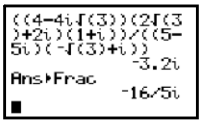
Let's revisit an "OLD QUESTION"...

EXERCISE: Express the following in the form "a + bi"...



$$\frac{\left(4-4i\sqrt{3}\right)\left(2\sqrt{3}+2i\right)\left(1+i\right)}{\left(5-5i\right)\left(-\sqrt{3}+i\right)}$$

Now - let's switch into **polar form** to solve the problem!!!

$4-4i\sqrt{3} \Rightarrow (4,-4\sqrt{3})$ Q4

$(4\sqrt{3})^2 = 16\cdot 3 = 48$

$r=\sqrt{16+48}$

$r=8$

$\tan\theta = \frac{4\sqrt{3}}{4}$

$\tan\theta = \sqrt{3}$

(Ref∡ = 60°)

$\theta = 300°$

Polar: $\boxed{8\text{cis}\,300°}$

$(2\sqrt{3},2) \to$ Q1

$r=\sqrt{12+4}$

$r=4$

$\tan\theta = \frac{1}{\sqrt{3}}$

(Ref∡ = 30°)

$\theta = 30°$

$\boxed{4\text{cis}\,30°}$

$(1,1) \to$ Q1

$r=\sqrt{1+1}$

$r=\sqrt{2}$

$\tan\theta = 1$

(Ref∡ = 45°)

$\theta = 45°$

$\boxed{\sqrt{2}\text{cis}\,45°}$

$(5,-5) \to$ Q4

$r=\sqrt{25+25}$

$r=\sqrt{50}$

$r=5\sqrt{2}$

$\tan\theta = \frac{5}{5} = 1$

(Ref∡ : 45°)

$\theta = 315°$

$= \boxed{5\sqrt{2}\text{cis}\,315°}$

$(-\sqrt{3},1) \to$ Q2

$r=\sqrt{3+1}$

$r=2$

$\tan\theta = \frac{1}{\sqrt{3}}$

(Ref∡ : 30°)

$\theta = 150°$

$= \boxed{2\text{cis}\,150°}$

Now in Polar Form

$$= \frac{(8\text{cis}300°)(4\text{cis}30°)(\sqrt{2}\text{cis}45°)}{(5\sqrt{2}\text{cis}315°)(2\text{cis}150°)}$$

$$= \frac{32\sqrt{2}\,\text{cis}(300°+30°+45°)}{10\sqrt{2}\,\text{cis}(315°+150°)}$$

$$= \frac{32\sqrt{2}\,\text{cis}(375°)}{10\sqrt{2}\,\text{cis}(465)}$$

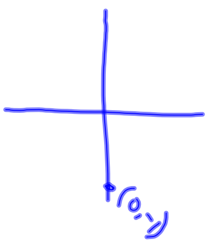
$$= \left(\frac{32\cancel{\sqrt{2}}}{10\cancel{\sqrt{2}}}\right)\text{cis}(-90°)$$

Back to Rectangular...

$= \frac{16}{5}\left(\cos(-90°) + i\sin(-90°)\right)$

$= \frac{16}{5}\left(0 + i(-1)\right)$

$= -\frac{16}{5}i$



What about something like this???

Evaluate the following:

Huh???

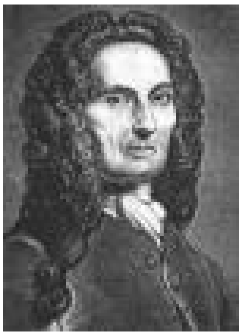
$(r \operatorname{cis} \theta)$

$$(-1+i)^{10}$$

$$\underbrace{(-1+i)(-1+i)(-1+i)\cdots(-1+i)}_{\text{10 of these}}$$

$$(r \operatorname{cis} \theta)^n$$

$$= r^n \operatorname{cis}(n\theta)$$



THEOREM!!!

