Warm Up

The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

at $x=0$
$y=\sqrt{4}$
$y=2$ $(0,2)$

Slope: $y' = \frac{1}{2}(4+\sin x)^{-1/2}(\cos x)$
at $x=0$: $y' = \frac{1}{2}(4+0)^{-1/2}(1) = \frac{1}{4}$

B

Tangent Line Approximation: $y-2 = \frac{1}{4}(x-0)$
$y = \frac{1}{4}x+2$
$y = \frac{1}{4}(0.12)+2 \approx 2.03$

Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

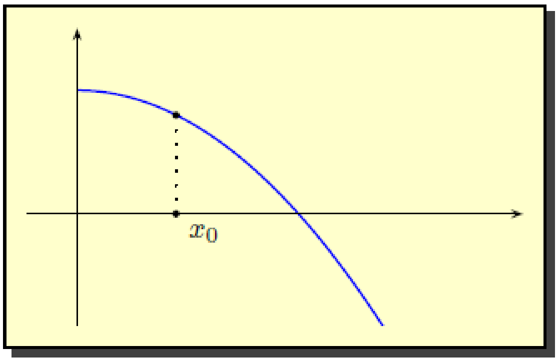
(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

$(3,2)$
$m=5$
$y-2=5(x-3)$
$y=5x-13$

Find zero: $(y=0)$ $0=5x-13$
$\frac{13}{5}=x$
$2.6=x$

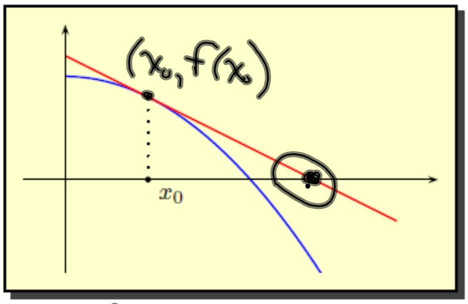
Newton's Method

Problem: Given an equation $f(x) = 0$, solve for x numerically.



- Make an *initial guess*: x_0. Now go up to the curve.

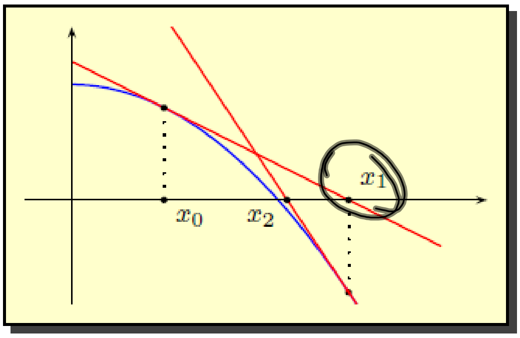
- Draw the tangent line.



- Let x_1 be in x-intercept of this tangent line.

$y - f(x_0) = f'(x_0)(x - x_0)$

Sub. $y = 0$ (x-Int)

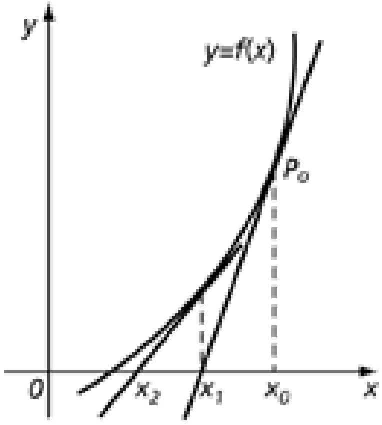


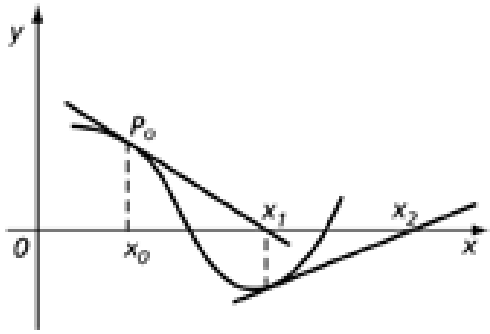
- This intercept is given by the formula: $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$.
- Now repeat using x_1 as the initial guess.
- The intercept x_2 is given by: $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$.

$$-f(x_0) = f'(x_0)(x) - f'(x_0)x_0$$

$$\frac{-f(x_0) + f'(x_0)(x_0)}{f'(x_0)} = x$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$


y
y=f(x)
P₀
0
x₂
x₁
x₀
x


y
P₀
x₁
x₂
0
x₀
x

Newton Iteration formula: $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

The initial guess x_0, and the Newton Iteration formula, together form an *algorithm* or a procedure of estimating the value of the root to the equation $f(x) = 0$.

