Warm Up

University of British Columbia: 2004

For the function

$$f(x)=\frac{x^2}{x^2-1}$$

determine the critical points, local maxima and minima, and intervals where f(x) is increasing or decreasing.

$$f'(x)=\frac{2x(x^2-1)-x^2(2x)}{(x^2-1)^2}$$

Critical Values: $f'(x)=0$ or $f'(x)$ undefined

$$f'(x)=\frac{2x^3-2x-2x^3}{(x^2-1)^2}$$

$$f'(x)=\frac{-2x}{(x^2-1)^2}$$

$f'(x)=0$ (Set Numerator $=0$)

$-2x=0$

$x=0$

(Set Den. $=0$)

$f'(x)$ undefined

$(x^2-1)^2=0$

$[(x-1)(x+1)]^2=0$

$x=1,-1$

First Derivative Sign Table

Intervals	$-2x$	$(x^2-1)^2$	f'	f
$(-\infty,-1)$	+	+	+	Inc
$(-1,0)$	+	+	+	Inc Dec
$(0,1)$	−	+	−	Dec
$(1,\infty)$	−	+	−	Dec

Intervals of...

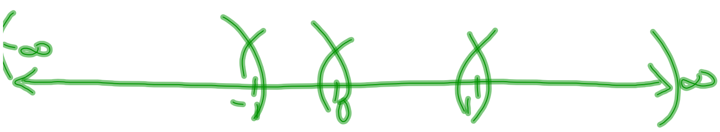
① Increase

$(-\infty,0)$

$x<0$

② Decrease

$(0,\infty)$



Local Max

$(0,0)$

Local Min.

$$\frac{(-1+i\sqrt{3})^5(-1-i)^4}{(2+2i\sqrt{3})^2}$$

Polar Form: $(-1, \sqrt{3})$ Quad. 2

Polar form: $\boxed{r\operatorname{cis}\theta}$

$180-\theta$

$r=\sqrt{1+3}$

$r=2$

$\tan\theta = \frac{\sqrt{3}}{1}$

(Ref∢: $60°$, Q2)

$\theta = 120°$

$=(2\operatorname{cis}120°)^5$

$=32\operatorname{cis}(600°)$

$-1-i \Rightarrow (-1,-1)$

$r=\sqrt{1+1}$

$r=\sqrt{2}$

$\tan\theta=1$

(Ref∢$=45°$, Q3)

$\theta = 180+45$

$\theta = 225°$

$\Rightarrow (\sqrt{2}\operatorname{cis}225°)^4$

$\underline{4\operatorname{cis}900°}$

$2+2i\sqrt{3} \Rightarrow (2, 2\sqrt{3})$ Q1

$r=\sqrt{4+12}$

$r=4$

$\tan\theta = \frac{2\sqrt{3}}{2}$

$=\sqrt{3}$

(Ref∢$=60°$, Q1)

$\theta = 60°$

$(4\operatorname{cis}60°)^2$

$=16\operatorname{cis}120°$

$$=\frac{(\cancel{32}^{\,2}\operatorname{cis}600°)(4\operatorname{cis}900°)}{\cancel{16}_{\,1}\operatorname{cis}120°}$$

