A

Functions and Relations 112
Review Activity: Rate of Change

B: $h(t) = -13t^2 + 312t + 20$
C: $h(t) = -14t^2 + 364t + 16$
D: $h(t) = -15t^2 + 360t + 40$

Instructions:
The following assignment is to be completed individually. All work must clearly be shown in the space provided.

The height of a model rocket in metres, t seconds after being launched is modeled by the quadratic function:

A: $h(t) = -12t^2 + 264t + 20$

1. Determine the height from which the rocket was initially launched. [1]

Let $t = 0$...
$h(0) = 20\text{ m}$

B: 20 m
C: 16 m
D: 40 m

2. Determine the average rate of change in the height of the rocket between 4 seconds and 20 seconds. Express your answer in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

$h(4) = 884\text{ m}$
$h(20) = 500\text{ m}$

$AROC = \frac{-384}{16}$
$= -24\text{ m/s}$

B: 0 m/s
C: 28 m/s
D: 0 m/s

3. Determine the instantaneous rate of change in the height of the rocket 18 seconds after being launched. Express your solution in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

$h(17.9) = 900.68\text{ m}$
$h(18.1) = 867.08$

$IROC = \frac{-33.6}{0.2}$
$= -169\text{ m/s}$

B: -156 m/s
C: -140 m/s
D: -120 m/s

4. Determine the instantaneous rate of change in the height of the rocket 5 seconds before it strikes the ground during its descent. [6]

$0 = -12t^2 + 264t + 20$

$t = \frac{-264 \pm \sqrt{264^2 - 4(-12)(20)}}{2(-12)}$

$= \frac{-264 \pm \sqrt{70656}}{-24}$

$t = -0.1, 22.1$

$\therefore$ Time $= 22.1 - 5$

$= 17.1$

IROC.... @ $t = 17.1$

$h(17.09) = 1026.9428$

$h(17.11) = 1024.0148$

$IROC = \frac{-2.928}{0.02}$

$= \boxed{-146.4 \text{ m/s}}$

B: IROC @ 19.1 sec
-194.6 m/s

C: IROC @ 21.0 sec
-224 m/s

D: IROC @ 19.1 sec
-213 m/s

5. Determine when the instantaneous rate of change in the height of the model rocket will be equal to 0 m/s. [4]

Occurs @ the max

$h(t) = -12(t^2 - 22t + 121) - 121(-12) + 20$

$= -12(t-11)^2 + 1472$

$\therefore$ vertex $(11, 1472)$

So $\boxed{t = 11 \text{ sec}}$

B: $h(t) = -13(t-12)^2 + 1392$
So $t = 12$ sec

C: $h(t) = -14(t-13)^2 + 2382$
So $t = 13$ sec

D: $h(t) = -15(t-12)^2 + 2200$
So $t = 12$ sec

6. Determine the average rate of change in the height of the model rocket as it travels between 1116.25 m on the way up and 546.25 m on the way down. Express your solution in m/s. [6]

$1116.25 = -12t^2 + 264t + 20$

$0 = -12t^2 + 264t - 1096.25$

$t = \frac{-264 \pm \sqrt{264^2 - 4(-12)(-1096.25)}}{2(-12)}$

$= \frac{-264 \pm \sqrt{17076}}{-24}$

$= 5.555, 16.444$

So $(5.555, 1116.25)$

$546.25 = -12t^2 + 264t + 20$

$0 = -12t^2 + 264t - 526.25$

$t = \frac{-264 \pm \sqrt{264^2 - 4(-12)(-526.25)}}{2(-12)}$

$= \frac{-264 \pm \sqrt{44436}}{-24}$

$= 2.217, 19.783$

So $(19.783, 542.25)$

$$AROC = \frac{-574}{14.228}$$

$$= \boxed{-40.3 \text{ m/s}}$$

B: $(4.275, 1116.25)$ & $(22.175, 546.25)$

$AROC = -31.9$ m/s

C: $(3.491, 1116.25)$ & $(24.451, 546.25)$

$AROC = -27.2$ m/s

D: $(3.5, 1116.25)$ & $(22.5, 546.25)$

$AROC = -30$ m/s

Functions and Relations 112
Review Activity: Rate of Change

B ☆

Instructions:
The following assignment is to be completed individually. All work must clearly be shown in the space provided.

