EXERCISE: Express the following in the form "a + bi"...

$$\boxed{\frac{\left(4-4i\sqrt{3}\right)\left(2\sqrt{3}+2i\right)\left(1+i\right)}{\left(5-5i\right)\left(-\sqrt{3}+i\right)}}$$

$$\frac{\left(8\sqrt{3}+8i-24i-8\sqrt{3}i^2\right)(1+i)}{\left(-5\sqrt{3}+5i+5\sqrt{3}i-5i^2\right)}$$

$$\frac{\left(16\sqrt{3}-16i\right)(1+i)}{5-5\sqrt{3}+5i+5\sqrt{3}i}$$

$$\frac{16\sqrt{3}+16\sqrt{3}i-16i-16i^2}{\left(5-5\sqrt{3}\right)+\left(5+5\sqrt{3}\right)i}$$

$$\frac{\left(16\sqrt{3}+16\right)+\left(16\sqrt{3}-16\right)i}{\left(5-5\sqrt{3}\right)+\left(5+5\sqrt{3}\right)i} \quad \left[\frac{\left(5-5\sqrt{3}\right)-\left(5+5\sqrt{3}\right)i}{\left(5-5\sqrt{3}\right)-\left(5+5\sqrt{3}\right)i}\right.$$

$$(a \quad + \quad bi)$$

$$\frac{\left(16\sqrt{3}+16\right)\left(5-5\sqrt{3}\right)-\left(16\sqrt{3}+16\right)\left(5+5\sqrt{3}\right)i+\left(16\sqrt{3}-16\right)\left(5-5\sqrt{3}\right)-\left(16\sqrt{3}-16\right)\left(5+5\sqrt{3}\right)i^2}{\left(5-5\sqrt{3}\right)^2-\left(5+5\sqrt{3}\right)^2i^2}$$

$$\frac{\cancel{80\sqrt{3}}-\cancel{80(3)}+\cancel{80}-\cancel{80\sqrt{3}}-\left(80\sqrt{3}+80(3)+80+80\sqrt{3}\right)i+\cancel{80\sqrt{3}}-\cancel{80(3)}-\cancel{80}+\cancel{80(3)}+80\sqrt{3}+\cancel{80(3)}-80-\cancel{80\sqrt{3}}}{25-\cancel{80\sqrt{3}}+75+25+\cancel{50\sqrt{3}}+75}$$

EXERCISE: Express the following in the form "a + b*i*"...

$$\boxed{\frac{\left(4-4i\sqrt{3}\right)\left(2\sqrt{3}+2i\right)\left(1+i\right)}{\left(5-5i\right)\left(-\sqrt{3}+i\right)}}$$

$$\frac{(4-4i\sqrt{3})(2\sqrt{3}+2i)(1+i)}{(5-5i)(-\sqrt{3}+i)} \quad \frac{(-\sqrt{3}-i)}{(-\sqrt{3}-i)}$$

$$\frac{(4-4i\sqrt{3})(2\sqrt{3}+2i)(1+i)(-\sqrt{3}-i)}{(5-5i)(3+1)} \quad \frac{(5+5i)}{(5+5i)}$$

$$\frac{(4-4i\sqrt{3})(2\sqrt{3}+2i)(1+i)(-\sqrt{3}-i)(5+5i)}{4(50)}$$

$$= \frac{\qquad}{200}$$

Product and Quotient of Complex Numbers in Polar Form

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\ \theta_2 + i\sin\ \theta_2)$…

Now let's examine $z_1 \bullet z_2$ and $\dfrac{z_1}{z_2}$

$z_1 \cdot z_2$

$[r_1(\cos\theta_1 + i\sin\theta_1)] \times [r_2(\cos\theta_2 + i\sin\theta_2)]$

$r_1 r_2 [(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)]$

$r_1 r_2 [\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 + \overset{-1}{i^2}\sin\theta_1\sin\theta_2]$

$r_1 r_2 [(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)]$

$\underline{r_1 r_2}[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

CONCLUSIONS...

