Here is another neat trick...

$$\int \sec x dx =$$

Hint 1...here is what the answer will look like with this trick

$$\ln|\sec(\mathbf{x} + \tan(\mathbf{x}))| + C$$

Serv dx Serxteanx

= In/Konxtrecx/+C

Hint 2...It is always legal to multiply by 1

# Warm Up

#### Evaluate each of the following:

$$\int \frac{1}{(16-x^2)^{\frac{3}{2}}} \, dx \quad \text{(UNB: 2005)}$$

$$\int \frac{\sqrt{2-x^2}}{x} \ dx \ \ (UNB: 2004)$$

$$\int (\csc^2 t \cot^2 t) \ dt \ \text{(UNB: 2004)}$$

$$\int \frac{\ln(\ln x)}{x} dx$$
 (StFX: 2005)

$$\frac{\sqrt{5}}{\sqrt{5}} \left( \frac{-\sqrt{5}}{\sqrt{5}} \sqrt{10} \right) + \frac{100}{5} + \frac{100}{5}$$

## Integration using Partial Fractions

Simplify: 
$$\frac{3}{x-5} + \frac{2}{x+4}$$

We want to reverse the process of finding a common denominator...

Express as partial fractions:  $\frac{5x+2}{x^2-x-20}$ 

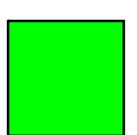
- 1. Factor the denominator:  $\frac{5x+2}{(x-5)(x+4)}$
- 2. Separate into partial fractions:

$$\frac{A}{x-5} + \frac{B}{x+4} = \frac{5x+2}{x^2 - x - 20}$$

3. Find common denominator and solve for A and B:

Now let's evaluate the following integral...

$$\int \frac{(5x+2)dx}{x^2 - x - 20}$$



Here is another example...

$$\int \frac{xdx}{x^2 - 3x + 2}$$

## Some special situations involving partial fractions...

### Note 1: \_\_\_\_

• If the degree of the numerator is the same as that of the denominator, or higher, we would have to take the preliminary step of first performing a long division.

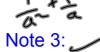
Ex. 
$$\frac{2x^3 - 11x^2 - 2x + 2}{2x^2 + x - 1}$$

$$2x^{2} + x + 1 \overline{\smash{\big)}\,2x^{3} - 11x^{2} - 2x + 2} = (x - 6) + \frac{5x - 4}{(x + 1)(2x - 1)}$$

### Note 2:

• If the denominator has more than two linear factors, we must include a term corresponding to each factor.

Ex. 
$$\frac{x+6}{x(x-3)(4x+5)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{4x+5}$$



$$\frac{1}{2}$$
  $\frac{1}{2}$   $\frac{3}{2}$   $\frac{4}{2}$   $\frac{1}{2}$   $\frac{1}$ 

• If a linear factor is repeated, we need to include extra terms in the partial fraction expression.

Ex. 
$$\frac{x}{(x+3)^2(x-2)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-2}$$

#### Note 4:

• When we factor the denominator as far as possible, it may happen that we end up with an irreducible quadratic factor of the form  $ax^2 + bx + c$ , where the discriminant is negative. Then the corresponding partial fraction is of the form...



$$\frac{Ax+B}{ax^2+bx+c}$$

$$\frac{Ax+B}{ax^2+bx+c} \qquad \frac{Ax+Bx+C}{ax^2+bx+c}$$

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where A and B are constants to be determined. This term can be integrated by completing the square and by using the integration formula...

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C \qquad (or \ a \ trig. \ substitution)$$

$$2x^{2} + x + 1) 2x^{3} - 11x^{2} - 2x + 2 = (x - 6) + \frac{5x - 4}{(x + 1)(2x - 1)}$$

$$2x^{2} + x^{2} - x$$

$$2x^{2} + x - 1 - 12x^{2} - x + 2$$

$$2x^{2} + 2x - x - 1 - 12x^{2} - 6x + 6$$

$$2x(x+1) - 1(x+1)$$

$$Ex = \begin{cases} 2x^{3} - 11x^{2} - 2x + 2 \\ 2x^{2} + x - 1 \end{cases}$$

$$(x - 6) + \begin{cases} 5x - 4 \\ (2x - 1)(x + 1) \end{cases}$$

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Ex. 
$$\frac{x+6}{x(x-3)(4x+5)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{4x+5}$$

 $A(x-3)(4x+5) + B(x)(4x+5) + C(x)(x-3) = \chi + 6$   $A(4x^2-7x-15) + B(4x^2+5x) + C(x^2-3x) = \chi + 6$  $4Ax^2-7Ax-15A+4Bx^2+5Bx+(x^2-3Cx=x+6)$ 

$$|7B = 3$$

$$|8 = \frac{3}{17}$$

Examples...

(a) 
$$\int \frac{x-1}{x(x^{2}+2x+1)} dx$$
(b) 
$$\int \frac{2x^{2}+x+16}{(x-2)(x^{2}+9)} dx$$

$$\int \frac{x-1}{x(x+1)^{2}} dx \qquad \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^{2}} = \frac{x-1}{x(x+1)^{2}}$$

$$A(x+1)^{2} + Bx(x+1) + Cx = x-1$$

$$A(x^{2}+2x+1) + Bx^{2}+Bx+Cx = x-1$$

$$A(x^{2}+2x+1) + Bx^{2}+Bx+Cx = x-1$$

$$A+B=0 \quad 2A+B+C=1 \quad A=-1$$

$$-1+B=0 \quad 2(-1)+1+C=1$$

$$B=1 \quad C=2$$

$$-1+B=0 \quad 2(-1)+1+C=1$$

$$C=2$$

$$-1+C=1$$

$$C=2$$

# Practice Questions...

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#9, 10, 11, 13, 14, 17, 19, 21, 23, 25, 27