

Warm-Up

SOLUTION!!!

Express the following as a complex number in standard form ($a + bi$).

$$3i^5 + (2i^6)^5 + \frac{(-1+2i) - 3(2+i)}{(-2+i)^2}$$

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3i^5+(2i^6)^5+((-1+2i)-3(2+i))/(-2+i)^2
-32.68+1.76i
Ans+Frac
-817/25+44/25i
    
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$$= 3(i^2)^2 i + 32(i^2)^5 + \frac{-1+2i-6-3i}{4-4i+i^2}$$

$$\rightarrow 32i^{30}$$

$$32(i^2)^{15}$$

$$= 3(1)i + 32(-1) + \frac{-7-i}{3-4i} \left(\frac{3+4i}{3+4i} \right)$$

$$= 3i - 32 + \frac{-21 - 28i - 3i - 4i^2}{9 - 16i^2}$$

$$= 3i - 32 + \frac{-17 - 31i}{25}$$

$$= \frac{3i}{1} - \frac{32}{1} - \frac{17}{25} - \frac{31}{25}i$$

$$= -\frac{800}{25} - \frac{17}{25} + \frac{75}{25}i - \frac{31}{25}i$$

$$= -\frac{817}{25} + \frac{44}{25}i$$

$$4. e) \frac{2+i\sqrt{5}}{1-3i} \left(\frac{1+3i}{1+3i} \right)$$

$$= \frac{2+6i+i\sqrt{5}+3\sqrt{5}i}{1-9i^2}$$

$$= \frac{2-3\sqrt{5}+(6+\sqrt{5})i}{10}$$

$$= \frac{2-3\sqrt{5}}{10} + \frac{6+\sqrt{5}}{10}i$$

Principle of Equality - "Comparison"

- comparison of left side versus right side.
- real parts must equal each other and the imaginary parts must be equal.

EXAMPLE #1:

$$3 - i + 2i = 6i - (2x + yi)$$

Solve for x & y :

$$3 + i = 6i - 2x - yi$$

Real = Real Im = Im ← Coefficients ONLY !!

$$\left. \begin{array}{l} \frac{3}{-2} = \frac{-2x}{-2} \\ x = -\frac{3}{2} \end{array} \right\} \begin{array}{l} 1 = 6 - y \\ y = 5 \end{array}$$

EXAMPLE #2:

$$4i(3x - y) = 3 - (3 - yi)i$$

$$12xi - 4yi = 3 - 3i + yi^2$$

$$12xi - 4yi = 3 - 3i - y$$

Real = Real Im = Im

$$0 = 3 - y$$

$y = 3$ →

$$12x - 4y = -3$$

$$12x - 4(3) = -3$$

$$12x = 9$$

$$x = \frac{9}{12}$$

$x = \frac{3}{4}$

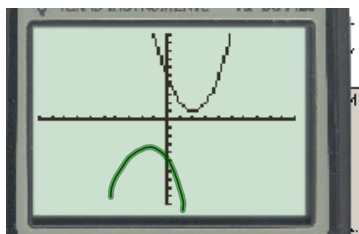
Complex Roots

- If it is not possible to factor a quadratic equation and you cannot use the completing the square or quadratic formula because there is a negative under the radical sign.....

There are no x-intercepts!!!!!!

EXAMPLE:

$$y = 1x^2 - 4x + 5, \quad x \in \mathbb{C}$$



Look, it doesn't cross the x-axis

What happens if I try to use the quadratic formula?

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(+5)}}{2(1)}$$

a =

b =

c =

$$x = \frac{4 \pm \sqrt{-4}}{2}$$

$-1 = i^2$
 No Solutions!!
 $\{x \in \mathbb{R}\}$

$$x = \frac{4 \pm \sqrt{4i^2}}{2}$$

$$x = \frac{4 \pm 2i}{2}$$

$$x = 2 \pm i \rightarrow x = 2 + i$$

Homework...

Worksheet

5, 6, 7

Complex Plane (Argand Diagrams)

We can represent complex numbers in the **complex plane**.

We use the **horizontal axis** for the **real part** and the **vertical axis** for the **imaginary part**.

