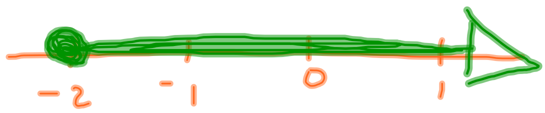
Bracket Notation



$x \geq -2$

$[-2, \infty)$

$(-1, 2]$

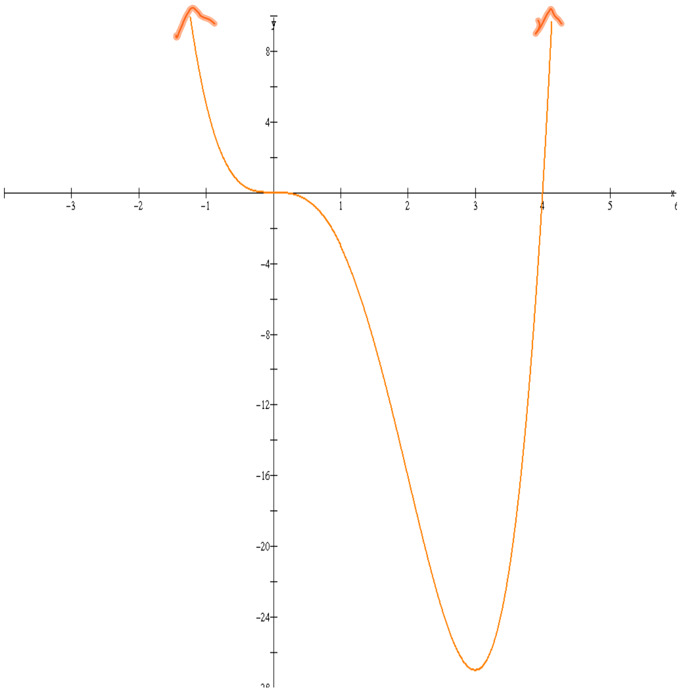
$(-\infty, 3)$

$(2, 3)$

Intervals of Increase and Decrease...

Given the function $f(x) = x^4 - 4x^3$, use the graph below to determine the intervals where *f(x)* is increasing and where *f(x)* is decreasing.

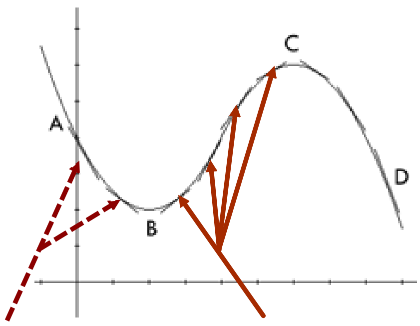
Graph...



The Calculus of Intervals of Increase and Decrease

- Examine this graph for intervals of increase and decrease...

Intervals of increase?



Intervals of decrease?

What do you notice about the slopes of these tangents?

What do you notice about the slopes of these tangents?

Critical Value(s):

Any value of x such that $f'(x) = 0$ or $f'(x)$ is undefined.

Where does $f(x)$ switch from increasing to decreasing?
How would this tie in with Calculus?

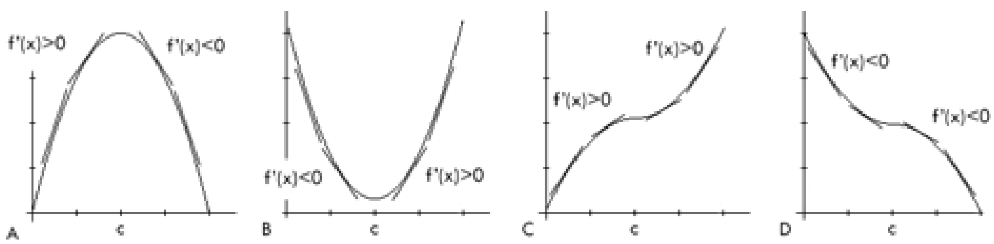
- *At the point where a function switches from increasing to decreasing, or decreasing to increasing, the derivative must be equal to 0 or undefined.*

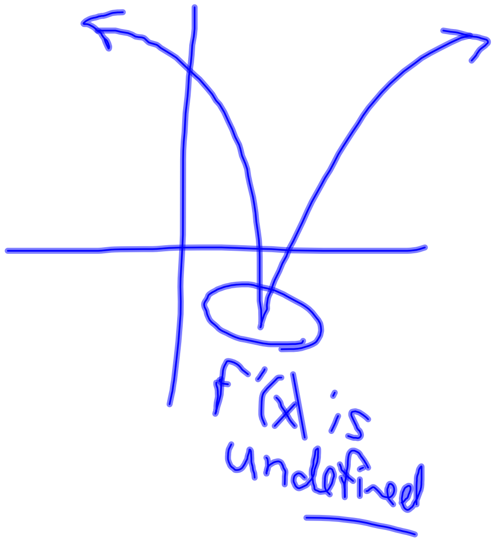
Let's summarize how Calculus could be used to identify regions of increase or decrease...

