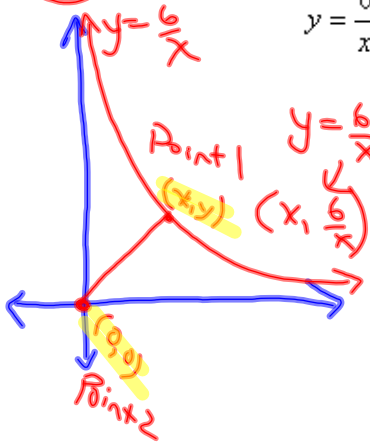
Warm Up

Determine the coordinates of the point closest to the origin on the graph of $y = \frac{6}{x}, \; x > 0$



Minimize Distance

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-0)^2 + (y-0)^2}$

$d = \sqrt{x^2 + y^2}$

$d = \sqrt{x^2 + \left(\frac{6}{x}\right)^2}$

$d = \sqrt{x^2 + 36x^{-2}}$

$d' = \frac{1}{2}(x^2 + 36x^{-2})^{-\frac{1}{2}}(2x - 72x^{-3})$

$0 = \frac{x - 36x^{-3}}{\sqrt{x^2 + 36x^{-2}}} \Rightarrow$ Numerator $= 0$

$\frac{2x - 72x^{-3}}{2\sqrt{x^2 + 36x^{-2}}} = 0$

$x - \frac{36}{x^3} = 0$

$x^4 - 36 = 0$

$x^4 = 36$

$x = \sqrt[4]{36}$

Undefined ($f'(x)$)

$\sqrt{x^2 + \frac{36}{x^2}} = 0$

$\emptyset$

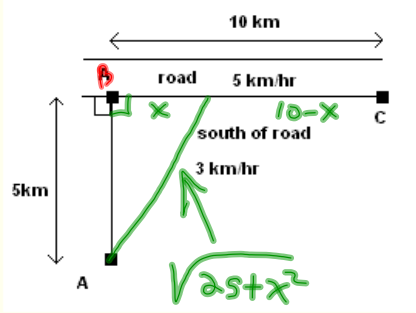
$y = \frac{6}{x}$

Point Closest

$\left(\sqrt[4]{36}, \frac{6}{\sqrt[4]{36}}\right)$

Example 5:

You decide to walk from point A (see figure below) to point C. To the south of the road through BC, the terrain is difficult and you can only walk at 3 km/hr. However, along the road BC you can walk at 5 km/hr. The distance from point A to the road is 5 km. The distance from B to C is 10 km. What path you have to follow in order to arrive at point C in the shortest (minimum) time possible?



$t = \frac{d}{v}$

$\frac{1}{h}$

Minimize TIME!!

$$\text{Time} = \frac{\sqrt{25+x^2}}{3} + \frac{10-x}{5}$$

Woods Road

$$\text{Time} = \frac{1}{3}(25+x^2)^{1/2} + 2 - \frac{1}{5}x$$

$$T' = \frac{1}{6}(25+x^2)^{-1/2}(2x) - \frac{1}{5}$$

$$0 = \frac{2x}{6\sqrt{25+x^2}} - \frac{1}{5}$$

$$\left(\frac{1}{5}\right)^2 = \left(\frac{2x}{6\sqrt{25+x^2}}\right)^2$$

$$\frac{1}{25} = \frac{4x^2}{36(25+x^2)}$$

$$36(25+x^2) = 100x^2$$

$$900 + 36x^2 = 100x^2$$

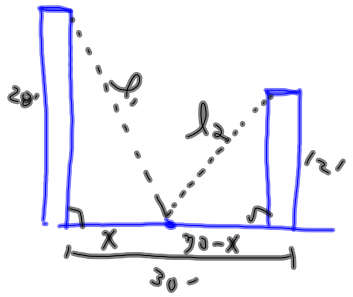
$$\frac{900}{64} = \frac{64x^2}{64}$$

$$\sqrt{14.06} = \sqrt{x^2}$$

$$3.75 \doteq x$$

Head 3.75 km [illegible]

Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached at a single stake, running from ground level to the top of each post. Where should the stake be placed to use the least amount of wire?



$$\text{Total Wire} = l_1 + l_2$$

$$W = \sqrt{x^2+784} + \sqrt{(30-x)^2+144}$$

$$W = \sqrt{x^2+784} + \sqrt{900-60x+x^2+144}$$

$$W = \sqrt{x^2+784} + \sqrt{1044-60x+x^2}$$

$$W' = \frac{1}{2}(x^2+784)^{-1/2}(2x) + \frac{1}{2}(1044-60x+x^2)^{-1/2}(-60+2x)$$

$$0 = \frac{x}{\sqrt{x^2+784}} + \frac{x-30}{\sqrt{x^2-60x+1044}}$$

$$\left[\frac{+(x-30)}{\sqrt{x^2-60x+1044}}\right]^2 = \left[\frac{x}{\sqrt{x^2+784}}\right]^2$$

$$\frac{(x-30)^2}{x^2-60x+1044} = \frac{x^2}{x^2+784}$$

$$x^4-60x^3+1044x^2 = (x-30)^2(x^2+784)$$

$$x^4-60x^3+1044x^2 = (x^2-60x+900)(x^2+784)$$

$$x^4-60x^3+1044x^2 = x^4+784x^2-60x^3-47040x+900x^2+705600$$

$$1044x^2-784x^2-900x^2+47040x-705600=0$$

$$-640x^2+47040x-705600=0$$

$$x = \frac{-47040 \pm \sqrt{(47040)^2-4(-640)(-705600)}}{2(-640)}$$

$$x = \frac{-47040 \pm 20160}{-1280}$$

$x = 21$ and $x = 52.5$

Continue practice problems...

Page 190

#10, 11, 12, 14, 16, 17, 19

Practice:
Pg. 187-191