900 square centimeters of material is to be used to make an open-topped box which has a square base. What should be the dimensions of the box in order that its volume will be as large as possible?

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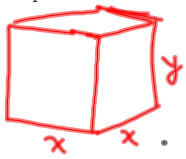

$V = x^2 y$ $\qquad x^2 + 4xy = 900$

$V = x^2\left(\frac{900 - x^2}{4x}\right)$ $\qquad y = \frac{900 - x^2}{4x}$

$V = \frac{900}{4}x - \frac{1}{4}x^3$

$V' = 225 - \frac{3}{4}x^2$

$0 = 225 - \frac{3}{4}x^2$

$-225 = -\frac{3}{4}x^2$

$\frac{-900}{-3} = x^2$

$x = \sqrt{300}$

$x = 10\sqrt{3}$

$y = \frac{900 - (10\sqrt{3})^2}{40\sqrt{3}}$

$y = \frac{600}{40\sqrt{3}} = \frac{15}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = 5\sqrt{3}$

Dimensions: $10\sqrt{3}$ cm by $10\sqrt{3}$ cm by $5\sqrt{3}$ cm

Let $f(x) = 2 - 2x - x^3$.

(a) What is the domain of f?

(b) Where does its graph cross the y-axis? (Don't try to calculate where it crosses the x-axis.)

(c) On what intervals is f increasing? (if none say so).

(d) On what intervals is f decreasing? (if none say so).

(e) Find the local minima of f (if any).

(f) Find the local maxima of f (if any).

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a) $x \in \mathbb{R}$

b) $y = 2 - 0 - 0$

$y = 2$

$(0, 2)$

c) $f'(x) = -2 - 3x^2$

$0 = -2 - 3x^2 \Rightarrow 2 = -3x^2$ $\quad \emptyset$

No critical Values

$f'(x) = -(2 + 3x^2)$

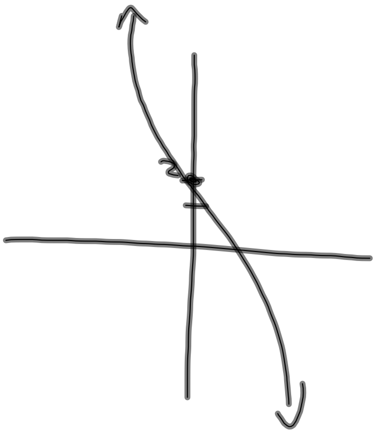
$f'(x) < 0$

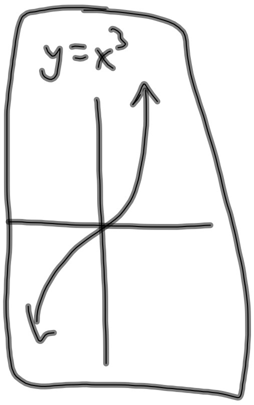
(c) Never

(d) $(-\infty, \infty)$

(e) None

(f) None





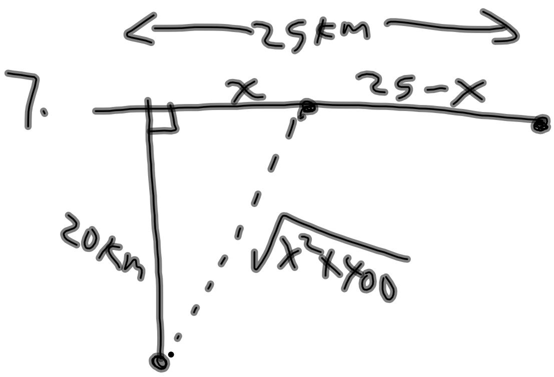
Practice Test:

6. Let x Rep. # of weeks

Profit = (Weight)(Price/lb) − storage costs

$$P(x) = (10000 - 100x)\Big(\underbrace{.16 + .001x}_{\text{Dollars}}\Big) - \underbrace{60x}_{\text{Dollars}}$$

7.