NOTE: $(x, y) = (\text{Re}, \text{Im})$

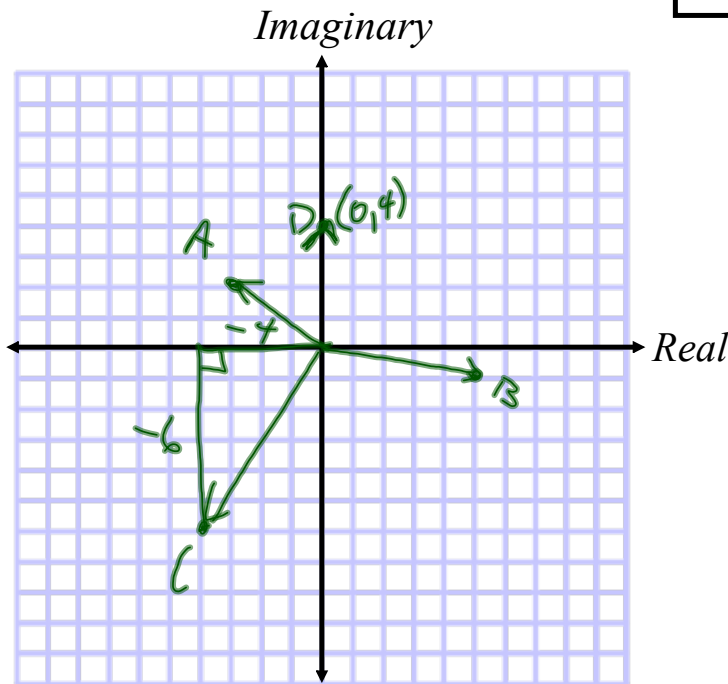
Examples:

A: $-3 + 2i \Rightarrow (-3, 2)$

B: $5 - i \Rightarrow (5, -1)$

C: $-4 - 6i \Rightarrow (-4, -6)$

D: $4i \Rightarrow (0, 4)$



Referred to as an ARGAND DIAGRAM

~~$|z| = r$~~

- the magnitude of a complex vector uses the notation $|a + bi|$ where the length is determined by the Pythagorean Theorem

