Determining Quadratic Functions From Parabolas

- **In order to determine the equation of a quadratic, you need to know...**

(1) the vertex and another point on the parabola

or

(2) any three points on the parabola

Vertex: $y=a(x-h)^2+k$ { Standard form: $y=ax^2+bx+c$

ex: Determine the equation of the parabola having its vertex at (3, -5) and passing through the point (5, -17).

(x, y)

$y=a(x-3)^2-5$

$-17=a(5-3)^2-5$

$-17=4a-5$

$-12=4a$

$-3=a$

$\boxed{y=-3(x-3)^2-5}$

- **STEPS:**

(1) Build the vertex form of the equation using the known vertex.

(2) Let "a" represent the unknown stretch factor.

(3) Substitute the known point into the equation and solve for "a".

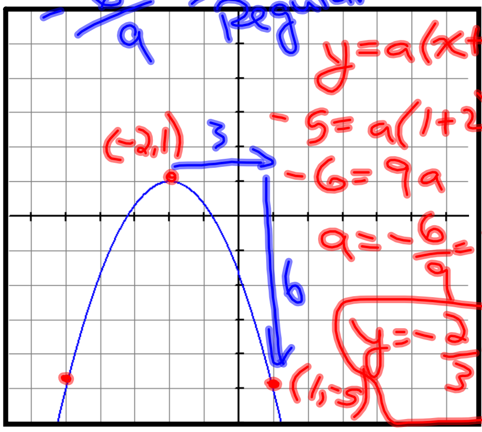
(4) Replace "a" value in the vertex form equation.

EXAMPLES: Determine the function that describes each of the following...

#1.

Actually Regular

$-\frac{6}{9}$



$y=a(x+2)^2+1$

$-5=a(1+2)^2+1$

$-6=9a$

$a=-\frac{6}{9}=-\frac{2}{3}$

$\boxed{y=-\frac{2}{3}(x+2)^2+1}$

#2.

X	Y1
-6	61
-5	33
-4	13
-3	1
-2	-3
-1	1
0	13

X=0

Vertex

$y=a(x+2)^2-3$

$13=a(0+2)^2-3$

$\frac{16}{4}=\frac{4a}{4}$

$a=4$

$\boxed{y=4(x+2)^2-3}$

#3. **vertex: (0, -3)**
point: (-4, 37)

$y=ax^2-3$ S.F. $=\frac{40}{16}$

$y=\frac{5}{2}x^2-3$

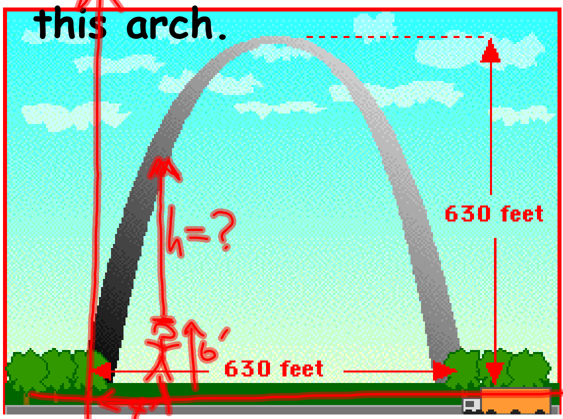
Example 4:

Determine the equation of the parabola that passes through the ordered pairs (-1,6), (0,1) and (2,3).

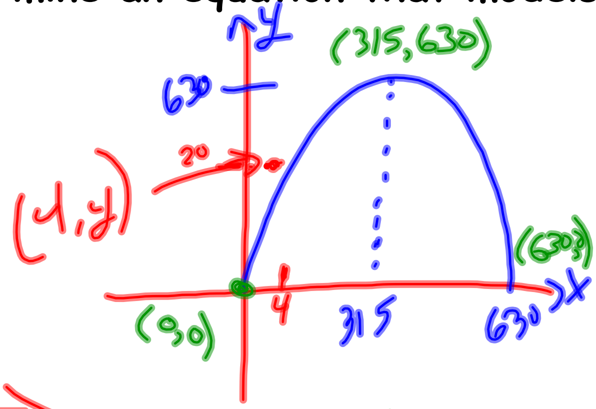
$y = a(x-h)^2 + k$

$y = ax^2 + bx + c$

St. Louis Gateway Arch - Determine an equation that models this arch.