$$t = \frac{d}{s}$$

$$t = \frac{\sqrt{x^2+400}}{15} + \frac{25-x}{35}$$

$$t = \frac{1}{15}(x^2+400)^{1/2} + \frac{25}{35} - \frac{1}{35}x$$

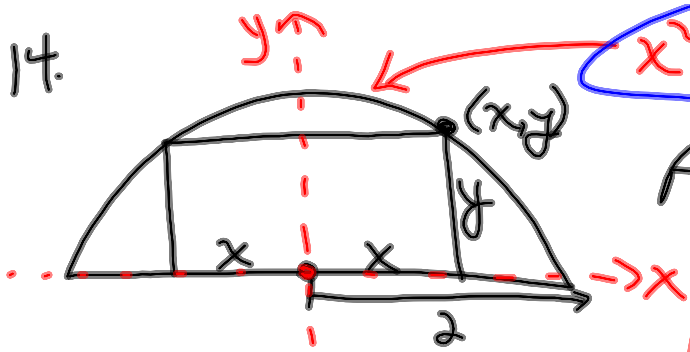
$$t' = \frac{1}{30}(x^2+400)^{-1/2}(2x) - \frac{1}{35}$$

$$0 = \frac{x}{15\sqrt{x^2+400}} - \frac{1}{35}$$

$$\left(\frac{1}{35}\right)^2 = \left(\frac{x}{15\sqrt{x^2+400}}\right)^2$$

⋮

14.



$\boxed{x^2+y^2=4}$

$y=\sqrt{4-x^2}$

$A=2xy$

$A=2x\left(\sqrt{4-x^2}\right)$

$A'=2\sqrt{4-x^2}+\cancel{2}x\left[\frac{1}{\cancel{2}}(4-x^2)^{-\frac{1}{2}}(-2x)\right]$

$\sqrt{4-x^2}\cdot 0=2\sqrt{4-x^2}\cdot\sqrt{4-x^2}-\frac{2x^2}{\cancel{\sqrt{4-x^2}}}\cdot\cancel{\sqrt{4-x^2}}$

$0=2(4-x^2)-2x^2$

$0=8-4x^2$

$4x^2=8$

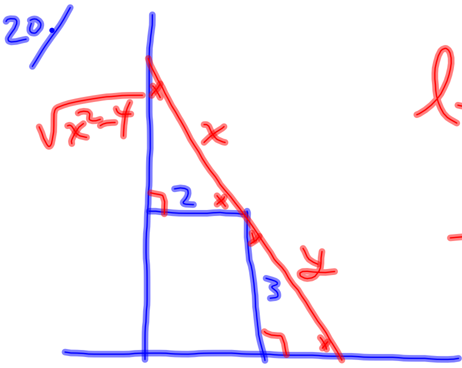
$x^2=2$

$x=\sqrt{2}$

$y=\sqrt{4-x^2}$

$y=\sqrt{4-2}$

$y=\sqrt{2}$



$$\text{length} = x + y$$

$$l = x + \frac{3x}{\sqrt{x^2-4}}$$

$$\frac{\sqrt{x^2-4}}{x} = \frac{3}{y}$$

$$y\sqrt{x^2-4} = 3x$$

$$y = \frac{3x}{\sqrt{x^2-4}}$$

$$l' = 1 + \frac{3\sqrt{x^2-4} - 3x\left(\frac{1}{2}(x^2-4)^{-\frac{1}{2}}(2x)\right)}{x^2-4}$$

$$0 = 1 + \frac{\left(\frac{3\sqrt{x^2-4}}{1} - \frac{3x^2}{\sqrt{x^2-4}}\right)}{x^2-4}$$

$$-1 = \left(\frac{3(x^2-4) - 3x^2}{\sqrt{x^2-4}}\right) \cdot \frac{1}{x^2-4}$$

$$\left[-1(x^2-4)\right]^2 = \left(\frac{-12}{\sqrt{x^2-4}}\right)^2$$

$$(x^2-4)^2 = \frac{144}{x^2-4}$$

$$(x^2-4)^2(x^2-4) - 144 = 0$$

$$\sqrt[3]{(x^2-4)^3} = \sqrt[3]{144}$$

$$x^2 - 4 = \sqrt[3]{144}$$

$$x = \sqrt{\sqrt[3]{144} + 4}$$

$$x = 3.03\text{m} \qquad y = \frac{3(3.03)}{\sqrt{(3.03)^2-4}}$$

$$y = 3.98\text{m}$$