Find the positive root of the equation $x^2 = 2$.

$$\underbrace{x^2 - 2}_{f(x)} = 0 \qquad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Guess: $x_0 = 1$ $\qquad f'(x) = 2x$

$$x_1 = 1 - \frac{(-1)}{(2)} \qquad (1)^2 - 2 \qquad 2(1)$$

$$x_1 = \frac{3}{2}$$

$$x_2 = \frac{3}{2} - \frac{0.25}{3} = 1.4167$$

$$x_3 = 1.4167 - \frac{0.00704}{2.8334} = 1.4142$$

$$x_4 = 1.4142 - \frac{-0.0000386}{2.8284} = 1.41421$$

X	Y1	Y2
1	-1	2
1.5	.25	3
1.4167	.00704	2.8334

X=

Newton's Method		
$f(x) = x^2 - 2$, $x_0 = 1.5$		
n	x_n	$f(x_n)$
0	1.50000000	0.25000000
1	1.41666667	0.00694445
2	1.41421568	0.00000600
3	1.41421356	0.00000000
4	1.41421356	0.00000000

Example 3.2. Find a solution to the equation $x^3 = x + 1$ that is near $x_0 = 1.5$.

$x^3 - x - 1 = 0$

$\therefore f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

$x_1 = 1.5 - \frac{0.875}{5.75} = 1.3478$

$x_2 = 1.3478 - \frac{0.10057}{4.4497} = 1.3252$

$x_3 = 1.3252 - \frac{0.00206}{4.2685} = 1.3247$

$x_4 = 1.3247 - \frac{-(0.000076)}{4.2645} = 1.3247$

Newton's Method		
$f(x) = x^3 - x - 1$,	$x_0 = 1.5$	
n	x_n	$f(x_n)$
0	1.50000000	0.87500000
1	1.34782608	0.10058217
2	1.32520039	0.00205836
3	1.32471817	0.00000092
4	1.32471795	0.00000000
5	1.32471795	0.00000000

Practice Problems...

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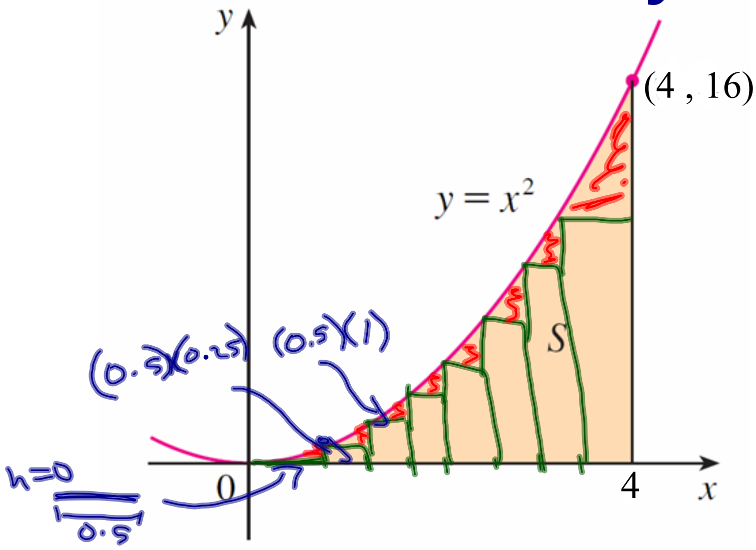
Areas bound by curves...

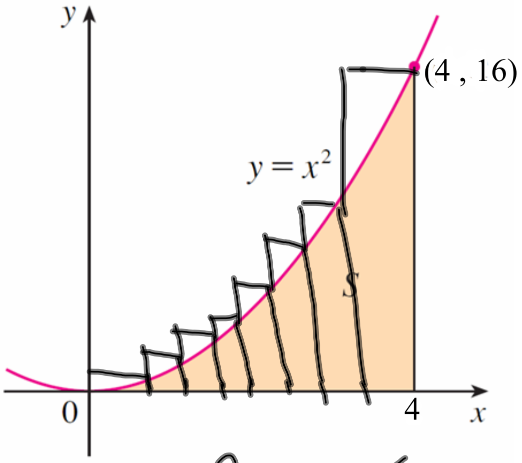


(1) Determine the area of region S by using the **left** endpoints of 8 equal subintervals.

$A = 0.5(0 + 0.25 + 1 + 2.25 + 4 + 6.25 + 9 + 12.25)$

$A \doteq 17.5$

(2) Determine the area of region S by using the **right** endpoints of 8 equal subintervals.



x	$f(x)$
0	0
0.5	0.25
1	1
1.5	2.25
2	4
2.5	6.25
3	9
3.5	12.25
4	16

$A = 0.5(0.25 + 1 + 2.25 + 4 + 6.25 + 9 + 12.25 + 16)$

$A \approx 25.5 u^2$

(3) Determine the area of region S by using 8 subintervals and the **trapezoidal rule**.

Attachments

Worksheet - Complex Numbers.doc

Worksheet - Converting Polar_Rectangular Coordinates.doc

Worksheet Solns - Complex Numbers.doc