Warm Up

Peter and Mary have purchased a home in an affluent neighbourhood for $225 000. The real estate agent informs them that homes in this area have generally appreciated by 10% every 5 years. Based on this, how much should they be able to sell their home for in 12 years?

$y = 225000(1.10)^{t/5}$

$y = 225000(1.10)^{12/5}$

$y = \$282829.67$

The Logarithmic Function

Logarithms have become important in the study of many geological phenomena. For instance, seismologists have devised a scale , called the Richter Scale, to describe the intensity of an earthquake. It measures the energy released at the centre of focus of an earthquake. The scale is logarithmic with values that record earthquakes from 1 to 10.

Richter Scale

http://www.youtube.com/watch?v=1WsB-gdvliA

The Logarithmic Function

A *Logarithm* is written in the form $\log_b a$, where *b* is referred to as the base and *a* is the argument.

Note:

- **When you write log 25, it means $\log_{10} 25$**
- **The log button on your calculator uses base 10**

log25
log2

Example

How many 2s do we multiply to get 8?

Answer: $\mathbf{2} \times \mathbf{2} \times \mathbf{2} = \mathbf{8}$, so we needed to multiply 3 of the **2**s to get **8**

So the logarithm is 3

How to Write it

We would write "the number of 2s you need to multiply to get 8 is 3" as

$$\log_2(8) = 3$$

So these two things are the same:

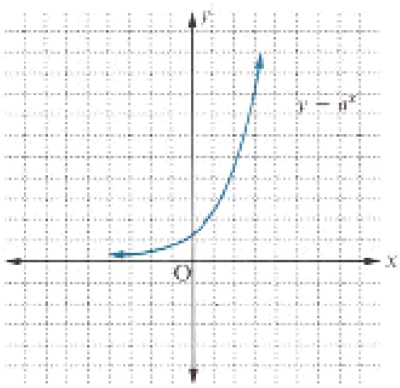
$$\underbrace{2 \times 2 \times 2}_{3} = 8 \quad \longleftrightarrow \quad \log_2(8) = 3$$

<u>Logarithms</u>

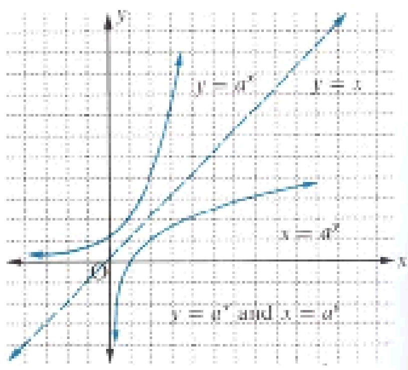
- is the <u>inverse</u> function of an exponential function.
- represented by the logarithmic symbol → $\log x$
- these functions are defined by equations of the form $y = \log_a x$, where $a > 0$ and $a \neq 1$.

Exponential Form		**Logarithmic Form**	
$x = a^y$	is written as…	$y = \log_a x$	← read as… "log of x to the base a"
Say, "the base a to the exponent y is x."		Say, "y is the exponent to which you raise the base a to get the answer x."	

The graph of an exponential function, $y = a^x$, is shown below. This graph will be reflected in the line $y = x$. (See the graph to the right.)



Since $y = a^x$ is reflected in the line $y = x$, $y = a^x$ becomes $x = a^y$. This inverse function describes certain scientific work.



- remember that logarithms can be written as exponentials because they are ***inverses*** (opposites) of each other.

$$\boxed{x = a^y \Leftrightarrow y = \log_a x}$$

Exponents to Logarithms

$5^{-2} = \frac{1}{25}$

$x = a^y$

Ex) a) $6^3 = 216$

$3 = \log_6(216)$

b) $\sqrt{81} = x$

$(81)^{1/2}$

$\frac{1}{2} = \log_{81}(x)$

c) $0.5^2 = 0.25$

$2 = \log_{0.5} 0.25$

d) $\frac{1}{8} = 2^{-3}$

$-3 = \log_2\left(\frac{1}{8}\right)$

e) $(x-3) = 2^2$

$2 = \log_2(x-3)$

$\log_5 25 = 2$

5 raised to what power gives you 25

$\log_3 27$ $\log_7 49$

$\log_4 64 = 3$

$\log_2 32$ $\log_2 \left(\frac{1}{8}\right)$

$2^x = \frac{1}{8}$

EXAMPLES...

exponential form

$x = a^y$

Say, "the base a to the exponent y is x."

logarithmic form

$y = \log_a x$

Say, "y is the exponent to which you raise base a to get the answer x."

logarithmic functions

These are functions defined by equations of the form $y = \log_a x$, where $a > 0$ and $a \neq 1$.

Example 1 Write each of the following in logarithmic form.

(a) $32 = 2^5$ (b) $2^{-5} = \dfrac{1}{32}$ (c) $x = 10^y$

$5 = \log_2 32$

Solutions...

When writing logarithms or evaluating expressions involving logarithms, you will find it useful to bear in mind the equivalent exponential form.

$$\boxed{x = a^y \leftrightarrow y = \log_a x}$$

Important skills for working with the properties of logarithmic functions are established by examples like the following.

Example 2 Evaluate each of the following.
(a) $\log_{10} 100$ (b) $\log_2 64$

Solutions...

Evaluating Logs

Step ***1***: Set the log equal to **x**

Step ***2***: Write the equation in exponential form ($x = a^y$)

Step ***3***: Find **x**

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Ex) a) $\log_{10}(100) = x$

$100 = 10^x$

$10^2 = 10^x$

$x = 2$

b) $\log_2(64) = x$

$64 = 2^x$

$2^6 = 2^x$

$x = 6$

Evaluate each of the following:

1. $\log_3 (81) = x$

$3^x = 81$

$x = 4$

2. $-\frac{2}{3} = \log_x 81$

$x^{-2/3} = 81$

$(x^{-2/3})^{-3/2} = 81^{-3/2}$

$x = (\sqrt{81})^{-3}$

$\frac{1}{9^3}$

$= 1/729$

3. $\log_5 \frac{1}{125} = x$

$5^x = \frac{1}{125}$

$x = -3$

4. $\log_2 8\sqrt{32} = x$

$2^x = 8\sqrt{32}$

$2^x = 8\sqrt{16 \cdot 2}$

$2^x = 8\sqrt{16} \cdot \sqrt{2}$

$2^x = (8)(4)(\sqrt{2})$

$2^x = 32\sqrt{2}$

$2^x = 2^5 \cdot 2^{1/2}$

$2^x = 2^{5.5}$

$x = 5.5$

Solve for x:

a) $log_{\sqrt{2}} 16 = x$ b) $-2 = log_4 x$ c) $\frac{5}{3} = log_x 32$

HOMWEORK...

Worksheet - Logarithms.doc

#1-5

Attachments

Worksheet - Logarithms.doc