Functions & Relations 112
Midterm – April 2007 (Version #1)

Name: Key

PART A – Multiple Choice (20 marks)
Shade in the letter corresponding to the correct solution on the scantron sheet that is provided.

1. Find the equation of the function represented by the following data:

x	-4	0	4	8	12
y	$\frac{3}{2}$	3	6	12	24

[A] $y = 2(-4)^{\frac{x}{4}}$ [B] $y = 2(-4)^{\frac{4}{x}}$ [C] $y = -4(2)^{\frac{x}{4}}$ [D] $y = -4(2)^{\frac{x}{4}}$

D

2. Evaluate the following: $\dfrac{2^{-5}}{2^{-2}+2^{-3}}$

A [A] $\frac{1}{12}$ [B] $\frac{3}{8}$ [C] 1 [D] 2

3. The table below shows the distances traveled over one-hour periods on a recent trip… During which one-hour time interval is the average rate of change the **greatest**?

Time (h)	Distance (km)
1	105
2	192
3	278
4	367
5	455

[A] 1 – 2 hours [B] 2 – 3 hours

C [C] 3 – 4 hours [D] 4 – 5 hours

4. How many times will the parabola $\frac{3}{2}(y-1) = (x-2)^2$ cross the x-axis?

A [A] 0 [B] 1 [C] 2 [D] 3

5. Simplify the following… $x^{-\frac{2}{3}} \bullet x^{\frac{1}{2}} \div x^{-\frac{1}{6}}$

A [A] 1 [B] x [C] $\frac{1}{\sqrt[3]{x}}$ [D] $\sqrt[3]{x^4}$

6. An astronaut jumps from her spacecraft to the surface of the moon. Her height, h, above the surface after t seconds is modeled by the function $h = -0.4t^2 + 8t + 1$. What is the maximum height reached by the astronaut?

B [A] 161 m [B] 41 m [C] 7.4 m [D] 2 m

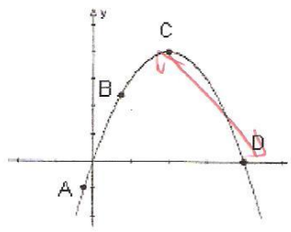
D 7. Given the roots of a quadratic equation are -3 & $\frac{2}{5}$, these answers were derived from which of the following equations?

[A] $y = 5x^2 - 13x - 6$ [B] $y = 2x^2 + x - 15$ [C] $y = 5x^2 - 13x + 6$ [D] $y = 5x^2 + 13x - 6$

8. A quadratic function has vertex (–2 , 11) and y-intercept of (0 , –1). Determine the standard form of the function.

B [A] $y = 3(x-2)^2 - 11$ [B] $y = -3(x+2)^2 + 11$ [C] $y = \frac{1}{3}(x+2)^2 + 11$ [D] $y = -3(x-2)^2 - 11$

9. Which secant will the average rate of change be **negative**?



A [A] CD [B] CA [C] DA [D] BC

10. The table of values below **best** represents which type of function?

Time (s)	0	1	2	3	4	5
Height (m)	- 5	- 5.4	- 6.6	- 8.6	- 11.4	- 15

B [A] Linear [B] Quadratic [C] Cubic [D] Exponential

11. What is the solution to the following equation: $9^{x-2} = \left(\dfrac{1}{27}\right)^{-2}$

D [A] $\frac{7}{2}$ [B] 4 [C] $\frac{9}{2}$ [D] 5

12. Determine the general term given the following: $t_1 = -11$ & $t_{65} = 245$

[A] $t_n = 4n - 15$ [B] $t_n = -11n + 15$ [C] $t_n = -11n + 256$ [D] $t_n = 4n - 7$

13. The solution to $4x^{\frac{2}{3}} - 5 = 11$ is...

[A] 0 [B] 8 [C] 16 [D] not possible

14. A student used the following steps to complete the square of the quadratic function $y = -\frac{3}{2}x^2 - 18x + 7$ ...

STEP 1 $y = -\frac{3}{2}(x^2 + 12x) + 7$

STEP 2 $y = -\frac{3}{2}(x^2 + 12x + 36) - 36\left(-\frac{3}{2}\right) + 7$

STEP 3 $y = -\frac{3}{2}(x+6)^2 - 54 + 7$

STEP 4 $y = -\frac{3}{2}(x+6)^2 - 47$

The student **first** made an error in...

[A] Step 1 [B] Step 2 [C] Step 3 [D] Step 4

15. Some summers in New Brunswick we experience a major drought due to the lack of rain. The water level at a local reservoir was monitored for 60 days. The height of the water in the reservoir can be modeled by the equation:

$$h = 0.0125(t - 21)^2 + 7.0$$

where h is the height in meters and t is the time in days. At what time will the instantaneous rate of change be **zero**?

[A] Day 0 [B] Day 7 [C] Day 21 [D] Day 42

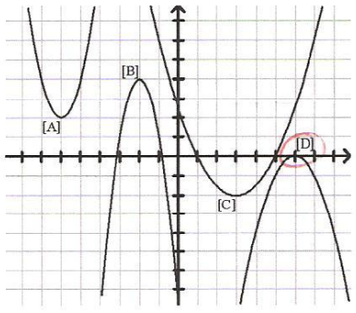
16. Simplify the following: $\dfrac{3x^{-4}y^2z^6}{12x^{-2}y^4z^{-2}}$

[A] $\dfrac{4z^8}{x^2y^2}$ [B] $\dfrac{x^2y^2z^8}{4}$ [C] $\dfrac{z^4}{4x^2y^2}$ [D] $\dfrac{z^8}{4x^2y^2}$

17. What is the y-intercept of the following equation: $y = 3(x-2)^2 - 5$

[A] 7 [B] -5 [C] -11 [D] –17

18. Which function has a corresponding quadratic equation with a discriminant value of **zero**?



19. A petri dish contains a bacteria population. A biologist records the growth over 14 hours.

Time (hours)	0	2	4	6	8	10	12	14
Number of Bacteria	100	260	340	400	525	662	814	1040

The average rate of change in the number of bacteria per hour between 6 hours and 12 hours would be...

