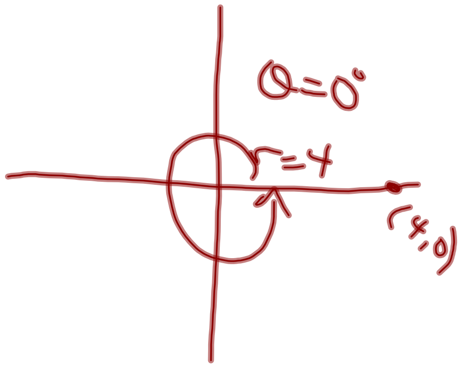
$= 233°$

$= 5(\cos 233° + i\sin 233°)$

$= 5\,\text{cis}\,233°$

3. Express the number 4 in polar form.

$\Rightarrow 4+0i \Rightarrow (4,0)$

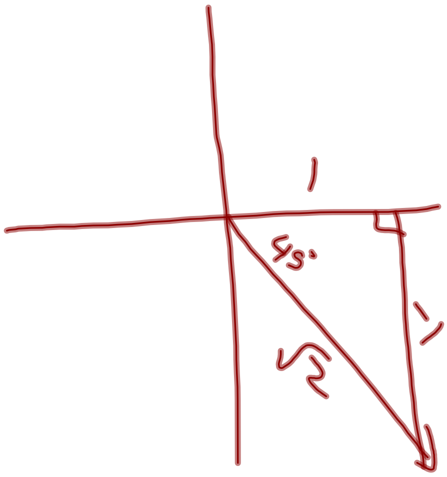


$= 4(\cos 0° + i\sin 0°)$

$= 4 \operatorname{cis} 360°$

4. Express the polar form $\sqrt{8}(cos1395° + isin1395°)$
as a complex number in rectangular form.

$1395° = \underline{PA}: 315°$



$= \sqrt{8}\left(\frac{1}{\sqrt{2}} + i\left(\frac{-1}{\sqrt{2}}\right)\right)$

$= \frac{\sqrt{8}}{\sqrt{2}} - \frac{\sqrt{8}}{\sqrt{2}} i$

$= 2 - 2i$

Product and Quotient of Complex Numbers in Polar Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\ \theta_2 + i\sin\ \theta_2)$...

Now let's examine $z_1 \bullet z_2$ and $\frac{z_1}{z_2}$

$z_1 \cdot z_2$

$[r_1(\cos\theta_1 + i\sin\theta_1)] \times [r_2(\cos\theta_2 + i\sin\theta_2)]$

$r_1 r_2 [(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)]$

$r_1 r_2 [\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + \overset{-1}{i^2}\sin\theta_1\sin\theta_2]$

$r_1 r_2 [(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)]$

$r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$$\boxed{\begin{aligned}\sin(A+B) &= \sin A\cos B + \cos A\sin B\\ \sin(A-B) &= \sin A\cos B - \cos A\cos B\\ \cos(A+B) &= \cos A\cos B - \sin A\sin B\\ \cos(A-B) &= \cos A\cos B + \sin A\sin B\end{aligned}}$$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

CONCLUSIONS...

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$...

$$\boxed{z_1 \bullet z_2 = r_1 r_2 \left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]}$$

- multiply all "r" values together
- add all angles together

&

$$\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]}$$

- divide "r" values
- subtract angles

Examples:

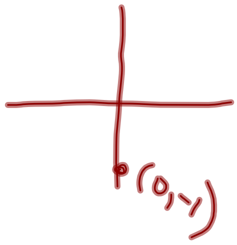
Evaluate:

$$\sqrt{2}\left(\cos 57^\circ + i\sin 57^\circ\right) \bullet 2\sqrt{6}\left(\cos 213^\circ + i\sin 213^\circ\right)$$

$= 2\sqrt{12}\left(\cos 270^\circ + i\sin 270^\circ\right)$

$= 4\sqrt{3}\left(0 - 1i\right)$

$= -4i\sqrt{3} \leftarrow \underline{a+bi}$



Evaluate: $$\frac{2\sqrt{2}\left(\cos 135^\circ + i\sin 135^\circ\right)}{6\left(\cos 300^\circ + i\sin 300^\circ\right)}$$

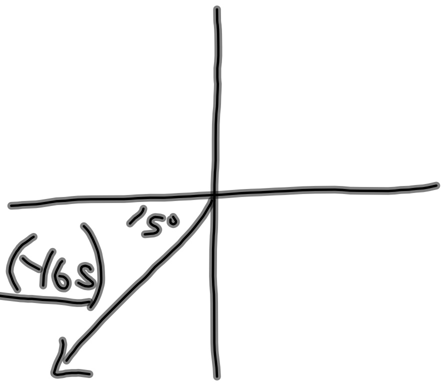
$= \frac{\sqrt{2}}{3}\,\text{cis}(-165^\circ)$

$\approx \cos(-165) + i\sin(-165)$

$\approx \frac{\sqrt{2}}{3}\left(-0.965 - 0.25i\right)$

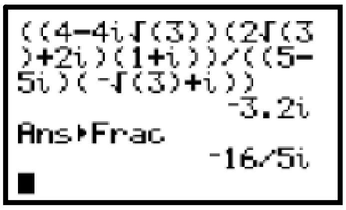
$\approx -0.5 - 0.1i$

Approx.



Let's revisit an "OLD QUESTION"...

EXERCISE: Express the following in the form "a + bi"...



$$\frac{\left(4-4i\sqrt{3}\right)\left(2\sqrt{3}+2i\right)\left(1+i\right)}{\left(5-5i\right)\left(-\sqrt{3}+i\right)}$$

Now - let's switch into **polar form** to solve the problem!!!

Attachments

Worksheet - General to Standard (a not 1).doc