The height of a model rocket in metres, t seconds after being launched is modeled by the quadratic function:

$$h(t) = -13t^2 + 312t + 20$$

1. Determine the height from which the rocket was initially launched. [1]

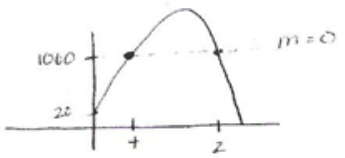
$h(0) = -13(0)^2 + 312(0) + 20$
$= 20$

Initially launched at 20 m

2. Determine the average rate of change in the height of the rocket between 4 seconds and 20 seconds. Express your answer in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

$h(4) = -13(4)^2 + 312(4) + 20 = 1060$ (4, 1060)
$h(20) = -13(20)^2 + 312(20) + 20 = 1060$ (20, 1060)

$AROC = m = \frac{h(20) - h(4)}{20 - 4} = \frac{1060 - 1060}{16} = 0$ m/s

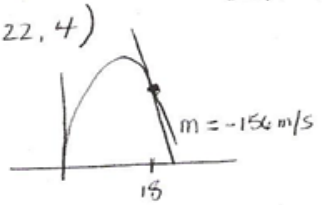


3. Determine the instantaneous rate of change in the height of the rocket 18 seconds after being launched. Express your solution in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

IROC @ 18 seconds

$h(17.99) = -13(17.99)^2 + 312(17.99) + 20 = 1425.6$
$h(18.01) = -13(18.01)^2 + 312(18.01) + 20 = 1422.4$

(17.99, 1425.6)
(18.01, 1422.4)



$IROC = m = \frac{h(18.01) - h(17.99)}{18.01 - 17.99}$
$= \frac{1422.4 - 1425.6}{18.01 - 17.99}$
$= -160$ m/s (-156 m/s using 184)

4. Determine the instantaneous rate of change in the height of the rocket 5 seconds before it strikes the ground during its descent. [6]

B

① hits the ground?
use Quad Formula.

$$t = \frac{-312 \pm \sqrt{(312)^2 - 4(-13)(20)}}{2(-13)}$$

$$= \frac{-312 \pm \sqrt{95364}}{-26}$$

$$= \frac{-312 \pm 313.6622}{-26}$$

$t = -0.06$ sec

$t = 24.064$ seconds

② 5 seconds before it hits the ground

$24.064 \text{ sec} - 5$

19.064

~ 19.1 seconds.

③ IROC @ 19.06 seconds

$(19.06, 1244.0332)$

$(19.07, 1242.1963)$

$$IROC = m = \frac{1242.1963 - 1244.0332}{19.07 - 1906}$$

$= -183.69$ m/s

(TI84 -183.66 m/s)

5. Determine when the instantaneous rate of change in the height of the model rocket will be equal to 0 m/s. [4]

IROC = 0 m/s at the maximum

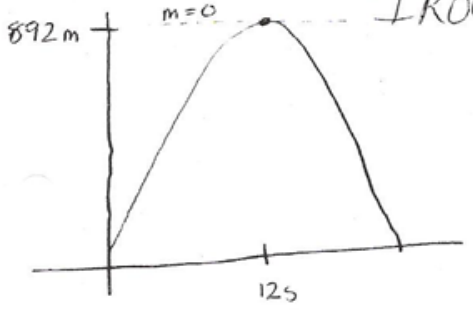
Complete the square to find vertex

$$h(t) = -13t^2 + 312t + 20$$

$$= -13(t^2 - 24t + 144) - 13(-144) + 20$$

$$= -13(t-12)^2 + 1872 + 20$$

$$= -13(t-12)^2 + 1892$$



IROC is 0 m/s at 12 seconds

6. Determine the average rate of change in the height of the model rocket as it travels between 1116.25 m on the way up and 546.25 m on the way down. Express your solution in m/s. [6]

B

$(\_\_\_\_, 1116.25)$

$h = -13t^2 + 312t + 20$

$1116.25 = -13t^2 + 312t + 20$

$0 = -13t^2 + 312t - 1096.25$

$t = \dfrac{-312 \pm \sqrt{(312)^2 - 4(-13)(-1096.25)}}{2(-13)}$

$= \dfrac{-312 \pm \sqrt{40339}}{-26}$

$t_1 = 4.28$ seconds (4.275164926) ← way up

$t_2 = 19.72$ seconds (19.72483507) ← way down

$(\_\_\_\_, 546.25)$

$h = -13t^2 + 312t + 20$

$546.25 = -13t^2 + 312t + 20$

$0 = -13t^2 + 312t - 526.25$

$t = \dfrac{-312 \pm \sqrt{(312)^2 - 4(-13)(-526.25)}}{2(-13)}$

$= \dfrac{-312 \pm \sqrt{69979}}{-26}$

$t_1 = 1.83$ seconds (1.825559925) ← way up

$t_2 = 22.17$ seconds (22.17444007) ← way down

$AROC = m = \dfrac{546.25 - 1116.25}{22.17444007 - 4.275164926}$

$(4.28, 1116.25)$

$(22.17, 546.25)$

$= \dfrac{-570}{17.89927514}$

$= -31.84$ m/s

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Review Activity: Rate of Change

C

Instructions:
The following assignment is to be completed individually. All work must clearly be shown in the space provided.