If $f'(x) > 0$ ⟶

If $f'(x) < 0$ ⟶

The graphs below illustrate the first derivative test.




f'(x) is undefined

Absolute Maxima/Minima

A function *f* has an absolute (or global) maximum at *c* if $f(c) \geq f(x)$ for all *x* in the domain D of *f*.

- The number $f(c)$ is called the maximum value of *f* on D.

A function *f* has an absolute (or global) minimum at *c* if $f(c) \leq f(x)$ for all *x* in the domain D of *f*.

- The number $f(c)$ is called the minimum value of *f* on D.

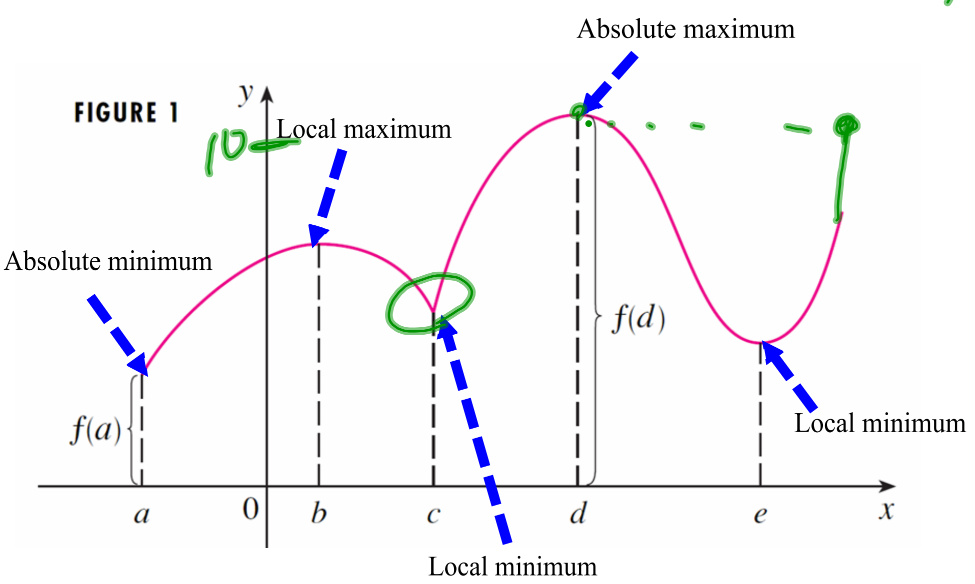
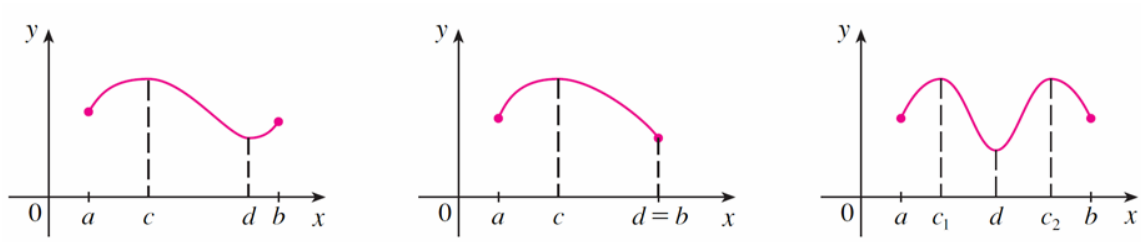


FIGURE 1

3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



Here are a couple of examples to reinforce that the function must be continuous over a closed interval.

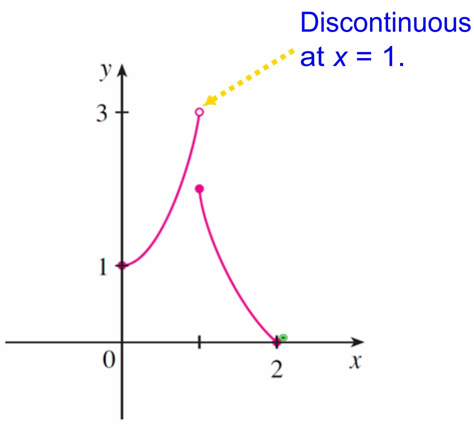


FIGURE 6
This function has minimum value $f(2) = 0$, but no maximum value.