EXAMPLE... (B) $|-2 - 4i|$

$$= \sqrt{(-2)^2 + (-4)^2}$$

$$= \sqrt{4 + 16}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5}$$

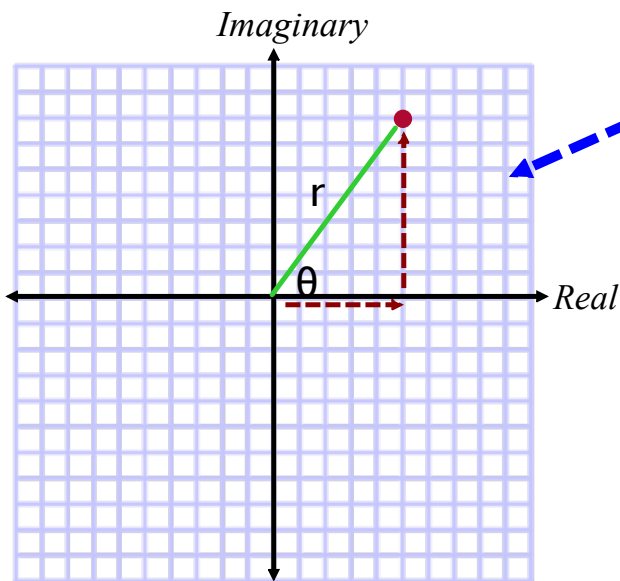
$$|a + bi| = \sqrt{a^2 + b^2}$$

Not
Absolute Value

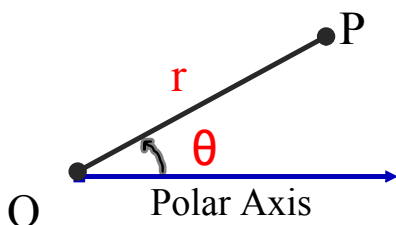
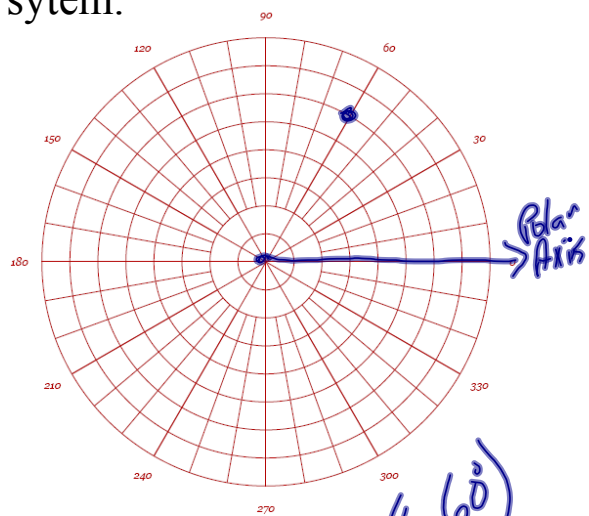
Polar Coordinate System

Graphing system that plots ordered pairs of the form (r, θ) .

- "r" is the absolute value or modulus. The distance from the origin to the point.
- θ is the angle of rotation from the starting position, referred to as the "pole".
- to locate a point, start with the point O, called the **pole** and a particular ray with its endpoint at O along the **polar axis**.



This is referred to as a rectangular coordinate system.

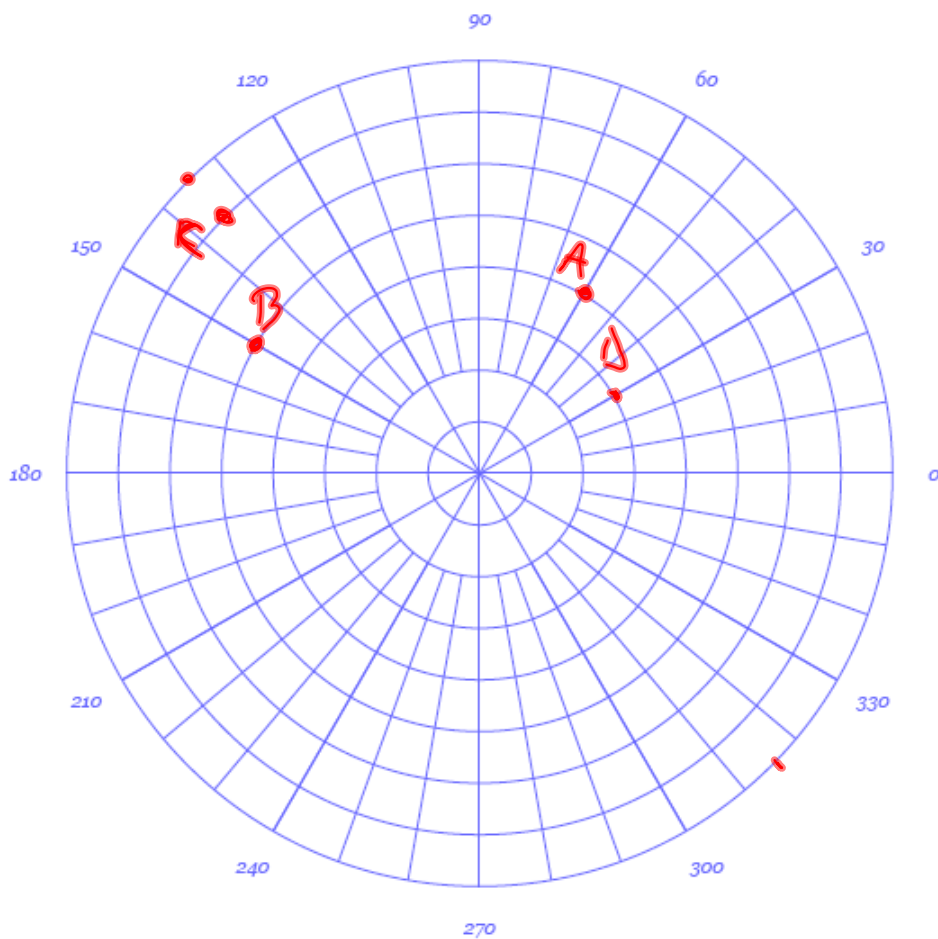


The ordered pair (r, θ) is referred to as the polar coordinates of P

Plotting Polar Coordinates:

Plot each of the following points:

A (4, 60°) B (5, -210°) C (-7, 315°) D (-3, -150°)



MORE EXAMPLES...

