Warm Up

Rewrite the following into exponential or logarithmic form

1. $3^{-4} = \frac{1}{81}$

$\log_3\left(\frac{1}{81}\right) = -4$

$\boxed{\log_b M = N \longleftrightarrow b^N = M}$

2. $log_{10}\left(\frac{1}{10}\right) = -1$

$10^{-1} = \frac{1}{10}$

Evaluate the following:

1. $\log_3 (9) = x$

$3^x = 9$

$x = 2$

2. $\log_x (64) = 3$

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$= 64^{1/3}$

3. $\log_2(x-1) = 3$

$2^3 = x-1$

$8 = x - 1$ (+1 on both sides)

$x = 9$

4. $\frac{5}{3} = log_x 32$

$(x^{5/3})^{3/5} = 32^{3/5}$

$x = (\sqrt[5]{32})^3$

$= 2^3$

$= 8$

<u>Laws of Logarithms</u>

Recall: $\boxed{\log_a x = y \Leftrightarrow a^y = x}$, a is the base and y is the exponent.

General Properties of Logarithms:

If $a > 0$ and $a \neq 1$, then…

(i) $\log_a 1 = 0$

(ii) $\log_a a^x = x$

(iii) $a^{\log_a x} = x$

*** Prove the above properties by changing them into the inverse form!!!

$\log_a 1 = 0$

$a^0 = 1$

$\log_a a^x = x$

$a^x = a^x$

$\log_3 3^2 = 2$

$\log_{99} 99^6 = 6$

$\log_x 49^2 = 2$

$a^{\log_a x} = x$

$\log_a x = \log_a x$

$5^{\log_5 7} = 7$

$x^{\log_3 5} = 5$

$x = 3$

Laws of Logarithms: If ***a > 0, M > 0, N > 0 and n ε R then…***

1) Product Law → the logarithm of a product is equal to the sum of the logarithms of the factors.

PROOF: Let $\log_a M = b$ and $\log_a N = c$

so $a^b = M$ $a^c = N$

then,

$$\log_a(MN) = \log_a(a^b \bullet a^c)$$
$$= \log_a(a^{b+c})$$
$$= b + c$$
$$\therefore \boxed{\log_a(MN) = \log_a M + \log_a N}$$

examples: a) $\log_{10}(6 \times 9)$

$= \log_{10}6 + \log_{10}9$

b) $\log_2 12 + \log_2 7$

$\log_2(12 \cdot 7)$

$\log_2(84) = x$

$2^x = 84$

c) Evaluate... $\log_3 54 + \log_3\left(\frac{3}{2}\right)$

$\log_3(54 \cdot \frac{3}{2})$

$\log_3(81) = x$

$3^x = 81$

$x = 4$

2) Quotient Law → the logarithm of a quotient is equal to the logarithm of the numerator minus the logarithm of the denominator.

PROOF: Let $\log_a M = b$ and $\log_a N = c$

so $a^b = M$ $a^c = N$

then,

$$\log_a\left(\frac{M}{N}\right) = \log_a\left(\frac{a^b}{a^c}\right)$$

$$= \log_a(a^{b-c})$$

$$= b - c$$

$$\therefore \quad \boxed{\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N}$$

examples: a) $\log_5\left(\frac{97}{62}\right)$ b) $\log_2 15 - \log_2 3$ c) $\log_{10}\left(\frac{1}{9}\right)$

$\log_5 97 - \log_5 62$

$\log_2\left(\frac{15}{3}\right)$

$\log_2(5)$

$\cancel{\log_{10} 1} -$

$\log_{10} 9$

$= -\log_{10} 9$

NOTE: $\log_a\left(\frac{1}{N}\right) = \log_a 1 - \log_a N = 0 - \log_a N = -\log_a N$

$\therefore \quad \boxed{\log_a\left(\frac{1}{N}\right) = -\log_a N}$

d) Evaluate... $\log_3 33 - \log_3 11$

$\log_3\left(\frac{33}{11}\right)$

$\log_3 3 = x$

$3^x = 3$

$x = 1$

e) Evaluate... $\log_5 50 + \log_5 2 - \log_5 4$

$\log_5(50 \cdot 2) - \log_5 4$

$\log_5(100) - \log_5 4$

$\log_5\left(\frac{100}{4}\right)$

$\log_5(25) = x$

$5^x = 25$

$x = 2$

3) Law of Logarithms for Powers → the logarithm of a power of a number is equal to the exponent multiplied by the logarithm of the number

PROOF: Let $\log_a M = b$

so $a^b = M$

then,

$\log_a x^2$

$2\log_a x$

$$\log_a M^p = \log_a (a^b)^p$$
$$= \log_a (a^{b \times p})$$
$$= b \times p$$

$$\therefore \quad \boxed{\log_a M^p = p \times \log_a M}$$

examples: a) $\log_{10} 8^9$ b) $2\log_3 5$ c) $\log_5 \sqrt{125}$

$9\log_{10} 8$

$\log_3 5^2$

$\log_3 25$

$125^{1/2}$

$\frac{1}{2}\log_5 125$

Note:

$$\log_a M^{\frac{p}{q}} = \frac{p}{q} \times \log_a M$$

$\log_5 4^{-2}$

$-2\log_5 4$

Find x if: Bonus (2)

$$2log_b(5)+\frac{1}{2}log_b 9-log_b 3=log_b x$$

Homework...

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Solutions...

11. (a) $3^2 = 9$ $\log_3 9 = 2$
(b) $4^2 = 16$ $\log_4 16 = 2$
(c) $3^4 = 81$ $\log_3 81 = 4$
(d) $2^{-4} = \frac{1}{16}$ $\log_2\left(\frac{1}{16}\right) = -4$
(e) $\left(\frac{1}{4}\right)^{-2} = 16$ $\log_{\frac{1}{4}} 16 = -2$
(f) $4^{-3} = \frac{1}{64}$ $\log_4\left(\frac{1}{64}\right) = -3$
(g) $16^{\frac{1}{2}} = 4$ $\log_{16} 4 = \frac{1}{2}$
(h) $2^{-3} = \frac{1}{8}$ $\log_2\left(\frac{1}{8}\right) = -3$
(i) $27^{\frac{1}{3}} = 3$ $\log_{27} 3 = \frac{1}{3}$
(j) $5^3 = 125$ $\log_5 125 = 3$

12. $\log_b a = c$

13. (a) $\log_{10} 10\,000 = 4$ (b) $\log_{10}\left(\frac{1}{100}\right) = -2$
(c) $\log_2 8 = 3$ (d) $\log_3\left(\frac{1}{81}\right) = -4$
(e) $10^2 = 100$ (f) $10^{-1} = \frac{1}{10}$
(g) $3^2 = 9$ (h) $2^{-3} = \frac{1}{8}$

14. (a) 3 (b) 2 (c) 1 (d) 0 (e) −1 (f) −5
(g) $\frac{1}{2}$ (h) $\frac{1}{3}$ (i) −3 (j) 3 (k) −3 (l) 1
(m) 2 (n) 8

15. (a) 32 (b) 15 (c) −3 (d) $\frac{1}{16}$
(e) $x^{\frac{5}{3}} = 32$
$\left(x^{\frac{5}{3}}\right)^{\frac{3}{5}} = (32)^{\frac{3}{5}}$
$x = 8$
(f) 9 (g) 6
(h) $(\sqrt{2})^x = 8$
$\left(2^{\frac{1}{2}}\right)^x = 8$
$x = 6$
(i) 2.17 (j) 1585 (k) 7.06 (l) −10