Warm Up

$d(\ln u) = \frac{1}{u}\,du$

If $\frac{dy}{dx} = \tan x$, then $y =$

$y' = \frac{-\sin x}{\cos x} \Rightarrow y = -\ln|\cos x| + C$

$\frac{\sec x \tan x}{\sec x}$

(A) $\frac{1}{2}\tan^2 x + C$

(B) $\sec^2 x + C$

(C) $\ln|\sec x| + C$

(D) $-\ln|\cos x| + C$

(E) $\sec x \tan x + C$

$y = \ln(\cos x)^{-1} + C$

$y = \ln\left(\frac{1}{\cos x}\right) + C$

$y = \ln|\sec x| + C$

If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\cancel{\frac{1}{2}e^{-2x+2}}$

(B) $\cancel{e^{-x-1}}$

(C) e^{1-x}

(D) $\cancel{e^{-x}}$

(E) $\cancel{-e^{x}}$

$f(x) = e^{1-x}$

$f'(x) = -e^{1-x}$

Review: Antiderivatives

Antidifferentiate each of the following: $\cancel{(u)^n du}$ $\cancel{d(\tan^{-1}u) = \frac{du}{1+u^2}}$

1. $f'(x) = 4xe^{x^2}$

$\boxed{d(e^u) = e^u \cdot du}$

$f'(x) = 2(e^{x^2}(2x))$

$f(x) = 2e^{x^2} + C$

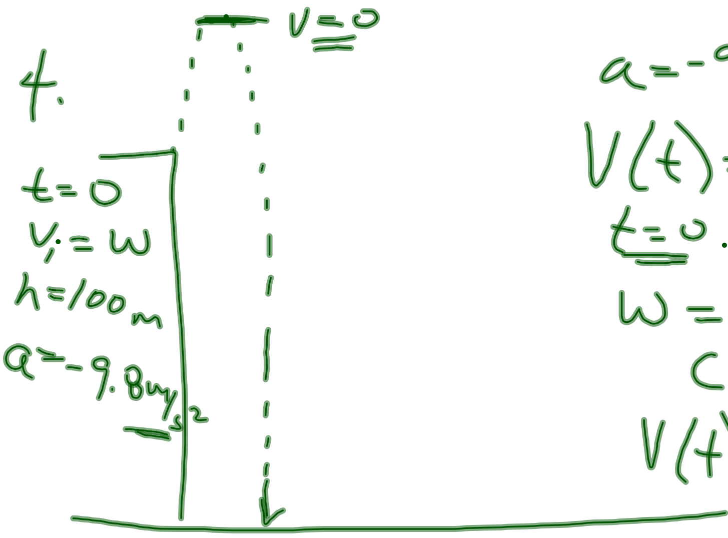
2. $y' = \dfrac{2}{(3x-5)^3} + \dfrac{5x}{x^2+1}$

$y' = \frac{2}{3}(3x-5)^{-3}(3) + \frac{5}{2}\left(\frac{2x}{x^2+1}\right)$ $\frac{du}{u}$

$y = \frac{2}{3}\left(\frac{1}{-2}\right)(3x-5)^{-2} + \frac{5}{2}\ln(x^2+1) + C$

$y = -\frac{1}{3}(3x-5)^{-2} + \frac{5}{2}\ln(x^2+1) + C$

4.



$a = -9.8\ m/s^2$

$V(t) = -9.8t + C$

$t = 0$:

$w = -9.8(0) + C$

$C = w$

$V(t) = -9.8t + w$

$h(t) = -4.9t^2 + wt + K$

$t = 0,\ h = 100$

$100 = -4.9(0)^2 + 0 + K$

$100 = K$

$$\boxed{h(t) = -4.9t^2 + wt + 100}$$

$t = 8,\ h = 0$

$0 = -4.9(8)^2 + w(8) + 100$

$213.6 = 8w$

$w = 26.7\ m/s$

$v(t) = -9.8t + 26.7$

$$\frac{-26.6}{-9.8} = \frac{-9.8t}{-9.8}$$

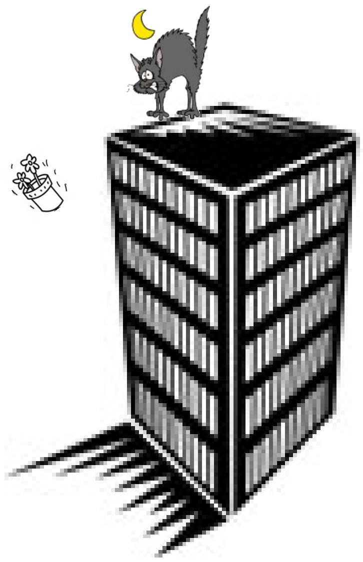
$t = 2.72$ sec

$h(2.72) = -4.9(2.72)^2 + 26.7(2.72) + 100$

$= 136.37\ m$

Motion Problems:

