Warm Up

1. Simplify: $\dfrac{\dfrac{1}{x^2}-\dfrac{1}{9}}{x-3}$

2. Factor each of the following:

$x^{27}-1$

$(x^2+1)^{\frac{1}{2}}+3(x^2+1)^{-\frac{1}{2}}$

3. Rationalize the denominator:

$$\frac{x+2}{\sqrt{x-4}-\sqrt{x-6}}$$

$(x^2-9)^{-1}$

1. Simplify: $\dfrac{\dfrac{1}{x^2}-\dfrac{1}{9}}{x-3}$ $\longrightarrow \left(\frac{1}{x}-\frac{1}{3}\right)\left(\frac{1}{x}+\frac{1}{3}\right)$

$\frac{7+9}{9+7}=1$

$(x^2-9)^2$

$\cancel{x^4 \times 9^2}$

$\left(\dfrac{9-x^2}{9x^2}\right) \div (x-3)$

$\dfrac{\overset{-1}{\cancel{(3-x)}}(3+x)}{9x^2} \cdot \dfrac{1}{\cancel{x-3}}$

$\frac{7-9}{9-7} = \frac{-2}{2}$

$=-1$

$= -\dfrac{3-x}{9x^2}$ OR $-\dfrac{(3+x)}{9x^2}$ OR $-\dfrac{3+x}{9x^2}$

$\dfrac{-(3+x)}{9x^2}$ $\quad -\dfrac{3+x}{9x^2}$

2. Factor each of the following:

$x^{27} - 1$

$(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$

$(x^9 - 1)(x^{18} + x^9 + 1)$

$(x^3 - 1)(x^6 + x^3 + 1)(x^{18} + x^9 + 1)$

$(x - 1)(x^2 + x + 1)(x^6 + x^3 + 1)(x^{18} + x^9 + 1)$

$$(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$$

$a^3 - a^5$

$a^3(1-a^2)$

$$(x^2+1)^{-1/2}\left[(x^2+1)^1 + 3\right]$$

$$(x^2+1)^{-1/2}(x^2+4)$$

$$(x^2+1)^{-1/2}\left(x^{2/3}+\sqrt[3]{4}\right)\left(x^{4/3} - \sqrt[3]{4}\,x^{2/3} + \left(\sqrt[3]{4}\right)^2\right)$$

$x+2$

$$\left(\sqrt[3]{x}+\sqrt[3]{2}\right)\left(\left(\sqrt[3]{x}\right)^2 - \sqrt[3]{2x} + \left(\sqrt[3]{2}\right)^2\right)$$

$x-7$

$$\left(\sqrt{x}-\sqrt{7}\right)\left(\sqrt{x}+\sqrt{7}\right)$$

3. Rationalize the denominator:

$$\frac{x+2}{\sqrt{x-4}-\sqrt{x-6}}\left(\frac{\sqrt{x-4}+\sqrt{x-6}}{\sqrt{x-4}+\sqrt{x-6}}\right)$$

$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{(x-4)-(x-6)}$$

$$\frac{(x+2)(\sqrt{x-4}+\sqrt{x-6})}{2}$$

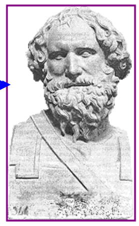
Limits

What is meant by a limit in Mathematics?

Let's explore and find out!

I bet he knows something about limits

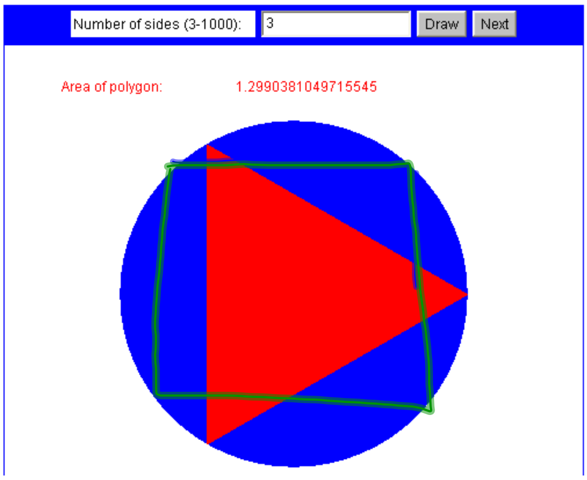
Archimedes



Have a look at these two scenarios:

Determining the area of a circle using polygons

Approximating the area of a unit circle with regular polygons



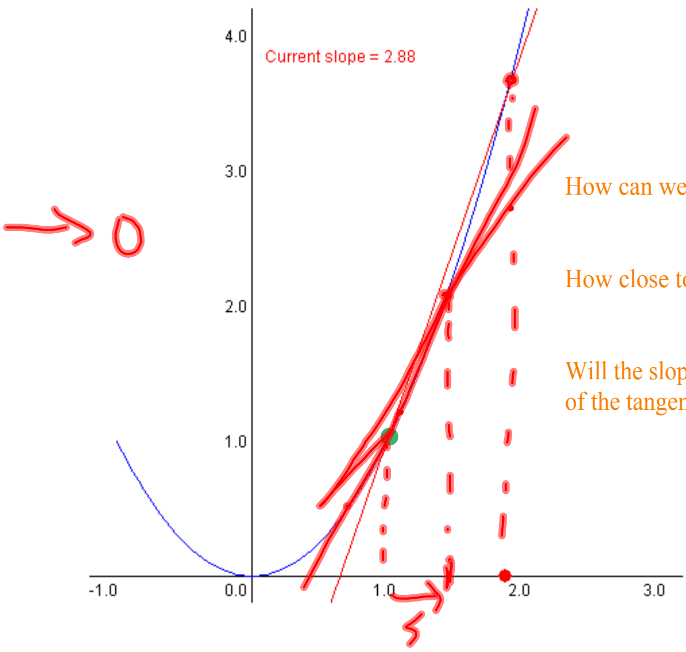
What should the area actually equal?

Will it ever equal this value?

What is the limit of the area of the polygon?

Determining the slope of a tangent to a curve

Secant line approximations to the tangent line



How can we find the slope of a secant?

How close to the point $x = 1$ should we choose?

Will the slope of the secant ever equal the exact slope of the tangent?

The common sense definition of a limit...

Click Me

What *is* a limit?

A formal definition of a limit...

We write $\lim\limits_{x \to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to L

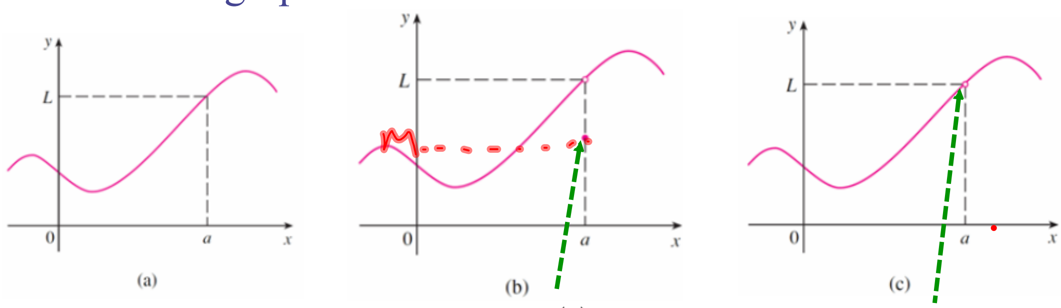
- (as close to L as we like)

by taking x to be sufficiently close to a

- (on either side of a)

but not equal to a .

Look at the graphs of these three functions...



Notice $f(a) \neq L$

Notice $f(a)$ is undefined

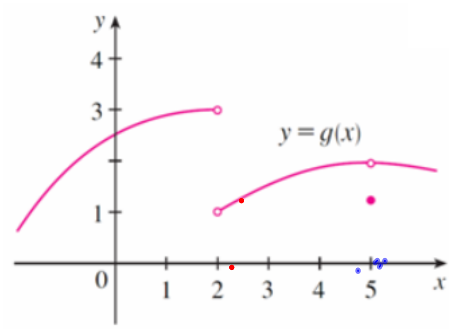
But in each case, regardless of what happens at a , it is true that

$$\lim_{x \to a} f(x) = L$$

One-sided limits

Use the graph shown below to evaluate the following limits:

$\lim\limits_{x\to 2^-}$



Does Not Exist

1. $\lim\limits_{x\to 2^-} g(x) = \boxed{3}$ "as x approaches 2 from the left"

2. $\lim\limits_{x\to 2^+} g(x) = \boxed{1}$ "as x approaches 2 from the right"

3. $\lim\limits_{x\to 2} g(x) = \boxed{\text{DNE}}$

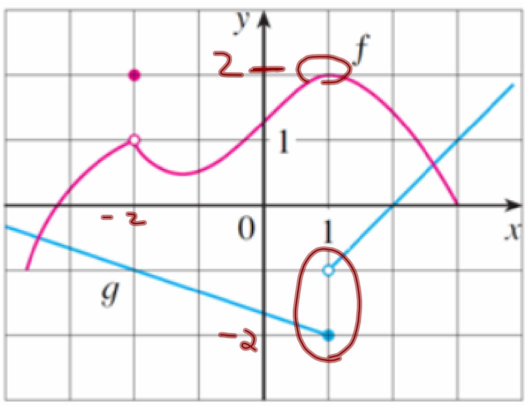
4. $\lim\limits_{x\to 5^-} g(x) = \boxed{2}$

5. $\lim\limits_{x\to 5^+} g(x) = \boxed{2}$

6. $\lim\limits_{x\to 5} g(x) = \boxed{2}$

Notice... $g(5) = 1$

Example:



Evaluate each of the following:

$f(-2) = 2$ $\quad$ $\lim\limits_{x\to 1^-} g(x) = -2$ $\quad$ $g(1) = -2$

$\lim\limits_{x\to 1^+} g(x) = -1$ $\quad$ $\lim\limits_{x\to 1} g(x) = \text{DNE}$ $\quad$ $\lim\limits_{x\to 1} f(x) = 2$

$\lim\limits_{x\to -2} f(x) =$