[A] 0.014 bacteria/hour [B] 69.0 bacteria /hour [C] 138.0 bacteria/hour [D] 676 bacteria/hour

20. Determine the discriminant value for the equation $y = 3x^2 - 4x - 5$

[A] 44 [B] 56 [C] 76 [D] –44

PART B – Open Response (30 marks)
Show all your work in the space that is provided. Be sure to include units when needed.

1. Mr. Remington was out at the Range shooting some skeet for the upcoming duck season. A machine projects the skeet into the air while a shooter tries to shoot the skeet. A clever physicist decided to implant a small transmitter into the skeet. The transmitter records the height of the skeet above the ground at one second intervals. The data received from one of the flights is given below…

Time (s)	1	2	3	4	5
Height (m)	14	24.5	32	36.5	38

D_1: 10.5, 7.5, 4.5, 1.5
D_2: −3, −3, −3

a) In the space above, prove that the data is **quadratic**. [1]

$D_2 = -3$ ①

b) Determine a quadratic equation that will describe the height, h, of the skeet in metres at any time, t, in seconds. [2]

$h(t) = -1.5t^2 + 15t + 0.5$ ②

c) Determine the average rate of change for each of the following intervals… [4]

(i) From 0 seconds to 5 seconds

$h(0) = 0.5$
$h(5) = 38$ ①
$AROC = \frac{38 - 0.5}{5 - 0}$
$= 7.5 \text{ m/s}$ ①

(ii) From 6 seconds to 8 seconds

$h(6) = 36.5$ ①
$h(8) = 24.5$
$AROC = \frac{24.5 - 36.5}{8 - 6}$
$= -6 \text{ m/s}$ ①

2. Determine the value(s) for x in each of the following exponential equations: [6]

a) $16^{5x+2}(4) = \frac{1}{32}$

$(2^4)^{5x+2} \cdot 2^2 = 2^{-5}$ ①
$2^{20x+8} \cdot 2^2 = 2^{-5}$
$2^{20x+10} = 2^{-5}$ ①
$20x + 10 = -5$
$20x = -15$
$x = \frac{-15}{20}$
$x = -\frac{3}{4}$ ①

b) $3^{2x} - 12(3^x) + 27 = 0$

Let $a = 3^x$
$a^2 - 12a + 27 = 0$ ①
$(a - 9)(a - 3) = 0$
$a = 9$ OR $a = 3$ ①
$3^x = 9 \quad 3^x = 3$
$x = 2$ OR $x = 1$ ①

3. Given the linear sequence below, determine the value of t_{130}. Clearly show or explain your solution. [2]

$12, 5, -2, -9, -16, -23, \ldots$

$t_1 = 12$
$d = -7$
$t_n = 12 - 7(n-1)$
$= 12 - 7n + 7$
$t_n = -7n + 19$ ①
$t_{130} = -7(130) + 19$
$= -891$ ①

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4. A Bald eagle swoops down toward an unsuspecting black bear in a parabolic flight path in an attempt to scare the bear away from it's nesting area. The height of the eagle above the bear in metres after t seconds is given by: $h = 4t^2 - 20t + 27$, where h is the height of the eagle in metres and t is the time in seconds.

a) Determine the minimum height that the eagle will reach during it's flight. [3]

$h = 4(t^2 - 5t) + 27$ ①

$= 4(t^2 - 5t + \frac{25}{4}) - \frac{25}{4}(4) + 27$ ①

$h = 4(t - \frac{5}{2})^2 + 2$

∴ min is 2 m ①

b) Determine the speed of the eagle at 1.5 seconds [3]

$t = 1.5$

$t = 1.49$ s, $h = 6.0804$ m ①

$t = 1.51$, $h = 5.9204$ ①

$IROC = \frac{5.9204 - 6.0804}{1.51 - 1.49}$

$= -8$ m/s ①

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5. Solve each of the following quadratic equations: *note: - if roots are non-real, express them as complex roots* [6]
- if roots are irrational, simplify the radical completely

a) $10x^2 - 13x - 3 = 0$

x → −30, + → −13 } −15, 2

$10x^2 - 15x + 2x - 3 = 0$

$5x(2x - 3) + 1(2x - 3) = 0$ ①

$(2x - 3)(5x + 1) = 0$

$x = \frac{3}{2}$ or $x = \frac{-1}{5}$ ① ①

b) $x^2 - 4x + 13 = 0$

x → 13, + → −4 } DNF

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$

$= \frac{4 \pm \sqrt{16 - 52}}{2}$ ①

$= \frac{4 \pm \sqrt{-36}}{2}$ ①

$x = \frac{4 \pm 6i}{2}$ ①

$x = 2 \pm 3i$

6

6. Given the function $y = 5(0.4)^x + 3$, answer the following questions: [3]

a) Does the function represent a growth or a decay situation? Decay ①

b) State the y-intercept. (0, 8) ①

c) State the equation of the horizontal asymptote. $y = 3$ ①

3

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