Related Rate Problems

If Q is a quantity that is varying with time, we know that the derivative measures how fast Q is increasing or decreasing. Specifically, if we let t stand for time, then we know the following.

Rate of Change of a Quantity

$$\text{Rate of change of } Q = \frac{dQ}{dt}$$

In a *related rates* problem, we are given the rate of change of certain quantities, and are required to find the rate of change of related quantities.

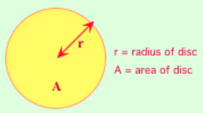
Example
The area of a circular doggie puddle is growing at a rate of 12 cm^2/s. How fast is the radius growing at the instant when it equals 10 cm?

$\frac{dA}{dt}$ $\quad$ $\frac{dr}{dt} = ?$

Solving a Related Rates Problem

Step 1: Identify the changing quantities, possibly with the aid of a sketch.
Step 2: Write down an equation that relates the changing quantities.
Step 3: Differentiate both sides of the equation with respect to t.
Step 4: Go through the whole problem and restate it in terms of the quantities and their rates of change. Rephrase all statements regarding changing quantities using the phrase "the rate of change of . . . ."
Last Step: Substitute the given values in the derived equation you obtained above, and solve for the required quantity.

Here is a little sketch of the puddle showing the changing quantities.



Note At this stage, we do not substitute values for the changing quantities. That comes at the end.

Important:
Never substitute a quantity that is changing until after finding the derivative!!!

$$A = \pi r^2 \qquad (r)^2$$

$$\frac{dA}{dt} = \pi(2r)\frac{dr}{dt} \qquad 2(r)^1\frac{dr}{dt}$$

$$12 = \pi(2)(10)\frac{dr}{dt}$$

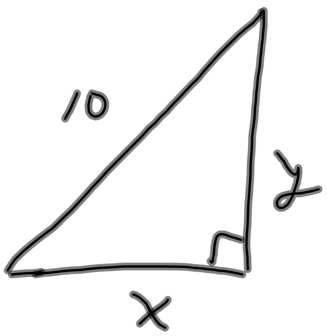
$$\frac{12}{(20\pi)} = \frac{dr}{dt}$$

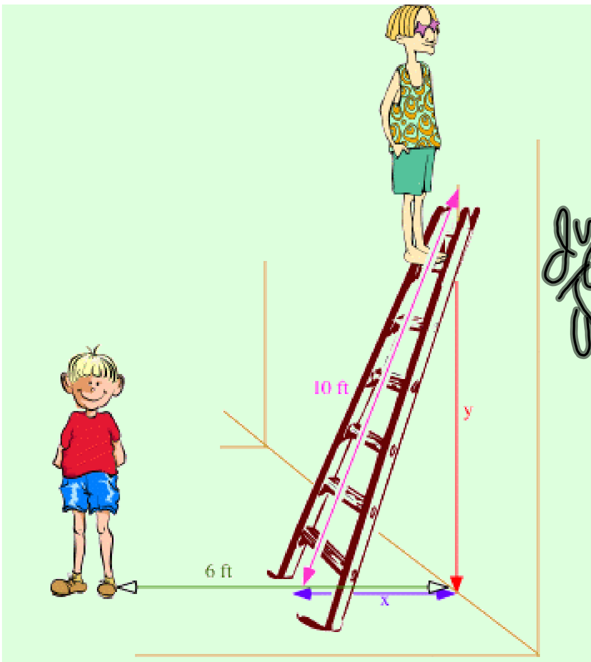
$$\frac{dr}{dt} = 0.19 \text{ cm/s}$$

Example 2:

Joey is perched precariously the top of a 10-foot ladder leaning against the back wall of an apartment building (spying on an enemy of his) when it starts to slide down the wall at a rate of 4 ft per minute. Joey's accomplice, Lou, is standing on the ground 6 ft. away from the wall. How fast is the base of the ladder moving when it hits Lou?

1 animation

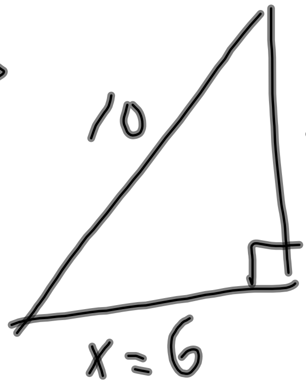




$\frac{dy}{dt} = -4$ ft/min

$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$



$y = \sqrt{100-36}$

$y = 8$

$2(6)\frac{dx}{dt} + 2(8)(-4) = 0$

$12\frac{dx}{dt} = 64$

$\frac{dx}{dt} = \frac{64}{12} = 5.\bar{3}$ ft./minute

Example:

Air is being pumped into a spherical shaped balloon at a rate of 100 cm^3/s. Determine the rate at which the radius of the balloon is increasing the instant the diameter is 50cm.

Sphere: $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$100 = \frac{4}{\cancel{3}}\pi(\cancel{3})(25)^2\frac{dr}{dt}$$

$$\frac{100}{[4\pi(25)^2]} = \frac{dr}{dt}$$

$$0.0127\,cm/sec = \frac{dr}{dt}$$

Warm Up

A rock is thrown into a pool of water. A circular wave leaves the point of impact and travels so that its radius increases at a rate of 25 cm/s. How fast is the circumference of the wave increasing when the radius of the wave is 1m ?

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$\frac{dC}{dt} = 2\pi(25)$$

$$= \underline{50\pi \text{ cm/sec}}$$

Pg. 145

#3 - 8

Sphere:

$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$