A (2, 45°)

B (-3, 225°)

C (1, 450°)

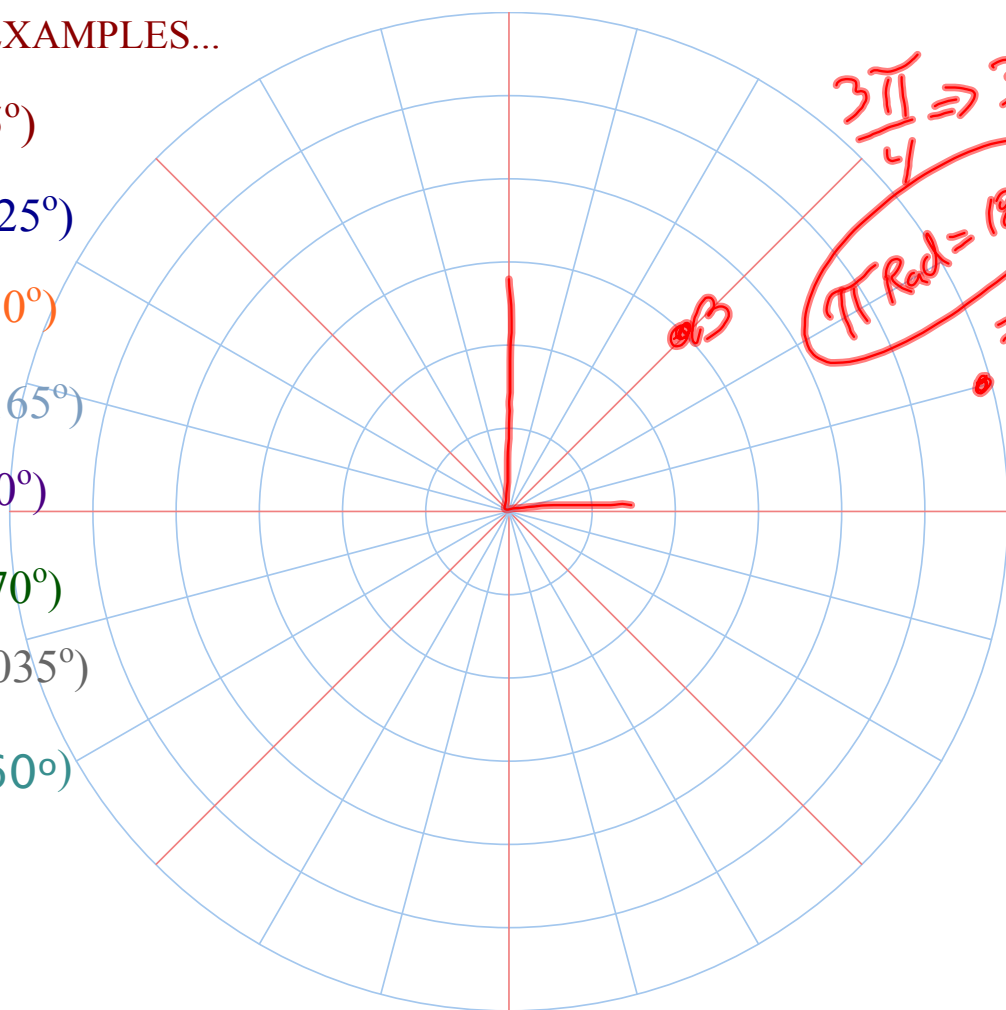
D (-6, -165°)

E (4, 240°)

F (-2, 270°)

G (4, -1035°)

H (-2, -60°)



$\frac{3\pi}{4} \Rightarrow 3 \frac{(180^\circ)}{4}$
 $\pi \text{ Rad} = 180^\circ$
 $= 135^\circ$

Homework...

Assignment - Plotting Polar Coordinates.doc

Attachments

Worksheet - Plotting Polar Coordinates.doc