Recall from our prior discussions that ...

1 Theorem $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$

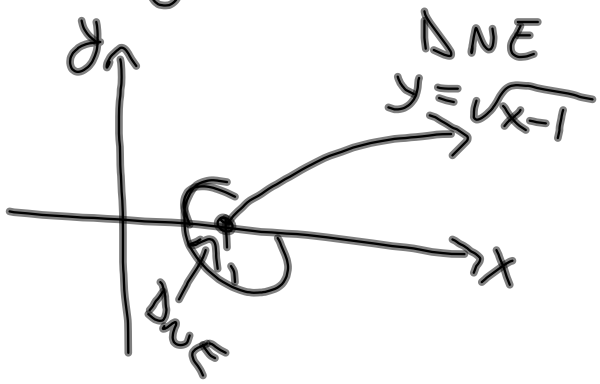
Let's look at a couple of unique functions:

* 1) $\lim_{x\to 1} \sqrt{x-1} = \underline{\underline{DNE}}$

$\lim_{x\to 1^+} \sqrt{1.000..1 - 1}$ $\qquad$ $\lim_{x\to 1} \sqrt{0.999.. - 1}$

$\sqrt{0.000...1}$ $\qquad$ $= \sqrt{-0.0000...1}$

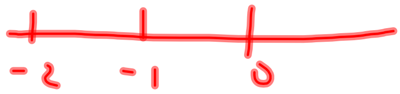
$= 0$



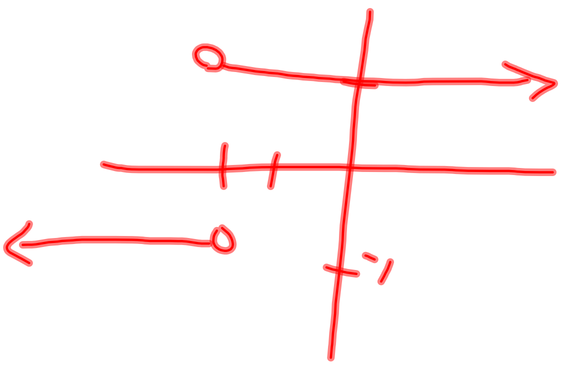
2) $\lim\limits_{x\to -2} \dfrac{|x+2|}{x+2} = \underline{\underline{DNE}}$

Absolute Value !!

$$\lim_{x\to -2^+} \frac{|-1.999\ldots+2|}{-1.999\ldots+2} = \frac{0.00\cancel{0\ldots1}}{0.00\cancel{0\ldots1}} = 1$$



$$\lim_{x\to -2^-} \frac{|-2.0000\ldots1+2|}{-2.000\ldots1+2} = \frac{0.00\cancel{0\ldots1}}{-0.00\cancel{0\ldots1}} = -1$$



Piecewise Defined Functions

Definition:
- **Functions defined by different formulas in different parts of their domains**

Example:

$$f(x) = \begin{cases} x+3 & \text{if } x \le 2 \\ x^2-2 & \text{if } x > 2 \end{cases}$$

$V(0,-2)$

$f(1) = 1+3$
$= 4$

$f(3) = 3^2 - 2$
$= 7$

$f(2) = 2+3$
$= 5$

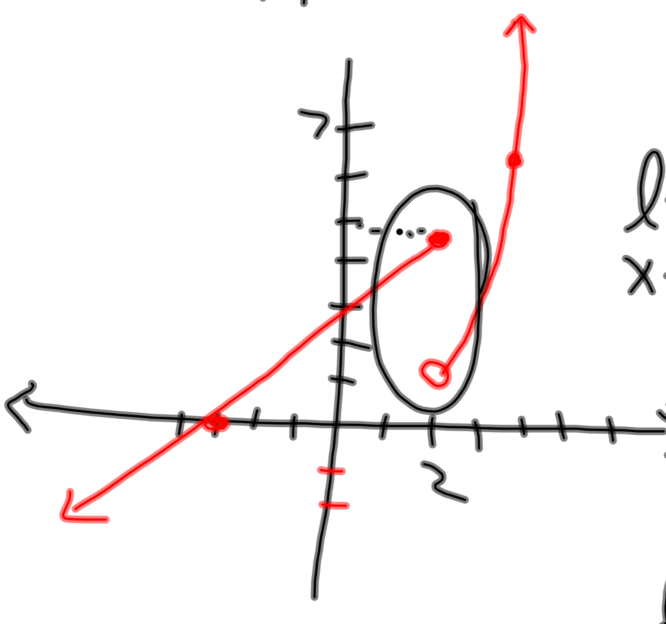
1) Determine *f(1)* , *f(3),* and *f(2)*.
2) Sketch *f(x)*.