The height of a model rocket in metres, t seconds after being launched is modeled by the quadratic function:

$$h(t) = -14t^2 + 364t + 16$$

1. Determine the height from which the rocket was initially launched. [1]

$t = 0$

$h(0) = -14(0)^2 + 364(0) + 16$

$= 16\ m$

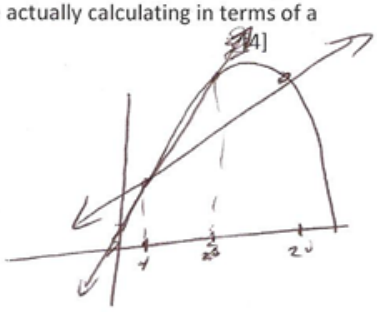
2. Determine the average rate of change in the height of the rocket between 4 seconds and 20 seconds. Express your answer in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

time	height
4	1248
20	1696

$ARC = \frac{1248 - 1696\ m}{4 - 20\ sec}$

$= \frac{-448}{-16}\ m/s$

$= 28\ m/s$



3. Determine the instantaneous rate of change in the height of the rocket 18 seconds after being launched. Express your solution in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

time	height
17.9	2045.86
18.1	2017.86

$IRC = \frac{2045.86 - 2017.86\ m}{17.9 - 18.1\ sec}$

$= \frac{28}{-0.2}$

$= -140\ m/s$

4. Determine the instantaneous rate of change in the height of the rocket 5 seconds before it strikes the ground

during its descent. [6]

Set $h = 0$:

$-14t^2 + 364t + 16 = 0$

$t = \dfrac{-364 \pm \sqrt{364^2 - 4(-14)(16)}}{2(-14)}$

$t = \dfrac{-364 \pm 365.228}{-28}$

$t = 26.04 \text{ sec}$ & ~~-0.04 sec~~

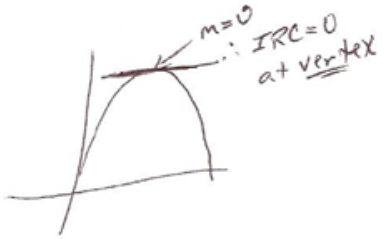
5 seconds earlier... 21.04 sec

time	height
21.03	1479.267
21.05	1474.765

$IRC = \dfrac{1479.267 - 1474.765 \text{ m}}{21.03 - 21.05 \text{ sec}}$

$= -225.1 \text{ m/s}$

5. Determine when the instantaneous rate of change in the height of the model rocket will be equal to 0 m/s. [4]



$h(t) = -14(t^2 - 26t + 169) + 16 + 2366$

$h(t) = -14(t - 13)^2 + 2382$

$V(13, 2382)$

$IRC = 0 \text{ m/s}$ @ $t = 13 \text{ seconds}$

6. Determine the average rate of change in the height of the model rocket as it travels between 1116.25 m on the way up and 546.25 m on the way down. Express your solution in m/s. [6]

$-14t^2+364t+16=1116.25$

$-14t^2+364t-1100.25=0$

$t=\frac{-364\pm\sqrt{364^2-4(-14)(-1100.25)}}{2(-14)}$

$t=\frac{-364\pm 266.237}{-28}$

$t=22.5\text{sec}$ or 3.5sec

$-14t^2+364t+16=546.25$

$-14t^2+364t-530.25=0$

$t=\frac{-364\pm 320.628}{-28}$

$t=24.5\text{sec}$ or 1.5sec

time	height
3.5	1116.25
24.5	546.25

$ARC=\frac{1116.25-546.25\ m}{3.5-24.5\ sec}$

$ARC=-27.14\ m/s$

Functions and Relations 112
Review Activity: Rate of Change

D

Instructions:
The following assignment is to be completed individually. All work must clearly be shown in the space provided.