The St. Louis Gateway Arch is an elegant monument to westward expansion in the USA. Located on the banks of the Mississippi River in St. Louis, Missouri, the 630-foot tall stainless steel arch rises above the city skyline. The Jefferson National Expansion Memorial consists of the Gateway Arch, the Museum of Westward Expansion, and St. Louis' Old Courthouse.



$y=a(x-h)^2+k$

$0=a(0-315)^2+630$

$-630=99225a$

$a=\frac{-630}{99225}=-\frac{2}{315}$

$y=-\frac{2}{315}(x-315)^2+630$

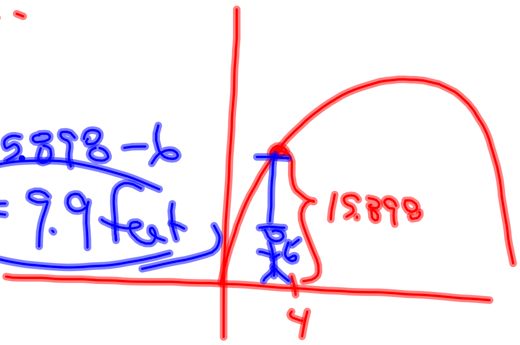
Sub. $x=4$...

$y=-\frac{2}{315}(4-315)^2+630$

$y=15.898..$

$15.898-6$

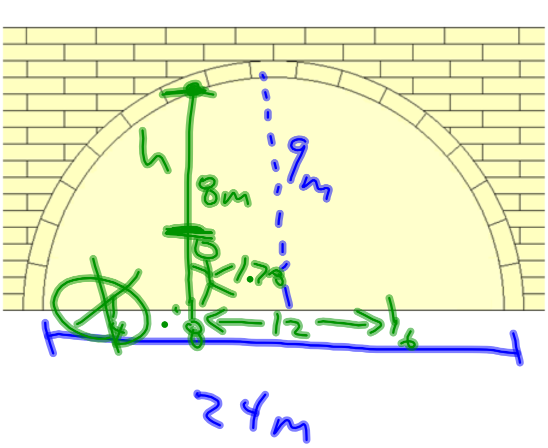
$\doteq 9.9$ feet

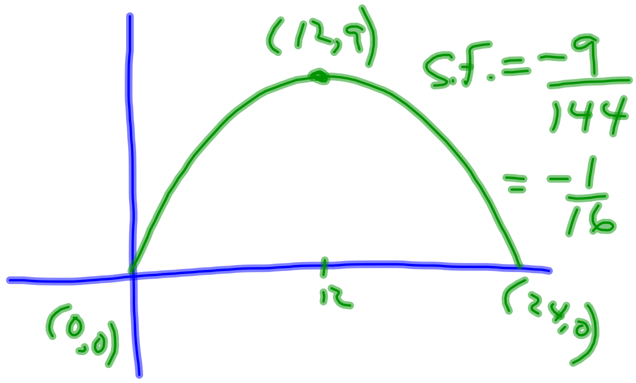


4. A parabolic arch supports a bridge that is spanning a walking tunnel. The arch is 24 m wide at the base and reaches a height of 9 m at the centre of the arch.

(a) Sketch this arch on a graph and determine a quadratic function that could describe the arch.

