Practice Problems...

A cubic block of ice (*which remains in the shape of a cube*) is melting so that its volume is *decreasing* at a rate of 2 cm^3/min. How fast is the length of a side changing (in cm/min) when the sides are 10 cm?

(a) $-\frac{2}{300}$ (b) None of these (c) $\frac{1}{600}$ (d) $\frac{2}{300}$ (e) $-\frac{1}{600}$

(Texas A & M Univ. Final Exam '12)

(10 pts) Water is poured into a conical cup at the rate of $\frac{5}{2}$ cubic inches per second. If the cup is 6 inches tall and the top of the cup has a radius of 2 inches, how fast does the water level rise when the water is 2 inches deep? Be sure to include units with your answer. NOTE: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

(Texas A & M Univ. Final Exam '08)

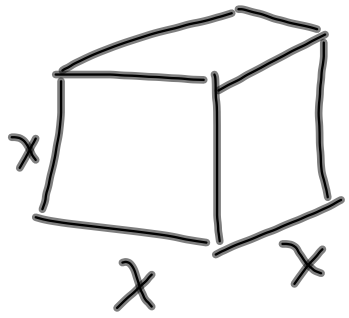
(7 pts) Cyclist A starts at point P and rides north at 15 mph. At the same time, cyclist B starts 10 miles east of point P and rides east at 15 mph. How fast is the distance between them changing after 2 hours?

(Texas A & M Univ. Final Exam '12)

15. (10 points) Two honeybees sitting together in a hive start flying in search of flower juice. One flies north at 3 ft/s and the other flies east at 4 ft/s. At what rate is the distance between the honeybees increasing 10 seconds later.

(Kansas State University: Final Exam 08)

b.



$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$-2 = 3(10)^2 \frac{dx}{dt}$$

$$-2 = 300 \frac{dx}{dt}$$

$$-\frac{2}{300} \text{ (m/min.)} = \frac{dx}{dt}$$

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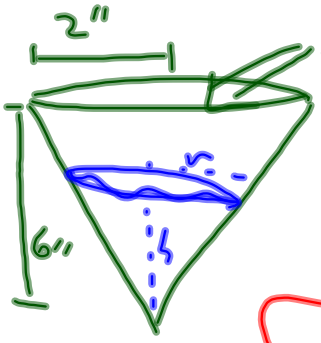


$\frac{dV}{dt} = \frac{5}{2}$ in.^{3}/sec

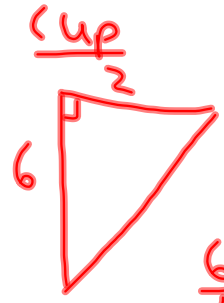
$\frac{dh}{dt} = ?$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{9}h^2\right)h$

$V = \frac{1}{27}\pi h^3$



$\frac{6}{h} = \frac{2}{r}$

$6r = 2h$

$r = \frac{2h}{6}$

$r = \frac{1}{3}h$

$\frac{dV}{dt} = \frac{1}{27}\pi\left(3h^2\frac{dh}{dt}\right)$

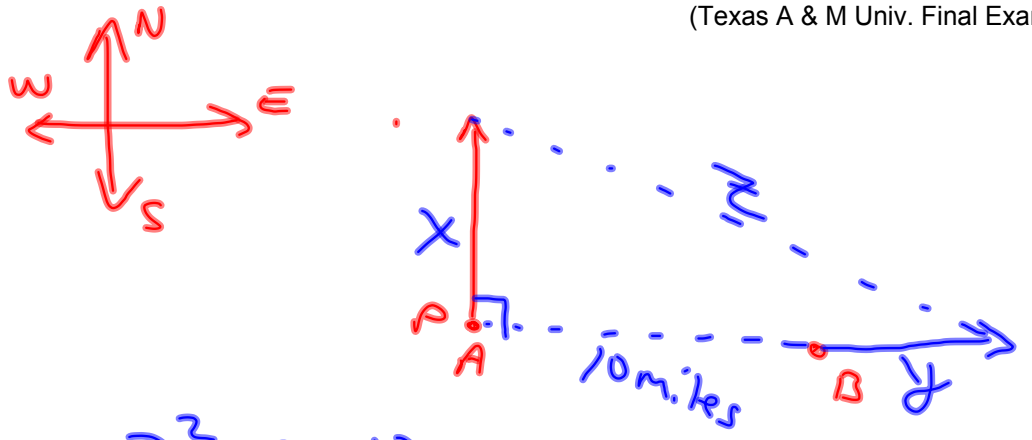
$\frac{5}{2} = \frac{1}{27}(\pi)(3)(2)^2\frac{dh}{dt}$

