# Warm Up

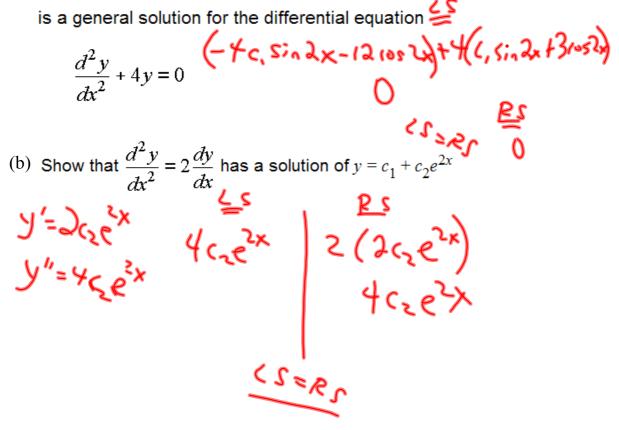
2. (a) Show that

$$y = c_1 \sin 2x + 3 \cos 2x$$

by that 
$$y' = 2c_1(\cos 2x - 6\sin 2x)$$
  
 $y = c_1 \sin 2x + 3 \cos 2x$   $y'' = -4c_1\sin 2x - 12\cos 2x$ 

is a general solution for the differential equation

$$\frac{d^2y}{dx^2} + 4y = 0$$
 (-4c, Sin 2x-12 (05 2x)+4(1, Sin 2x+31052x)



## Warm Up

### 1997 AB6/BC6

Let v(t) be the velocity, in feet per second, of a skydiver at time t seconds,  $t \ge 0$ . After her parachute opens, her velocity satisfies the differential equation  $\frac{dv}{dt} = -2v - 32$ , with initial condition v(0) = -50.

- (a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.
- (b) Terminal velocity is defined as  $\lim_{t\to\infty} v(t)$ . Find the terminal velocity of the skydiver to the nearest foot per second.
- (c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

## 1997 AB6/BC6 Solution

(a) 
$$\frac{dv}{dt} = -2v - 32 = -2(v + 16)$$
$$\frac{dv}{v + 16} = -2dt$$
$$\ln|v + 16| = -2t + A$$
$$|v + 16| = e^{-2t + A} = e^{A}e^{-2t}$$
$$v + 16 = Ce^{-2t}$$
$$-50 + 16 = Ce^{0}; C = -34$$
$$v = -34e^{-2t} - 16$$

(b) 
$$\lim_{t \to \infty} v(t) = \lim_{t \to \infty} (-34e^{-2t} - 16) = -16$$

(c) 
$$v(t) = -34e^{-2t} - 16 = -20$$
  
 $e^{-2t} = \frac{2}{17}; \ t = -\frac{1}{2}\ln\left(\frac{2}{17}\right) = 1.070$ 

#### 1997 AB6/BC6

Let v(t) be the velocity, in feet per second, of a skydiver at time t seconds,  $t \ge 0$ . After her parachute opens, her velocity satisfies the differential equation  $\frac{dv}{dt} = -2v - 32$ , with

- (a) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.
   (b) Terminal velocity is defined as lim v(t). Find the terminal velocity of the skydiver.
- (c) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

(a) 
$$\frac{dv}{-2v-3z} = \frac{1}{2v-3z}$$
 $\frac{1}{2}\ln|2v+3z| = \frac{1}{2}\ln|2v+3z| = \frac{1}{2}\ln|2v$ 

# Separable differential equations:

# Practice...

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**Problem** Using separation of variables, solve  $y' = 6e^{2x-y}$ , y(0) = 0.

$$\frac{dy}{dx} = \frac{6e^{2x}}{e^{4}}$$

$$e^{4}dy = \frac{6e^{2x}}{e^{4}}$$

$$e^{4}dy = \frac{6e^{2x}}{e^{2x}}$$

## Warm-Up...

Solve each of the following differential equations:

$$1. \frac{dy}{dx} = xye^{x^2}$$

2. 
$$\frac{dy}{dx} = 1 + y^2 + x^2 + x^2 y^2$$

$$y = \frac{1}{4} \left( \frac{x^3}{3} + x + \frac{77}{4} \right)$$

$$y = \tan\left(\frac{x^3}{3} + x + \frac{\pi}{4}\right)$$