Warm-Up

1. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^3 = 1$

2. Find all roots $z \in C$ of the equation $z^2 - 3iz + 10 = 0$.

3. Express the following in polar form:

(a) $(-1, -\sqrt{2})$ (b) $(0, -1)$

4. Convert the following to rectangular coordinates:

(a) $(3, 135°)$ (b) $(-8, -300°)$

1. Prove that $\left(\frac{-1+i\sqrt{3}}{2}\right)^3 = 1$

$$\frac{(-1+i\sqrt{3})^2(-1+i\sqrt{3})}{8}$$

$$\frac{(1-2i\sqrt{3}+3i^2)(-1+i\sqrt{3})}{8}$$

$$\frac{(-2-2i\sqrt{3})(-1+i\sqrt{3})}{8}$$

$$\frac{2-2i\sqrt{3}+2i\sqrt{3}-2i^2(3)}{8}$$

$$\frac{2+6}{8}$$

$$\frac{8}{8}$$

$$= 1$$

2. Find all roots $z \in C$ of the equation $z^2 - 3iz + 10 = 0$.

$a = 1$
$b = -3i$
$c = 10$

$$z = \frac{3i \pm \sqrt{(-3i)^2 - 4(1)(10)}}{2} \rightarrow 9i^2$$

$$z = \frac{3i \pm \sqrt{-9 - 40}}{2}$$

$$z = \frac{3i \pm \sqrt{-49}}{2}$$

$$z = \frac{3i \pm \sqrt{49i^2}}{2}$$

$$z = \frac{3i \pm 7i}{2}$$

$z = 5i$ OR $z = -2i$

3. Express the following in polar form:

(a) $(-1, -\sqrt{2}) \Rightarrow (r, \theta)$

$r = \sqrt{x^2 + y^2}$ Quad: 3

$r = \sqrt{(1)^2 + (\sqrt{2})^2}$

$r = \sqrt{3}$

$180 + \theta$

Ref. $\measuredangle$: $\tan\theta = \frac{\sqrt{2}}{1}$

(Ref $\measuredangle$: $55°$)

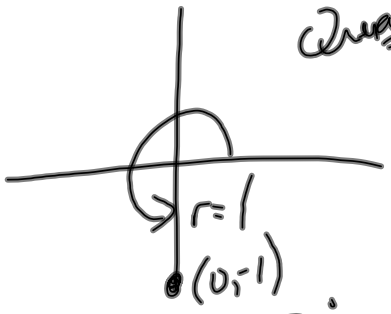
$\theta = 180 + 55°$

$\theta = 235°$

$(\sqrt{3}, 235°)$

(b) $(0, -1)$

Quadrantal



$\theta = 270°$

$(1, 270°)$

4. Convert the following to rectangular coordinates:

(a) $(3, 135°)$

$x = r\cos\theta \quad y = r\sin\theta$

$x = 3\cos 135° \quad y = 3\sin 135°$

$x = -2.1 \quad y = 2.1$

$(-2.1, 2.1)$

(b) $(-8, -300°)$

$x = -8\cos(-300°)$

$= -4$

$y = -8\sin(-300°)$

$y = -6.9$

$(-4, -6.9)$

$a+bi = r(\cos\theta + i\sin\theta)$

1. Express $1-\sqrt{3}i$ in polar form.

$(Re, Im) \to (1, -\sqrt{3})$ Q4 $\longrightarrow$ $r = \sqrt{1+3}$

Rect $\to$ Polar $\qquad r = 2$

$180-\theta$	θ
$180+\theta$	$360-\theta$

Ref Δ $\tan\theta = \sqrt{3}$, $\theta = 60°$

$\Rightarrow (2, 300°)$

$\theta = 360° - 60° = 300°$

$= 2(\cos 300° + i\sin 300°)$

$= 2\,\mathrm{cis}\,300°$

2. Express $-3-4i$ in polar form.

$(-3,-4) \to$ Q3

$r = \sqrt{9+16}$ $\qquad \tan\theta = \frac{4}{3}$

$r = 5$ $\qquad (\theta = 53°)$ Ref. $\angle$

$\theta = 180° + 53° = 233°$

$= 5(\cos 233° + i\sin 233°)$

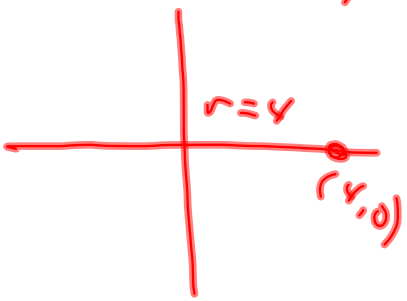
$= 5\,\mathrm{cis}\,233°$

3. Express the number 4 in polar form.

$= 4 + 0i$

$(4, 0°)$ $= 4(\cos 0° + i \sin 0°)$

$= 4\,\mathrm{cis}\,0°$



4. Express the polar form $\sqrt{8}(cos1395° + isin1395°)$ as a complex number in rectangular form. $(a+bi)$

$= \sqrt{8}\left(\frac{1}{\sqrt{2}} + i\left(\frac{-1}{\sqrt{2}}\right)\right)$

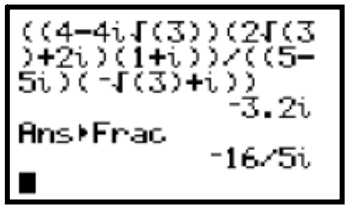
$= 2 - 2i$

Homework...

Worksheet - Converting Polar_Rectangular Coordinates.doc

Finish Worksheet

EXERCISE: Express the following in the form "a + b*i*"...



$$\frac{\left(4-4i\sqrt{3}\right)\left(2\sqrt{3}+2i\right)\left(1+i\right)}{\left(5-5i\right)\left(-\sqrt{3}+i\right)}$$

[5]

$-\frac{16}{5}i$

Worksheet - Complex Numbers.doc

Worksheet Solns - Complex Numbers.doc

BONUS

Attachments

Worksheet - Complex Numbers.doc

Worksheet - Converting Polar_Rectangular Coordinates.doc

Worksheet Solns - Complex Numbers.doc