(b) A person 1.78 m tall is standing on the walkway below the arch at a point situated 4 m from the centre of the arch. Determine if this person could stand upright without touching their head on the top of the tunnel. If they are able to stand upright, how much clearance will there be between the top of their head and the arch?





$$s.f. = \frac{-9}{144} = -\frac{1}{16}$$

$$h = 8 - 1.78m$$
$$h = 6.22m$$

$$y = a(x-12)^2 + 9$$
$$y = \frac{-1}{16}(x-12)^2 + 9$$
$$y = -\frac{1}{16}(8-12)^2 + 9$$
$$y = 8$$

Quadratic Equations

Solving Quadratic Equations by Factoring

To solve a quadratic equation, you could graph the corresponding quadratic function OR use the following principle:

If ab = 0, then a= 0 or b = 0

Example 1,

a)Solve $x^2 - 4x + 3 = 0$

$x^2 - 4x + 3 = 0$

Simple Trinomial
Factor by Inspection

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

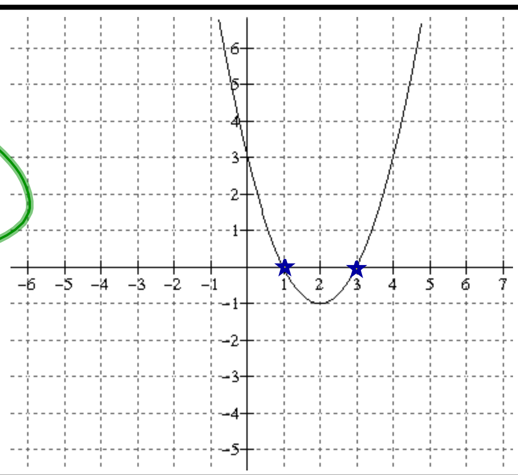
$(x - 1)(x - 3) = 0$

$x-1=0$ **OR** $x-3=0$

$x=1$ $\quad$ $x=3$

The roots are 1 and 3

b) Graph $y = x^2 - 4x + 3$



The x-intercepts of the graph are the same as the roots of the equation.

To solve a quadratic equation, the trinomial expression must be **equated to zero!!!**

Different Ways of Expressing........

What are the roots of...

Determine the zeroes of...

Solve....

Find the solution set of....

Find the x-intercepts of....

Another Example...

Solving Equations by Factoring:

Solve: $x^2 - 10 = 3x$

1. Get one side = 0 $\quad x^2 - 3x - 10 = 0$
2. Factor completely $\quad (x - 5)(x + 2) = 0$
3. Set each factor = 0 $\quad x - 5 = 0 \;\; OR \;\; x + 2 = 0$
4. Solve each equation $\quad x = 5 \;\; OR \;\; x = -2$
5. Check in original $\quad (5)^2 - 10 = 3(5)?$ $\quad (-2)^2 - 10 = 3(-2)?$

   $25 - 10 = 15 \;\; YES$ $\quad 4 - 10 = -6 \;\; YES$
6. Solution set $\quad \{5, -2\}$

