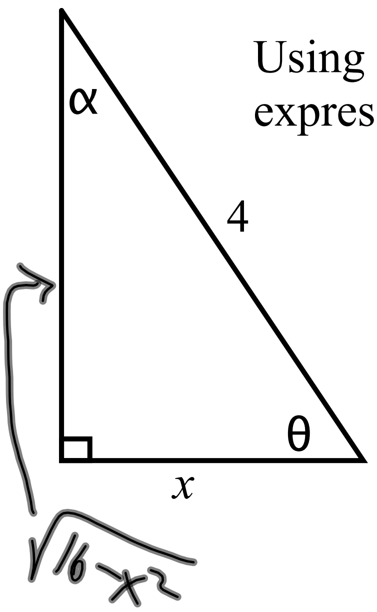
Warm Up



Using the diagram shown determine an expression for each of the following:

$\sin\theta = \frac{\sqrt{16-x^2}}{4}$ $\sec\alpha = \frac{4}{\sqrt{16-x^2}}$

$\tan\alpha = \frac{x}{\sqrt{16-x^2}}$ $\tan\theta = \frac{\sqrt{16-x^2}}{x}$

$\cos^{-1}\left(\frac{x}{4}\right) = \theta$ $\sec^{-1}\left(\frac{4}{\sqrt{16-x^2}}\right) = \alpha$

Derivatives of Transcendental Functions

transcendental functions

(*mathematics*) Functions which cannot be given by any algebraic expression involving only their variables and constants.

Examples include the functions log x, sin x, cos x, e^x and any functions containing them.

Inverse Trigonometric Functions

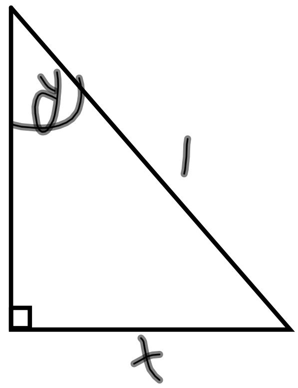
Let's review the definition of an inverse trigonometric function: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$

$$y = \sin^{-1} x \qquad \text{or} \qquad y = Arc\sin x$$

What do the above statements mean verbally?

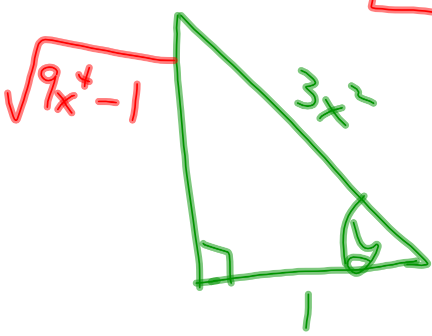
" y is an angle whose sine Ratio is $\frac{x}{1}$ "

Express this visually:



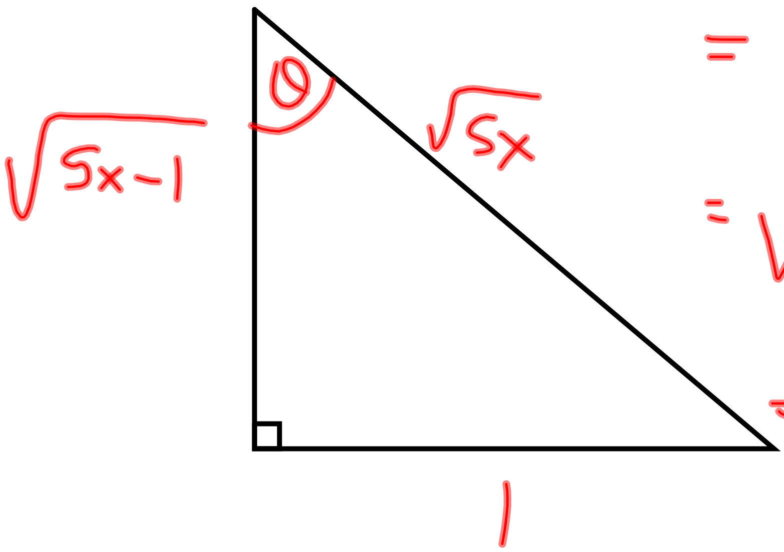
Example:

- Represent the inverse trigonometric function $y = \sec^{-1}(3x^2)$ with a diagram.
- Evaluate: $w = \tan\underbrace{\left[\sec^{-1}(3x^2)\right]}_{y} = \sqrt{9x^4 - 1}$