$\frac{5}{2}\left(\frac{9}{4\pi}\right) = \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{45}{8\pi} = 1.8$ inches/sec

(7 pts) Cyclist A starts at point P and rides north at 15 mph. At the same time, cyclist B starts 10 miles east of point P and rides east at 15 mph. How fast is the distance between them changing after 2 hours?

(Texas A & M Univ. Final Exam '12)



$$x^2+(10+y)^2=z^2$$

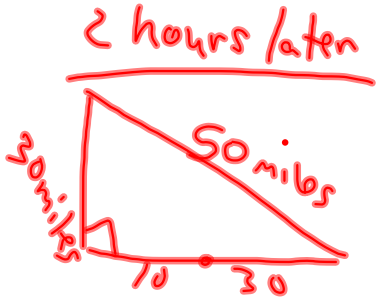
$$2x\frac{dx}{dt}+2(10+y)^1\left(\frac{dy}{dt}\right)=2z\frac{dz}{dt}$$

$$30(15)+(10+30)(15)=50\frac{dz}{dt}$$

$$450+600=50\frac{dz}{dt}$$

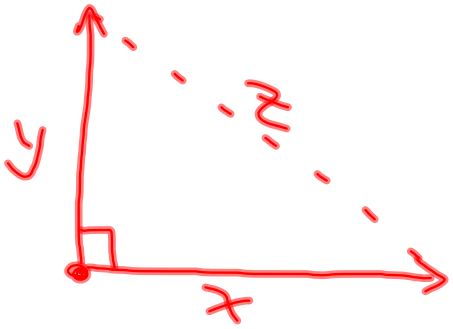
$$\frac{1050}{50}=\frac{dz}{dt}$$

$$21\text{ mph.}=\frac{dz}{dt}$$



15. (10 points) Two honeybees sitting together in a hive start flying in search of flower juice. One flies north at 3 ft/s and the other flies east at 4 ft/s. At what rate is the distance between the honeybees increasing 10 seconds later.

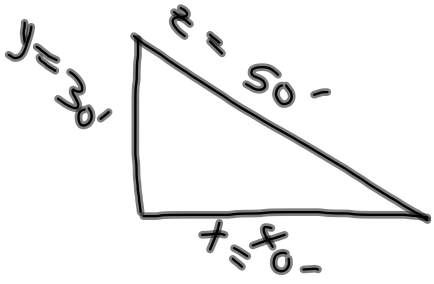
(Kansas State University: Final Exam 08)



$x^2+y^2=z^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

10 sec. later ...



