## Warm Up

Solve the following separable differential equations:

$$2xy^{2}\frac{dy}{dx} = y^{2} + 1$$

$$\frac{dy}{dx} = \frac{4\sqrt{y \ln x}}{x}, y(e) = 1$$

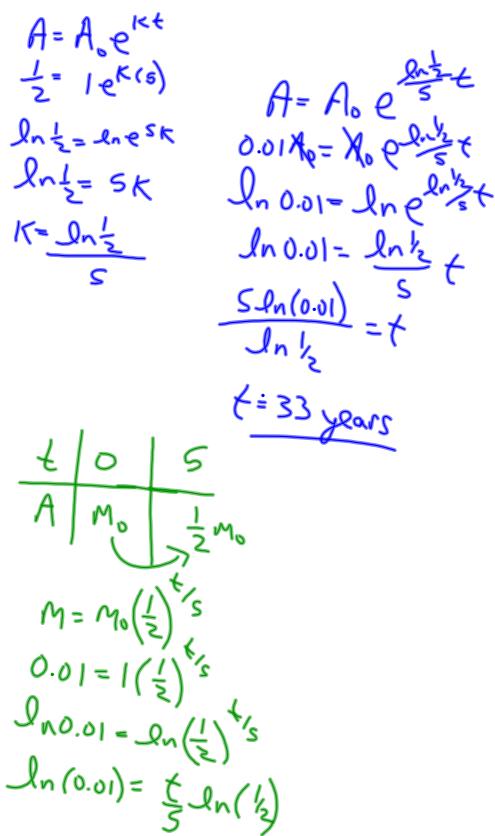
$$\frac{1}{\sqrt{y}} dy = \frac{1}{\sqrt{x}} dx$$

$$\frac{1}{\sqrt{x}} dx = \frac{1}{$$

2. One hundred fruit flies are placed in a breeding container that can support a population of at most 5000 fruit flies. If the population grows exponentially at the rate of 2% per day, how long will it take for the container to reach capacity?

$$N = N_0 e^{Kt}$$
 $5000 = 1000 e^{0.02t}$ 
 $50 = e^{0.02t}$ 
 $100 = 100 e^{0.02t}$ 
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3. A radioactive substance decays at a rate proportional to the amount present. Assuming the "half life" is 5 years, how onlg will it be until only 1% of the original substance remains?



4. A certain substance decomposes at a rate proportional to its weight. If 9 grams ofthe substance are initially present and 1 gram decomposes in the first hour, how long will it take before 5 grams have decomposed?

Mass 
$$M = M_0 e^{Kt}$$

Mass  $M = Q e^{K(1)}$ 

Remaining  $M = Q e^{K(1)}$ 
 $M = M_0 e^$ 

## **Newton's Law of Cooling**

The rate at which a hot body cools to the temperature of its surroundings is proportional to the temperature difference between the body and its surroundings.

ie. A cup of tea at 60 degrees above its surroundings cools degree by degree twice as rapidly as when it is 30 degrees above its surroundings.

\*Note: This law also applies to warming

If we let T(t) be the temperature of the object at time t and  $T_s$  be the temperature of the surroundings, then we can formulate Newton's Law of Cooling as a differential equation:

$$\frac{dT}{dt} = k(\vec{r} - T_s)$$
 Surrounding Temp. (loneton)

where k is a constant. This could be solved as a separable differential equation, but an easier method would be to change variables...

Let 
$$y(t) = T(t) - T_s$$
  

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$$y'(t) = T'(t) - 0$$

$$\therefore \frac{dy}{dt} = \frac{dT}{dt}$$

This means that our initial differential equation can be expressed as...

$$\frac{dy}{dt} = ky$$

Note: If the object is cooling k must be negative because the temperature is decreasing as time passes.

## Example:

A cup of water with a temperature of 95°C is placed in a room with a constant temperature of  $21^{\circ}$ C.

- (a) Assuming Newton's Law of Cooling applies, set up and solve an initial-value problem whose solution is the temperature of the water *t* minutes after it is placed in the room.
- (b) How many minutes will it take for the water to reach a temperature of 51°C if it cools to 85°C in 1 minute?

(a) 
$$\frac{dT}{dt} = k(T - T_s)$$

Let 
$$y(t) = T(t) - T_s$$

then 
$$\frac{dy}{dt} = ky$$
, where  $y(0) = 95^{\circ}C - 21^{\circ}C = 74^{\circ}C$ 

$$\therefore y = Ae^{kt}$$

$$A - T_i - T_s$$

$$74 = Ae^0$$
,  $A = 74$ 

$$y = 74e^{kt}$$
 now re-substitute  $y(t) = T(t) - T_s$ 

$$T(t) - T_s = 74e^{kt}$$

$$T(t) = 74e^{kt} + 21$$
 or  $T(t) = 74e^{-kt} + 21$  (decreasing)

Newton's Law of Cooling	Solution
$rac{dT}{dt} = -k(T - T_a).$ $T(0) = T_o$	$T(t) = T_a + (T_o - T_a)e^{-kt}$

(b)  $T(t) = 74e^{kt} + 21$  Must first determine the proportionality constant

$$T(1) = 74e^k + 21 = 85$$

$$\therefore 74e^k = 64$$

$$e^k = \frac{64}{74}$$

$$k = \ln \frac{64}{74} = -0.145$$

$$T(t) = 74e^{-0.145t} + 21$$

$$51 = 74e^{-0.145t} + 21$$

$$30 = 74e^{-0.145t}$$

$$e^{-0.145t} = \frac{30}{74} \therefore -0.145t = \ln \frac{30}{64}$$
$$t = 6.227 \text{ minutes}$$