Example:

Evaluate the following: $y = \cos\left[\underbrace{\csc^{-1}\sqrt{5x}}_{\theta}\right] = \dfrac{\sqrt{5x-1}}{\sqrt{5x}}$



$= \sqrt{\dfrac{5x-1}{5x}}$

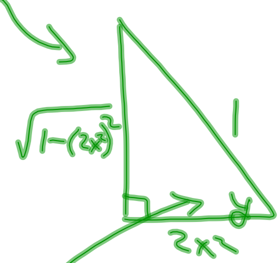
$= \sqrt{1-\dfrac{1}{5x}}$

$= \sqrt{1-(5x)^{-1}}$

$= \dfrac{\sqrt{25x^2-5x}}{5x}$

Differentiating Inverse Trigonometric Functions

Ex. Differentiate $y = \cos^{-1}(2x^2)$



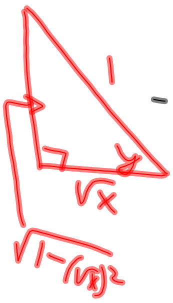
$$\cos y = 2x^2$$

$$(-\sin y)\frac{dy}{dx} = 4x$$

$$\frac{dy}{dx} = \frac{4x}{-\sin y}$$

$$\frac{dy}{dx} = \frac{-4x}{\sqrt{1-(2x^2)^2}}$$

Ex. Differentiate $y = \cos^{-1}(\sqrt{x})$



$$\cos y = \sqrt{x}$$

$$-\sin y \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$

$$\frac{dy}{dx} = \frac{-\left(\frac{1}{2}x^{-1/2}\right)}{\sin y}$$

$$\frac{dy}{dx} = \frac{-\left(\frac{1}{2}x^{-1/2}\right)}{\sqrt{1-(\sqrt{x})^2}}$$

$$y = \cos^{-1}(2x^2) \qquad\qquad y = \cos^{-1}(\sqrt{x})$$

$$y' = \frac{-4x}{\sqrt{1-(\underbrace{2x^2}_{u})^2}} \qquad\qquad y' = \frac{-\frac{1}{2}x^{-1/2}}{\sqrt{1-(\sqrt{x})^2}}$$

$$y = \cos^{-1}(u)$$

$$y' = \frac{-du}{\sqrt{1-u^2}}$$

These two examples lead to the following set of rules for differentiating inverse trigonometric functions:

$$\frac{d\left(\sin^{-1}u\right)}{du}=\frac{1}{\sqrt{1-u^2}}du \qquad \frac{d\left(\csc^{-1}u\right)}{du}=\frac{-1}{u\sqrt{u^2-1}}du$$

$$\frac{d\left(\cos^{-1}u\right)}{du}=\frac{-1}{\sqrt{1-u^2}}du \qquad \frac{d\left(\sec^{-1}u\right)}{du}=\frac{1}{u\sqrt{u^2-1}}du$$

$$\frac{d\left(\tan^{-1}u\right)}{du}=\frac{1}{1+u^2}du \qquad \frac{d\left(\cot^{-1}u\right)}{du}=\frac{-1}{u^2+1}du$$

Examples:

Differentiate each of the following...

$$f(x)=x^3\sin^{-1}\left(3x^2\right)$$

$$f'(x)=3x^2\sin^{-1}(3x^2)+x^3\left[\frac{6x}{\sqrt{1-9x^4}}\right]$$

$$f(x)=\sqrt{3x-\tan^{-1}\sqrt{x}}$$

$$f'(x)=\frac{1}{2}\left[3x-\tan^{-1}\sqrt{x}\right]^{-1/2}\left[3-\frac{\frac{1}{2}x^{-1/2}}{1+x}\right]$$

$$f(x)=\frac{\cot^3 5x}{\cot^{-1}(5x)}$$

$(\cot 5x)^3$

$\neq (\cot 5x)^{-1}$

$$f'(x)=\frac{\left[3(\cot 5x)^2(-\csc^2 5x)(5)\right]\cot^{-1}5x-\cot^3(5x)\left[\frac{-5}{1+25x^2}\right]}{\left[\cot^{-1}(5x)\right]^2}$$

$$f(x)=\tan\left[\underbrace{\operatorname{arc}\csc\left(x^5\right)}_{u}\right]$$

$$f'(x)=\sec^2(\csc^{-1}x^5)\left[\frac{-5x^4}{x^5\sqrt{x^{10}-1}}\right]$$

Homework:

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