①

x	y
2	5
-3	0

②

x	y
2	2
3	7



$\lim_{x \to 2^-} f(x) = 5$

$\lim_{x \to 2^+} f(x) = 2$

$\lim_{x \to 2} f(x)$ DNE

Sketch the following piecewise function:

$$f(x)=\begin{cases}\frac{1}{2}x-2 & if\ x<-2\\ -1 & if\ -2\le x\le 1\\ (x-2)^2+1 & if\ x>1\end{cases}$$

① $V(2,1)$

slightly!!

$x<-2$ $\quad$ $x>-2$

$a(x-h)^2+K$

$\underline{x=-2}$ $\quad$ $\underline{x=1}$ $\quad$ $V(h,k)$

$\lim_{x\to -2^-} f(x) = \frac{1}{2}(-2)-2 = -3$

$\lim_{x\to -2^+} f(x) = -1$

$\lim_{x\to 1^-} f(x) = -1$

$\lim_{x\to 1^+} f(x) = (1-2)^2+1 = 2$

Piece 1:

x	y
-2	-3
-4	-4

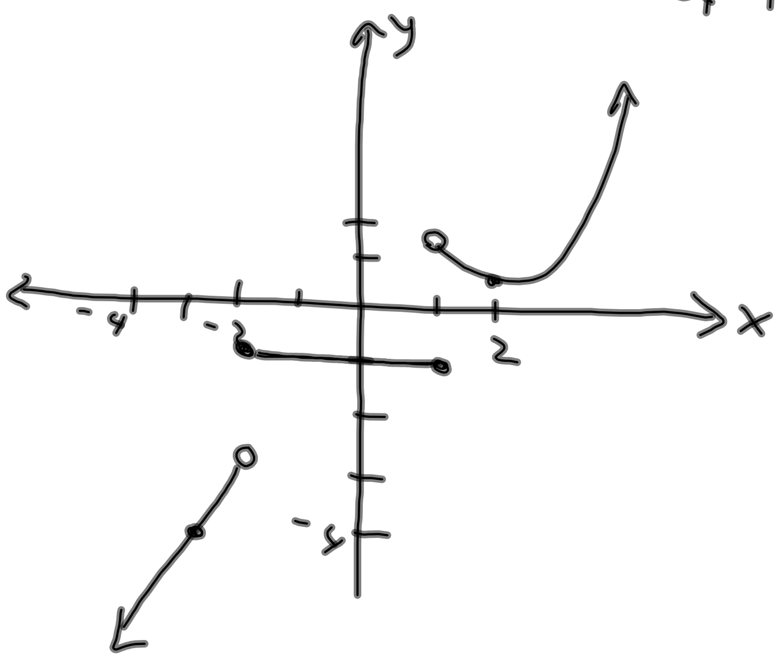
Piece 2:

$y=-1$

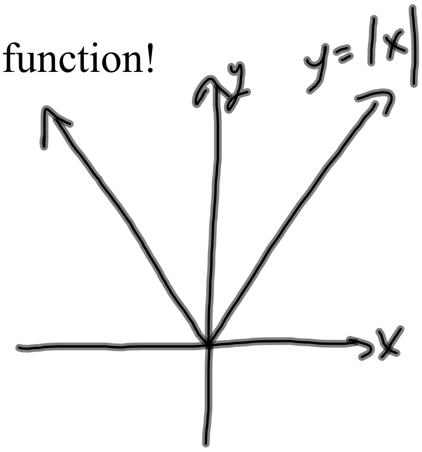
Piece 3

x	y
1	2
2	1

← Vertex



Absolute value function...the hidden piecewise function!



What about this function? $\longrightarrow$ $f(x) = |x|$

$y = -|x-3| + 2$

$V(3,2)$ opens Down

$y = |6| = 6$

Between Bars(−)

$y = |x|$

$x < 0$

$y = |-6| = 6$

$y = |-10| = 10$

$$y = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$$

More Practice...

- Express the following absolute value function as a piecewise function
- Sketch the function

① Between Bars (-) $f(x) = |x-3|$

$x-3<0$

$x<3$

$\therefore f(x) = -(x-3)$

② Between Bars (+)

$x-3>0$

$x>3$

$f(x) = x-3$

$$f(x) = \begin{cases} -x+3 & \text{if } x \leq 3 \\ x-3 & \text{if } x > 3 \end{cases}$$

ex: $f(x) = |x+6| - 2$

$|-10|$

$-(-10)$

① BBP

$x+6>0$

$x>-6$

② BBN

$\left.\begin{array}{l} x+6<0 \\ x<-6 \end{array}\right\}$ $f(x) = -(x+6)-2$

$= -x-8$

$$f(x) = \begin{cases} -x-8 & \text{if } x < -6 \\ x+4 & \text{if } x \geq -6 \end{cases}$$