EXAMPLE: **Determine the zeroes of the following...**

$$y = 5x^2 - 17x + 6$$

$0 = 5x^2 - 17x + 6$

$0 = 5x^2 - 15x - 2x + 6$

$0 = 5x(x-3) - 2(x-3)$

$0 = (x-3)(5x-2) \rightarrow 5x-2=0$

$5x = 2$

$x = 2/5$

$x = 3$ or $x = \frac{2}{5}$

$x = \frac{17 \pm \sqrt{289 - 4(5)(6)}}{2(5)}$

$x = \frac{17 \pm 13}{10}$

$x = \frac{30}{10}$ or $\frac{4}{10}$

$= 3$ $\quad = \frac{2}{5}$

EXAMPLE: **Solve each of the following:**

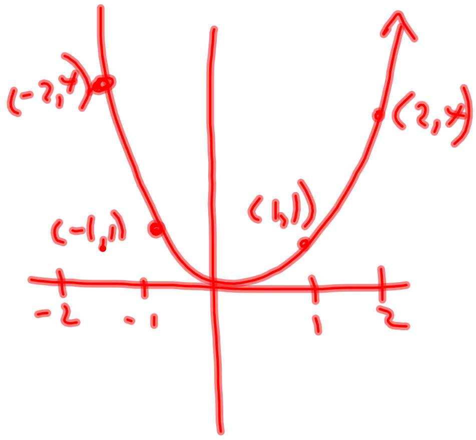
(a) $2 - \frac{1}{x} = \frac{3}{x+2}$

(b) $x^4 - 20x^2 + 64 = 0$

Hint: Look for the pattern of a quadratic...substitute a new variable

$y = x^2$

x	y
3	9
2	4
1	1
0	0
-1	1
-2	4
-3	9



$V(-2, 6)$

① Pt. $(-4, 13)$

$6 \text{ t } 13 \rightarrow$ up 7

$S.F. = \frac{7}{4}$

② $(-7, -34)$

$S.F. = \frac{-40}{25} = \frac{-8}{5}$

Vertex: $(-7, 10)$

Thru: $(-3, 23)$

$$y = a(x-h)^2 + K$$

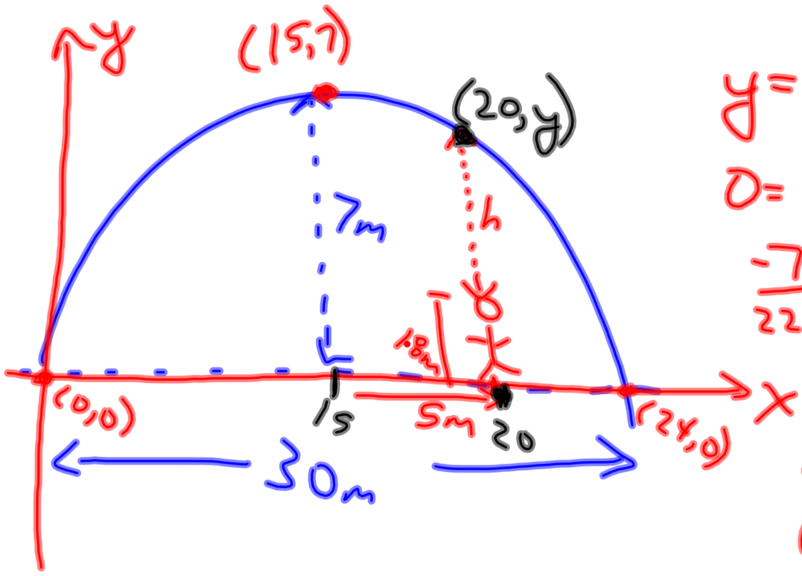
$$y = a(x+7)^2 + 10$$

$$23 = a(-3+7)^2 + 10$$

$$13 = 16a$$

$$a = \frac{13}{16}$$

$$\boxed{y = \frac{13}{16}(x+7)^2 + 10}$$



$y = a(x-15)^2+7$

$0 = a(0-15)^2+7$

$\frac{-7}{225} = a$

$y = \frac{-7}{225}(x-15)^2+7$

at $x=20$

$y = \frac{-7}{225}(20-15)^2+7$

$y = \underline{6.2\ m}$

$\therefore$ Clearance: $6.2m - 1.8m$

$= 4.4m$

$y=-2x^2+16x-5$

$\boxed{\text{Vertex?} \quad y=a(x-h)^2+k}$

-32

$y=-2(x^2-8x+\underline{16})-5+32$

$y=-2(x-4)^2+27$

$V(4,27)$

$y=3x^2+30x+1$

$+75$

$y=3(x^2+10x+\underline{25})+1-75$

$y=3(x+5)^2-74$

$V(-5,-74)$

$y=3x^2-15x+11 \rightarrow \left(\frac{5}{2}\right)^2$

$y=3(x^2-5x+\underline{\frac{25}{4}})+\frac{11}{1}-\frac{75}{4}$

$y=3(x-\frac{5}{2})^2-\frac{31}{4}$