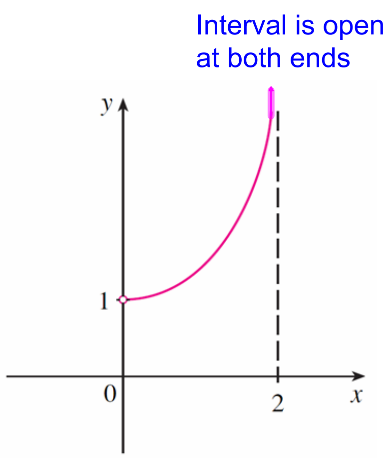


FIGURE 7
This continuous function g has no maximum or minimum.

5 Definition A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

Example:

Find the critical values of $f(x) = x^{\frac{3}{5}}(4-x)$ and determine all intervals of increase and decrease as well as any local extrema.

$$f'(x) = \frac{3}{5}x^{-2/5}(4-x) + x^{3/5}(-1)$$

$$0 = \frac{3(4-x)}{5x^{2/5}} - x^{3/5}$$

$$0 = \frac{3(4-x) - x^{3/5}(5x^{2/5})}{5x^{2/5}} = \frac{12-3x-5x}{5x^{2/5}}$$

$$0 = \frac{12-8x}{5x^{2/5}}$$

Set Numerator = 0

$$12-8x=0$$

$$\frac{12}{8} = \frac{8x}{8}$$

$$\frac{3}{2} = x$$

$x=0$ $f'(x) = \text{und}$

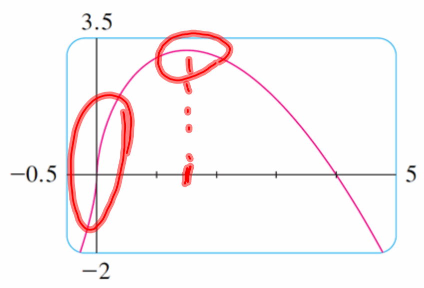


FIGURE 11

Sign Table:

	$12-8x$	$5x^{2/5}$	f'	f
$(-\infty, 0)$	+	+	+	Inc
$(0, \frac{3}{2})$	+	+	+	Inc
$(\frac{3}{2}, \infty)$	−	+	−	Dec

Local MAX. $(\frac{3}{2}, y)$

How do we determine absolute maximum and minmum values?

The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b). & $f(b)$
2. Find the values of f at the endpoints of the interval. $f(a)$
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Example:

Solution

Given the function...

$$f(x) = 3x^4 - 16x^3 + 18x^2 \ , \ -1 \le x \le 4$$

Determine the absolute maximum and minimum values of the function.

Critical Values:

$f'(x) = 12x^3 - 48x^2 + 36x$ $\quad [-1, 4]$

$0 = 12x(x^2 - 4x + 3)$

$0 = 12x(x-3)(x-1)$

$x = 0, 1, 3$ $\quad [-1, 4]$

$-1 \le x \le 4$

x	y
-1	37
0	0
1	5
3	-27
4	32

Abs Max = 37

Abs. Min. = -27

Example 2:

Using Calculus methods determine the absolute maximum and minimum values of the function given below:

$f(x) = x^3 + 2x^2 + x - 1$ over the interval [-1 , 1]

$f'(x) = 3x^2 + 4x + 1$

$0 = 3x^2 + 4x + 1$

$0 = 3x^2 + 3x + x + 1$

$0 = 3x(x+1) + 1(x+1)$

$(x+1)(3x+1) = 0$

$x = -1, -\frac{1}{3}$

$\left(-\frac{1}{3}\right)^3 + 2\left(-\frac{1}{3}\right)^2 + -\frac{1}{3} - 1$

$-\frac{1}{27} + \frac{2}{9} - \frac{1}{3} - 1$

x	y
-1	-1
$-\frac{1}{3}$	-1.148
1	3

Abs. Max. = 3

Abs. Min. = -1.148

Warm-Up

Given the function $f(x) = 4x^4 - 8x^2 + 1$ determine ...

(a) the absolute maximum and minimum values on the interval [0 , 3].

(b) the intervals of increase/decrease and local extreme values on the interval (-∞ , ∞).

(a) $f'(x) = 16x^3 - 16x$

$0 = 16x(x^2 - 1)$

$0 = 16x(x-1)(x+1)$

Critical Values:

$x = 0, 1, -1$ outside $(0, 3)$

x	y
0	1
1	-3
3	253

Abs. Max = 253

Abs. Min. = -3

(b)

	16x	x-1	x+1	f'	f
(-∞, -1)	−	−	−	−	Dec
(-1, 0)	−	−	+	+	Inc
(0, 1)	+	−	+	−	Dec
(1, ∞)	+	+	+	+	Inc

Min.

MAX

Min

Homework

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#3 d, f, l
#4 d, f, h, i
#6
#7