The height of a model rocket in metres, t seconds after being launched is modeled by the quadratic function:

$$h(t) = -15t^2 + 360t + 40$$

1. Determine the height from which the rocket was initially launched. [1]

$h(0) = -15(0)^2 + 360(0) + 40$

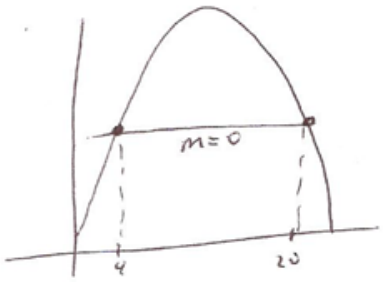
$= 40\ m$

2. Determine the average rate of change in the height of the rocket between 4 seconds and 20 seconds. Express your answer in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

time	height
4	1240
20	1240

$ARC = \frac{1240 - 1240\ m}{4 - 20\ sec}$

$= 0\ m/s$

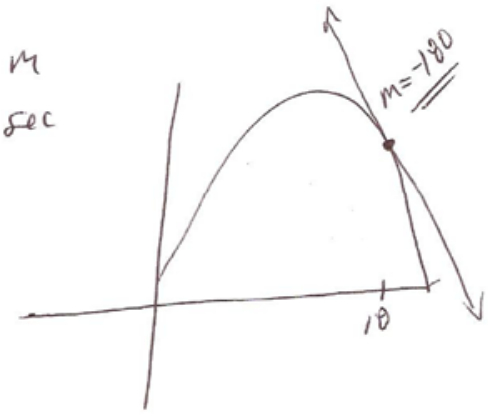


3. Determine the instantaneous rate of change in the height of the rocket 18 seconds after being launched. Express your solution in m/s. Provide a rough sketch that demonstrates what you are actually calculating in terms of a slope value. [4]

time	height
18.1	1641.85
17.9	1677.85

$IRC = \frac{1641.85 - 1677.85\ m}{18.1 - 17.9\ sec}$

$= -180\ m/s$



4. Determine the instantaneous rate of change in the height of the rocket 5 seconds before it strikes the ground during its descent. [6]

ground: Set $h = 0$

$-15t^2 + 360t + 40 = 0$

$t = \dfrac{-360 \pm \sqrt{360^2 - 4(-15)(40)}}{2(-15)}$

$t = \dfrac{-360 \pm 363.319}{-30}$

$t = 24.11$ sec or $t = -0.11$ sec

↑ 5 seconds earlier... $t = 19.11$ sec

IRC @ 19.11 sec ...

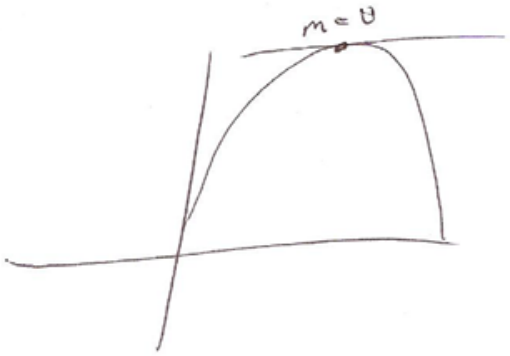
time	height
19.12	1439.584
19.10	1443.85

$IRC = \dfrac{1439.584 - 1443.85 \text{ m}}{19.12 - 19.10 \text{ sec}}$

$= -213.3$ m/s

5. Determine when the instantaneous rate of change in the height of the model rocket will be equal to 0 m/s. [4]

IRC = 0 m/s at Vertex



$h(t) = -15(t^2 - 24t + 144) + 40 + 2160$

$h(t) = -15(t - 12)^2 + 2200$

$V(12, 2200)$

∴ IRC = 0 m/s @ 12 seconds

6. Determine the average rate of change in the height of the model rocket as it travels between 1116.25 m on the way up and 546.25 m on the way down. Express your solution in m/s. [6]

$-15t^2 + 360t + 40 = 1116.25$

$-15t^2 + 360t - 1076.25 = 0$

$t = \frac{-360 \pm 255}{-30}$

$t = 20.5\text{sec}$ & 3.5sec

$-15t^2 + 360t + 40 = 546.25$

$-15t^2 + 360t - 506.25 = 0$

$t = \frac{-360 \pm 315}{-30}$

$t = 22.5\text{sec}$ & 1.5sec

time	height
3.5	1116.25
22.5	546.25

$ARC = \frac{1116.25 - 546.25 \text{ m}}{3.5 - 22.5 \text{ sec}}$

$= -30 \text{ m/s}$

Attachments

Integration Tables.pdf