Review - Complex Numbers...

- operations using complex numbers.
  - add/subutract, multiplication, division (complex conjugate)
  - comparion method (Re = Re & Im = Im)
  - describing complex roots

- plotting polar coordinates (both in degrees and radians)

- switching forms: rectangular ⇒ polar & polar ⇒ rectangular

$\boxed{\tan\theta = \left|\frac{y}{x}\right|}$ $\boxed{r = \sqrt{x^2 + y^2}}$

$$\boxed{\cos\theta = \frac{x}{r} \quad so \quad x = r\cos\theta}$$

$$\boxed{\sin\theta = \frac{y}{r} \quad so \quad y = r\sin\theta}$$

- switching forms: complex number ⇒ polar form (r cis θ)

$$\boxed{\begin{array}{l} a + bi \Rightarrow r\cos\theta + (r\sin\theta)i \\ \therefore \quad polar\ form is... \qquad r\underline{cis}\,\theta = r\,\underline{\cos}\,\theta + \underline{i\sin}\,\theta \end{array}}$$

$\boxed{\tan\theta = \left|\frac{y}{x}\right|}$ $\boxed{r = \sqrt{x^2 + y^2}}$

- operations in polar form

$$\boxed{z_1 \bullet z_2 = r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]}$$

- multiply all "r" values together
- add all angles together

$$\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]}$$

- divide "r" values
- subtract angles

- DeMoivre's Theorem

$$\boxed{\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]}$$

Quick Review to get the dust off...

Find the complex conjugate of $\dfrac{(1+2i)(6-5i)}{(3+3i)}$

$$= \frac{6-5i+12i-10i^2}{3+3i}$$

$$= \frac{16+7i}{3+3i}\left(\frac{3-3i}{3-3i}\right)$$

$$= \frac{48-48i+21i-21i^2}{9-9i^2}$$

Take conjugate

$$= \frac{69-27i}{18} = \frac{23}{6} - \frac{3}{2}i = \frac{23}{6} + \frac{3}{2}i$$

Solve the following equation for the real variables x and y ....

$$(3+4i)^2 - 2(x-iy) = x+iy$$

$$9+24i+16i^2-2x+2iy = x+iy$$

$$9+24i-16-2x+2iy = x+iy$$

Real = Real

$$9-16-2x = x$$

$$-7 = 3x$$

$$x = \frac{-7}{3}$$

Im = Im

$$24+2y = y$$

$$24 = -y$$

$$y = -24$$

Express the following in the form $a + bi$:

$$3(2-5i)^2 - 5i^{21} + 7i^{10} + (2i^3)^5$$

$= 3(4-20i+25i^2) - 5(i^2)^{10}i + 7(i^2)^5 + 32(i^2)^7 i$

$= 12-60i-75-5i-7-32i$

$=-70-97i$

Convert each of the following... $-750°$

a) polar coordinates $\left(-2\sqrt{3}, \dfrac{-25\pi}{6}\right)$ to rectangular coordinates.

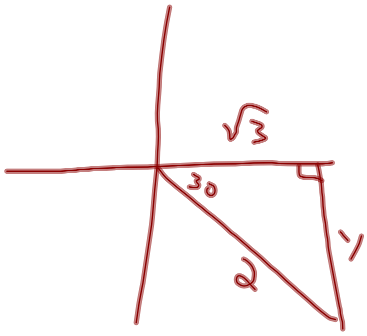
b) rectangular coordinates $\left(-3\sqrt{5}, 7\right)$ to polar coordinates.

a) $x = r\cos\theta$ $\quad y = r\sin\theta$

$x = -2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$ $\quad y = -2\sqrt{3}\left(\frac{-1}{2}\right)$

$x = 3$ $\quad y = \sqrt{3}$

$(-3, \sqrt{3})$



b) $(-3\sqrt{5}, 7) \leftarrow$ Q2

$r = \sqrt{45+49}$

$r = \sqrt{94}$

$\tan\theta = \dfrac{7}{3\sqrt{5}}$

$\tan^{-1}(7/(3\sqrt{5}))$

(Ref ∢: 46°)

$\theta = 180 - 46$

$\theta = 134°$

$\boxed{(\sqrt{94}, 134°)}$

coordinates to evaluate the following expression: $\dfrac{\left(-1+i\sqrt{3}\right)^5\left(-1-i\right)^4}{\left(2+2i\sqrt{3}\right)^2}$

$(-1,\sqrt{3}) \to Q2$

$r=2$

$\tan\theta=\sqrt{3}$

(Ref∡: $60°$)

$\theta=120°$

$=\left[2\operatorname{cis}120°\right]^5$

$=2^5\operatorname{cis}(600°)$

$(-1,-1) \to Q3$

$r=\sqrt{2}$

$\tan\theta=1$

(Ref∡: $45°$)

$\theta=225°$

$=\left[\sqrt{2}\operatorname{cis}225°\right]^4$

$=2^2\operatorname{cis}(900°)$

$(2,2\sqrt{3})$ Q1

$r=\sqrt{4+12}$

$r=4$

$\tan\theta=\dfrac{2\sqrt{3}}{2}=\sqrt{3}$

(Ref∡: $60°$)

$\theta=60°$

$=\left[4\operatorname{cis}60°\right]^2$

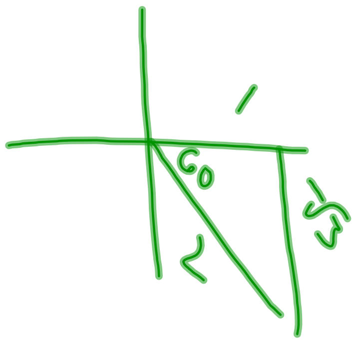
$=4^2\operatorname{cis}120°$

$$=\frac{(\cancel{32}\,2\operatorname{cis}600)(4\operatorname{cis}900)}{\cancel{16}\operatorname{cis}120}$$

$=8\operatorname{cis}1380° \longrightarrow 1380° \to$ P.A. $300°$

$=8\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$=4-4i\sqrt{3}$



Review Time!!!

Review - Complex Numbers.doc

Review Solns - Complex Numbers.doc

Attachments

Text Solns_p. 38 Ques. 36 - 45.doc

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