Middlebury College 2012

1. Evaluate each of the following integrals. If there is a particular technique that you use, name it.

(a) $\int x \sec^2(x)\, dx$

$u = x \quad dv = \sec^2 x\, dx$

$du = dx \quad v = \tan x$

$= x\tan x - \int \tan x\, dx$

$= x\tan x - \int \frac{\sin x}{\cos x}\, dx$

$= x\tan x + \ln|\cos x| + C$

(b) $\int \tan^2 x \sec^4 x\, dx$

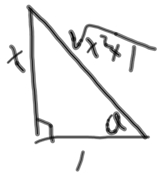
$\int \tan^2 x(\sec^2 x)\sec^2 x\, dx$

$\int \tan^2 x(1+\tan^2 x)\sec^2 x\, dx$

$\int \tan^2 x \sec^2 x\, dx + \int \tan^4 x \sec^2 x\, dx$

$= \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$

(c) $\int_0^1 \frac{1}{(x^2+1)^2}\, dx$ (Hint: use a trigonometric substitution.)



$\tan\theta = x$

$\sec\theta = \sqrt{x^2+1}$

$\sec^2\theta = x^2+1$

$\sec^2\theta\, d\theta = dx$

$= \int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^2}$

$\int \frac{1}{\sec^2\theta}\, d\theta$

$\int \cos^2\theta\, d\theta \longrightarrow \cos 2\theta = 2\cos^2\theta - 1$

$\frac{1}{2}(\cos 2\theta + 1) = \cos^2\theta$

$\frac{25}{7}$

$7\overline{)25}$ 3, 21, 4

$= 3 + \frac{4}{7}$

$\left(\frac{1}{2}\right)\frac{1}{2}\int \cos 2\theta\,(2)\, d\theta + \frac{1}{2}\int d\theta$

$\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta + C$

$= \frac{1}{4}(2\sin\theta\cos\theta) + \frac{1}{2}\tan^{-1}x$

$= \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) + \frac{1}{2}\tan^{-1}x \Big|_0^1$

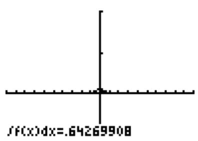
$= \frac{1}{2}\frac{x}{(x^2+1)} + \frac{1}{2}\tan^{-1}x = \frac{1}{2}\left(\frac{x}{x^2+1} + \tan^{-1}x\right)$

$= \frac{1}{2}\left[\left(\frac{1}{2} + \tan^{-1}(1)\right) - \left(0 + \tan^{-1}0\right)\right]$

$= \frac{1}{2}\left[\left(\frac{1}{2} + \frac{\pi}{4}\right) - (0+0)\right]$

$= \frac{1}{2}\left(\frac{2+\pi}{4}\right)$

$= \frac{2+\pi}{8}$



(2+π)/8

.6426990817

(d) $\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$

$$\begin{array}{r} x+1 \\ x^2-x-6 \overline{)\,x^3-4x-10} \\ \underline{x^3-x^2-6x} \\ x^2+2x-10 \\ \underline{x^2-x-6} \\ 3x-4 \end{array}$$

$$\int\left[(x+1)+\frac{3x-4}{(x-3)(x+2)}\right]dx$$

$$\int x\,dx+\int 1\,dx+\int\frac{3x-4}{(x-3)(x+2)}dx$$

$$\frac{A}{x-3}+\frac{B}{x+2}=\frac{3x-4}{(x-3)(x+2)}$$

$$A(x+2)+B(x-3)=3x-4$$

$\times 2$ $(A+B=3)$ $\quad 2A-3B=-4$

$$\underline{2A+2B=6}$$

$$-5B=-10$$

$A+2=3$ $\quad B=2$

$A=1$

$$\int x\,dx+\int 1\,dx+\int\frac{dx}{x-3}+2\int\frac{dx}{x+2}$$

$$\frac{x^2}{2}+x+\ln|x-3|+2\ln|x+2|\Big|_0^1$$

$$=\left(\frac{1}{2}+1+\ln 2+2\ln 3\right)-\left(0+0+\ln 3+2\ln 2\right)$$

$$=\frac{3}{2}+\ln 2+2\ln 3-\ln 3-2\ln 2$$

$$=\frac{3}{2}-\ln 2+\ln 3$$

OR

$$=\frac{3}{2}+\ln\frac{3}{2}$$

$$\frac{3}{2}+(\ln 3-\ln 2)$$