Demoivre's

$$\boxed{[r(\cos\theta + i\sin\theta)]^n = r^n[\cos(n\theta) + i\sin(n\theta)]}$$

$$(rcis\theta)^n$$

Example: Simplify the following using the polar form of a complex number.

$$(1-i)^6$$

```
(1-i)^6
                8i
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$(1,-1)$ Q4

$r = \sqrt{2}$

$\tan\theta = 1$

(Ref ∡: $45°$)

$\theta = 315°$

$= \sqrt{2}\,cis 315°$

$(2^{1/2})^6$

$(\sqrt{2}\,cis 315°)^6$

$= (2^3\,cis(6 \times 315°))$

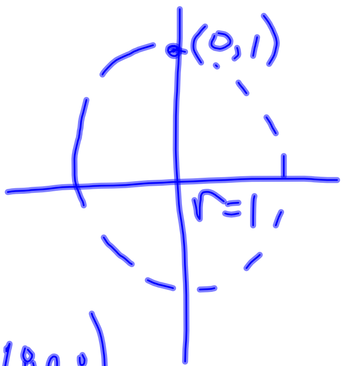
$= 8\,cis(1890°)$

$= 8(\cos 1890° + i\sin 1890°)$

$= 8(0 + 1i)$

$= 8i$

$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$



$\underline{\underline{1890°}}$

P.A. $= 1890 \div 360$

$= 5.25$

-5

0.25×360

$= 90°$

ple:

$$\frac{\left(-1+i\sqrt{3}\right)^9(4i)^8(3+3i)}{\left(-4\sqrt{3}-4i\right)^6(1-i)^8}$$

SOLUTION:
-24 - 24i

$(-1,\sqrt{3})$ Q2
$r=2$
$\tan\theta=\sqrt{3}$
(Ref ∢: 60°)
$\theta=120°$
$=\underline{2\text{cis}120°}$

$4i \Rightarrow (0,4)$

$=\underline{4\text{cis}90°}$

$3+3i \Rightarrow (3,3)$ Q1
$r=\sqrt{18}$
$r=3\sqrt{2}$
$\tan\theta=1$
(Ref ∢ = 45°)
$\theta=45°$
$=\underline{3\sqrt{2}\text{cis}45°}$

$(-4\sqrt{3},-4) \Rightarrow$ Q3
$r=\sqrt{48+16}$
$r=8$
$\tan\theta=\frac{4}{4\sqrt{3}}=\frac{1}{\sqrt{3}}$
(Ref ∢ = 30°)
$\theta=210°$
$=\underline{8\text{cis}210°}$

$1-i=(1,-1)$ Q4
$r=\sqrt{2}$
$\tan\theta=1$
Ref ∢ = 45°
$\theta=315°$
$=\underline{\sqrt{2}\text{cis}315°}$

$$=\frac{(2\text{cis}120°)^9(4\text{cis}90°)^8(3\sqrt{2}\text{cis}45°)}{(8\text{cis}210°)^6(\sqrt{2}\text{cis}315°)^8}$$

$(2^2)^8$ $(2^3)^6$ $(2^{1/2})^8$

$$=\frac{(2^9\text{cis}1080°)(2^{16}\text{cis}720°)(3\sqrt{2}\text{cis}45°)}{(2^{18}\text{cis}1260°)(2^4\text{cis}2520°)}$$

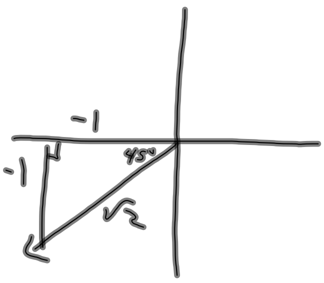
$$=\frac{2^{25}(3\sqrt{2})\text{cis}(1845°)}{2^{22}\text{cis}(3780°)}$$

$$=2^3(3\sqrt{2})\ \text{cis}(-1935°)$$

co-terminal
$-135°$

$$=24\sqrt{2}\left(-\frac{1}{\sqrt{2}}+\left(-\frac{1}{\sqrt{2}}\right)i\right)$$

$$=-24-24i$$



Homework...

Worksheet - Polar Form

We just did #5 together!!!

#6 and #7

#6) $20\sqrt{3}-20i$

#7) $-\frac{5\sqrt{3}}{48}-\frac{5}{48}i$

#8) $-\frac{1}{300}-\frac{1}{300}i$

Attachments

Worksheet - DeMoivres Theorem.doc