A cat, walking along the window ledge of a New York apartment, knocks off a flowerpot that falls to the street 122.5m below. How fast is the flowerpot travelling when it hits the street below?



Indeterminate forms and L'Hospital's Rule

Suppose we want to examine the following limit:

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \frac{0}{0}$$

What is the limit of both the denominator and numerator as x approaches 2?

Determine the value of this limit.

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$= 4$$

The Indeterminate Form 0/0

- In general, a limit of the form $\lim_{x \to a} \frac{f(x)}{g(x)}$ where both $f(x) \to 0$ and $g(x) \to 0$ as $x \to a$ may or may not exist.
- Such a limit is called an *indeterminate form of type* $\frac{0}{0}$; similarly, we also consider the indeterminate form $\frac{\infty}{\infty}$.

L'Hospital's Rule

L'Hospital's Rule Suppose f and g are differentiable and $g'(x) \neq 0$ near a (except possibly at a). Suppose that

$$\lim_{x \to a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \to a} g(x) = 0$$

or that

$$\lim_{x \to a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x \to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $\frac{0}{0}$ or ∞/∞.) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$).

L'Hospital's Rule says that the limit of a quotient of functions is equal to the limit of the quotient of their derivatives.

L'Hospital's Rule is also valid for one-sided limits and for limits at $\pm\infty$.

Examples:

$$\lim_{x\to 1}\frac{\ln x}{x^2 - x} = \frac{0}{0}$$

$$\lim_{x\to 1}\frac{\left(\frac{1}{x}\right)}{2x-1}$$ l'hospital's

$$= \frac{1}{1}$$

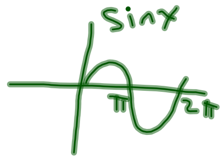
$$= 1$$

$$\lim_{x\to 0}\frac{e^{5x}-1}{3x} = \frac{0}{0}$$

$$\lim_{x\to 0}\frac{e^{5x}(5)}{3}$$

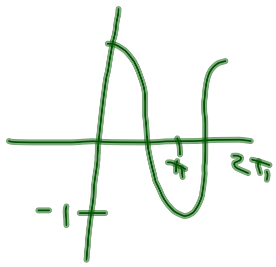
$$= \frac{5}{3}$$

$$\lim_{x\to 0}\frac{4x-\sin 4x}{x^3}$$



$$\lim_{x\to 0}\frac{4-\cos 4x\,(4)}{3x^2}$$

$$\lim_{x\to 0}\frac{+\sin(4x)(16)}{6x}$$



$$\lim_{x\to 0}\frac{+\cos(4x)(64)}{6}$$

$$= +\frac{64}{6}$$

$$= +\frac{32}{3}$$

$$\lim_{x\to \pi}\frac{\sin x}{1-\cos x} = \frac{0}{1-(-1)}$$

$$= \frac{0}{2}$$

$$= \frac{0}{2}$$

Also works for limits of quotients approaching ∞/∞

$$\lim_{x\to\infty} \frac{2x^3 - x^2 + 1}{1 - 5x^3} = \frac{\infty}{-\infty}$$

$$\lim_{x\to\infty} \frac{6x^2 - 2x}{-15x^2}$$

$$\lim_{x\to\infty} \frac{12x - 2}{-30x}$$

$$\lim_{x\to\infty} \frac{12}{-30}$$

$$= \frac{2}{-5}$$

$$\lim_{x\to\infty} \frac{e^x}{2x^2} \to \infty$$

D.N.E

$$\lim_{x\to\infty} \frac{e^x}{4x}$$

$$\lim_{x\to\infty} \frac{e^x}{4}$$

$$\lim_{x\to\infty} \frac{e^x}{0}$$

undefined

Practice Problems...

Page 303

#1 - 40

Odd numbered questions