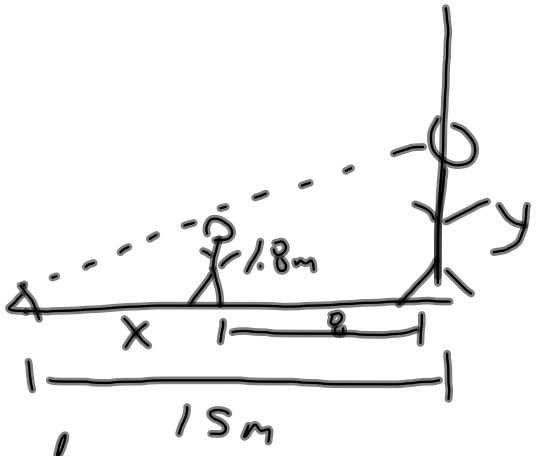
$40(4)+30(3)=50\frac{dz}{dt}$

$160+90=50\frac{dz}{dt}$

$\frac{250}{50}=\frac{dz}{dt}$

$5\,ft./sec=\frac{dz}{dt}$

6/



$\frac{dx}{dt} = -2 \text{ m/s}$

$\frac{1.8}{y} = \frac{x}{15}$

$xy = 27$

$\frac{dx}{dt} y + x \frac{dy}{dt} = 0$

$(-2)\left(\frac{5}{2}\right) + (7)\frac{dy}{dt} = 0$

$7\frac{dy}{dt} = 5$

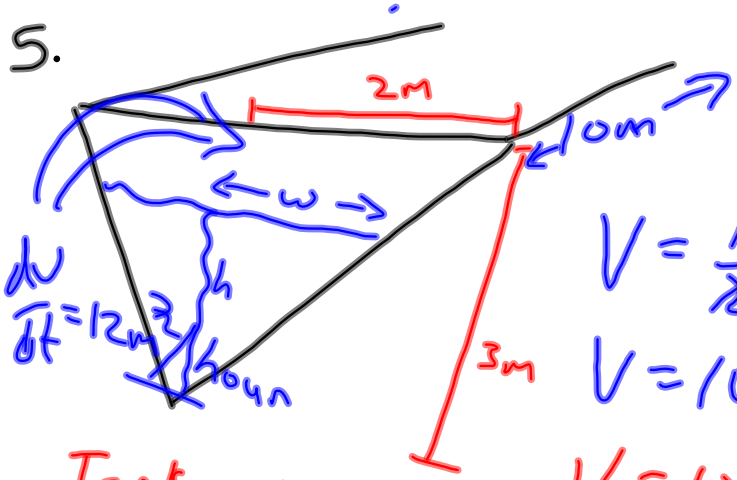
$\frac{dy}{dt} = \frac{5}{7} \text{ m/s}$

$xy = 27$

$7y = 27$

$y = \frac{27}{7}$

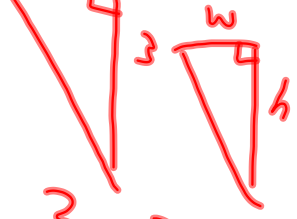
5.



$V=\frac{1}{2}(2w)h(10)$

$V=10wh$

Tank



Water

$\frac{2}{w}=\frac{3}{h}$

$3w=2h$

$w=\frac{2}{3}h$

$V=10\left(\frac{2}{3}h\right)h$

$V=\frac{20}{3}h^2$

$\frac{dV}{dt}=\frac{20}{3}h\frac{dh}{dt}$

$12=\frac{20}{3}(1)\frac{dh}{dt}$

$\frac{36\ m/hour}{40}=\frac{dh}{dt}$

$\frac{9}{10}=\frac{dh}{dt}$

$0.9\ m/h=\frac{dh}{dt}$

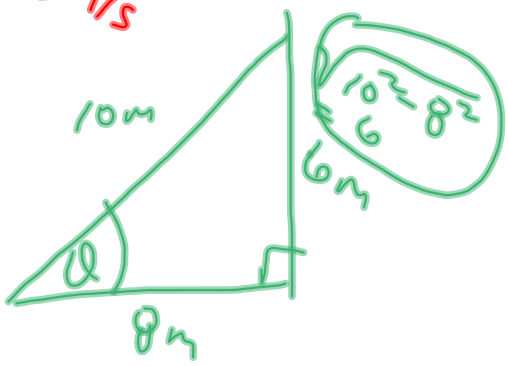
8.

$\frac{dx}{dt} = 3 m/s$

$\frac{d\theta}{dt} = ?$

$\cos\theta = \frac{x}{10}$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$



$-\frac{6}{10}\frac{d\theta}{dt} = \frac{1}{10}(3)$

$\frac{d\theta}{dt} = \frac{3}{10}\left(\frac{(10)}{-6}\right)$

$\frac{d\theta}{dt} = -\frac{1}{2}$ Rad/sec