$= 8\operatorname{cis}(600+900-120)$

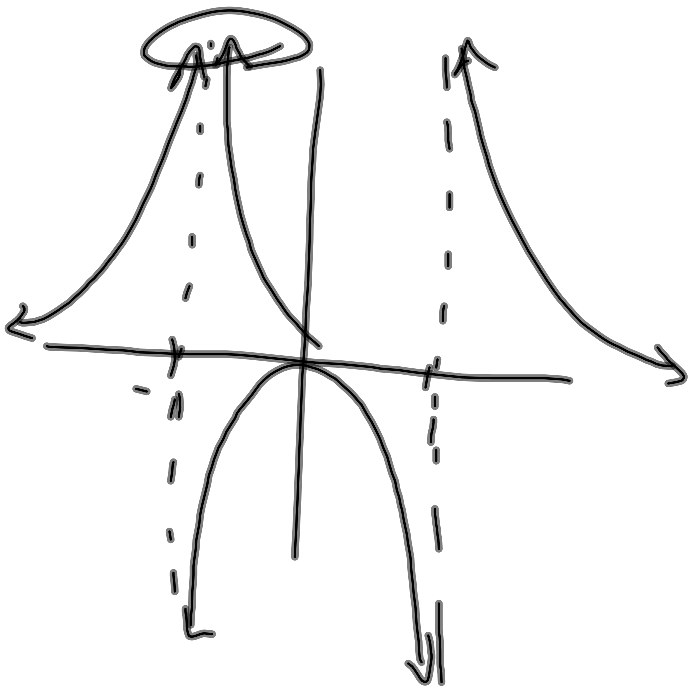
Rectangular

$= 8\operatorname{cis}(1380°)$

$= 8(\cos 1380° + i\sin 1380°)$

$= 8\left(\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 4-4i\sqrt{3}$



6. a) $y = \left(\frac{x+3}{3x-6}\right)^3$ at $\underline{x=3}$

$y' = 3\left(\frac{x+3}{3x-6}\right)^2 \left[\frac{(1)(3x-6)-(x+3)(3)}{(3x-6)^2}\right]$

$M = 3\left(\frac{6}{3}\right)^2 \left(\frac{1(3)-(6)(3)}{(3)^2}\right)$

$m = \cancel{3}(4)\left(-\frac{\cancel{15}\;5}{\cancel{9}\;3}\right)$

$m = -\frac{20}{1}$ (tangent)

Normal . . .

$m = \frac{1}{20}$

$y = \left(\frac{3+3}{3(3)-6}\right)^3 = 8$

$(3, 8)$

$y - 8 = \frac{1}{20}(x-3)$

b) $f(x) = \begin{cases} 3 & , x \leq -2 \\ x^2 & , -2 < x < 1 \\ 3-k & , x \geq 1 \end{cases}$

(1) $\lim_{x \to -2^-} f(x)$ ← <u>less than</u>

$= 3$

(2) $\lim_{x \to -2^+} f(x)$ ← Right side (greater)

$= (-2)^2 = 4$

(3) $f(-2)$

$= 3$

(iv) ① $f(1)$ must exist

② $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

③ $f(1) = \lim_{x \to 1} f(x)$

$\lim_{x \to 1^-} f(x)$

$= (1)^2$

$= 1$

$\lim_{x \to 1^+} f(x) = 1$

$3 - k = 1$

$k = 2$

7. $s(t) = 40t^2 + 60t$ $\Rightarrow$ $40 = 40t^2 + 60t$

$40t^2 + 60t - 40 = 0$

$2t^2 + 3t - 2 = 0$

$2t^2 + 4t - t - 2 = 0$

$2t(t+2) - 1(t+2) = 0$

$(t+2)(2t-1) = 0$

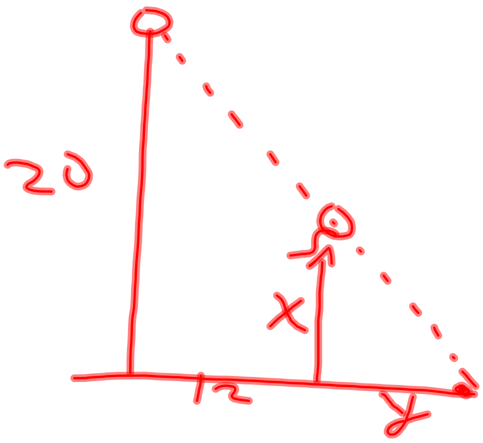
$\cancel{t = -2}$ $\quad t = \frac{1}{2}$

$s'(t) = 80t + 60$

$s'\left(\frac{1}{2}\right) = 80\left(\frac{1}{2}\right) + 60$

$= \underline{100 \text{ cm/s}}$

5.



$$\frac{dx}{dt} = 8 \text{ ft./sec}$$

$$\frac{20}{x} = \frac{12+y}{y}$$

$$20y = 12x + xy$$

$$20\frac{dy}{dt} = 12\frac{dx}{dt} + \frac{dx}{dt}y + x\frac{dy}{dt}$$