Warm Up

2. (a) Show that

$$y = c_1 \sin 2x + 3 \cos 2x$$

$y' = 2c_1 \cos 2x - 6 \sin 2x$

$y'' = -4c_1 \sin 2x - 12 \cos 2x$

is a general solution for the differential equation

$$\frac{d^2y}{dx^2} + 4y = 0$$

LS

$(-4c_1 \sin 2x - 12 \cos 2x) + 4(c_1 \sin 2x + 3\cos 2x)$

0

RS

0

LS = RS

(b) Show that $\frac{d^2y}{dx^2} = 2\frac{dy}{dx}$ has a solution of $y = c_1 + c_2e^{2x}$

$y' = 2c_2e^{2x}$

$y'' = 4c_2e^{2x}$

LS	RS
$4c_2e^{2x}$	$2(2c_2e^{2x})$
	$4c_2e^{2x}$

LS = RS

1997 AB6/BC6

Warm Up

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

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Solution

(a) $\frac{dv}{dt} = -2v - 32 = -2(v+16)$

$$\frac{dv}{v+16} = -2dt$$

$$\ln|v+16| = -2t + A$$

$$|v+16| = e^{-2t+A} = e^A e^{-2t}$$

$$v + 16 = Ce^{-2t}$$

$$-50 + 16 = Ce^0; \ C = -34$$

$$v = -34e^{-2t} - 16$$

(b) $\lim_{t \to \infty} v(t) = \lim_{t \to \infty} \left(-34e^{-2t} - 16\right) = -16$

(c) $v(t) = -34e^{-2t} - 16 = -20$

$$e^{-2t} = \frac{2}{17}; \ t = -\frac{1}{2}\ln\left(\frac{2}{17}\right) = 1.070$$

1997 AB6/BC6

Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$, with initial condition $v(0) = -50$.

(a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

$v = \ldots (t)$

(b) Terminal velocity is defined as $\lim_{t \to \infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

(c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

(a) $\frac{dv}{-2v-32} = \frac{\cancel{(-2v-32)}\,dt}{\cancel{-2v-32}}$

$-\frac{1}{2}\int \frac{(2)\,dv}{2v+32} = \int dt$

$-\frac{1}{2}\ln|2v+32| = t + C$

$v(0) = -50$

$-\frac{1}{2}\ln|-100+32| = C$

$-\frac{1}{2}\ln 68 = C$

$-\frac{1}{2}\ln 68 = C$

$C = e^{-\frac{1}{2}\ln 68}$

$-\frac{1}{2}\ln|2v+32| = t - \frac{1}{2}\ln 68$

$\ln_e|2v+32| = -2t + \ln 68$

$\log_b M = N$

$b^N = M$

$e^{-2t+\ln|68|} = 2v+32$

$v = \frac{e^{-2t+\ln|68|}}{2} - \frac{32}{2}$

$b^{\log_b x} = x$

$v = \frac{(e^{-2t})(e^{\ln|68|})}{2} - 16$

$v = \frac{e^{-2t}(68)}{2} - 16$

$v = 34e^{-2t} - 16$

b) $\lim_{t \to \infty} \frac{-34}{e^{2t}} - 16$

$= 0 - 16$

$= -16$ ft./second

$t = 1.070$ sec

c) $-20 = -34e^{-2t} - 16$

$-4 = -34e^{-2t}$

$\frac{2}{17} = e^{-2t}$

$\ln\frac{2}{17} = \ln e^{-2t}$

$\ln\frac{2}{17} = -2t$

$t = -\frac{1}{2}\ln\left(\frac{2}{17}\right)$

$t = 1.070$ sec

Separable differential equations:

Practice...

Page 519

#1 - 16

Problem Using separation of variables, solve $y' = 6e^{2x-y}$, $y(0) = 0$.

$$\frac{dy}{dx} = \frac{6e^{2x}}{e^y}$$

$$\int e^y \, dy = \int \frac{6}{2} e^{2x} (2) dx$$

$$e^y + C = 3\int e^{2x}(2)\,dx$$

$$e^y + C = 3e^{2x} \quad (0,0)$$

$$e^0 + C = 3e^0$$

$$C = 2$$

$$e^y + 2 = 3e^{2x}$$

$$\ln e^y = \ln(3e^{2x} - 2)$$

$$y = \ln|3e^{2x} - 2|$$

Warm-Up...

Solve each of the following differential equations:

1. $\dfrac{dy}{dx} = xye^{x^2}$

$\boxed{y = e^{\frac{1}{2}x^2 + C}}$

2. $\dfrac{dy}{dx} = 1 + y^2 + x^2 + x^2y^2$

$\boxed{y = \tan\left(\frac{x^3}{3} + x + \frac{\pi}{4}\right)}$