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\ \theta_2 + i\sin\ \theta_2)$...

$$\boxed{z_1 \bullet z_2 = r_1 r_2\left[\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2)\right]}$$

- multiply all "r" values together
- add all angles together

&

$$\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2)\right]}$$

- divide "r" values
- subtract angles

EXERCISE: Express the following in the form "a + bi"...

$$\boxed{\frac{\left(4-4i\sqrt{3}\right)^{6}\left(2\sqrt{3}+2i\right)^{10}\left(1+i\right)^{8}}{\left(5-5i\right)^{4}\left(-\sqrt{3}+i\right)^{5}}}$$

① $4-4i\sqrt{3} \Rightarrow r\operatorname{cis}\theta$

$(4,-4\sqrt{3})$ (Q4)

$r=\sqrt{4^2+(4\sqrt{3})^2}$

$r=\sqrt{16+48}$

$r=8$

$\tan\theta=\frac{4\sqrt{3}}{4}$

$\tan^{-1}\sqrt{3}=\theta$

(Ref ∡ $=60^\circ$)

$360-\theta$ $\therefore \theta=300^\circ$

Polar Form:

$=8(\cos 300^\circ+i\sin 300^\circ)$

$=8\operatorname{cis}300^\circ$

② $(2\sqrt{3},2)\rightarrow$ Q1

$r=\sqrt{12+4}$

$r=4$

$\tan\theta=\frac{2}{2\sqrt{3}}$

(Ref ∡ $=30^\circ$)

$\theta=30^\circ$

$=4\operatorname{cis}30^\circ$

③ $(1,1)$ Q1

$r=\sqrt{1+1}$

$r=\sqrt{2}$

$\tan\theta=1$

(Ref ∡ $=45^\circ$)

$\theta=45^\circ$

$=\sqrt{2}\operatorname{cis}45^\circ$

④ $(5,-5)$, Q4

$r=\sqrt{25+25}$

$r=\sqrt{50}$

$r=5\sqrt{2}$

$\tan\theta=\frac{5}{5}=1$

(Ref ∡ $=45^\circ$)

$\theta=360-45^\circ$

$\theta=315^\circ$

$=5\sqrt{2}\operatorname{cis}315^\circ$

⑤ $(-\sqrt{3},1)$ Q2

$r=\sqrt{3+1}$

$r=2$

$\tan\theta=\frac{1}{\sqrt{3}}$

(Ref ∡ $=30^\circ$)

$\theta=180-30$

$=150^\circ$

$=2\operatorname{cis}150^\circ$

$$=\frac{(8\operatorname{cis}300^\circ)(4\operatorname{cis}30^\circ)(\sqrt{2}\operatorname{cis}45^\circ)}{(5\sqrt{2}\operatorname{cis}315^\circ)(2\operatorname{cis}150^\circ)}$$

$$=\frac{(8\cdot 4\cdot\sqrt{2})\operatorname{cis}(300+30+45)^\circ}{(5\sqrt{2}\cdot 2)\operatorname{cis}(315^\circ+150^\circ)}$$

$$=\frac{32\sqrt{2}\operatorname{cis}375^\circ}{10\sqrt{2}\operatorname{cis}465^\circ}$$

$$=\left(\frac{32\cancel{\sqrt{2}}}{10\cancel{\sqrt{2}}}\right)\operatorname{cis}(375-465)$$

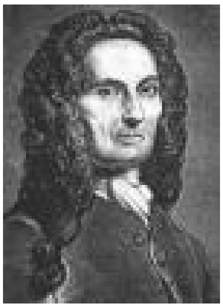
$$=\frac{16}{5}\operatorname{cis}(-90^\circ)$$

$$=\frac{16}{5}\left(\cos(-90^\circ)+i\sin(-90^\circ)\right)$$

$$=\frac{16}{5}\left(0+i(-1)\right)$$

$$=0-\frac{16}{5}i$$

$$=-\frac{16}{5}i$$



THEOREM!!!

Demoivre's

$$\boxed{[r(\cos\theta + i\sin\theta)]^n = r^n[\cos(n\theta) + i\sin(n\theta)]}$$

$(rcis\theta)^n$

Example: Simplify the following using the polar form of a complex number.

$$(1-i)^6$$

```
(1-i)^6
            8i
```

$r=\sqrt{1+1}$

$r=\sqrt{2}$

$(1,-1) \Rightarrow Q4$

$\tan\theta = 1$

(Ref ∡ $=45°$)

$\theta = 315°$

$= (\sqrt{2}\ cis\ 315°)^6$

$= \left((2^{\frac{1}{2}})^6\right) cis(6 \times 315°)$

$= 2^3\ cis(1890°)$

$= 8(\cos 1890° + i\sin 1890°)$

$= 8(0+1i)$

$= 8i$

Homework...

Worksheet - Polar Form

We just did #5 together!!!

#6 and #7

Attachments

Worksheet - DeMoivres Theorem.doc