Sequences

- an ordered arrangement of numbers, symbols, or pictures in which each item or term follows another according to a rule.

The first number sequence that you encountered was probably the natural number sequence.

Each number in the sequence is called a TERM of the sequence.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, . . .

The following symbols are used to represent the terms in any sequence.

$t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, \ldots, t_n, \ldots$

As a result, for the natural number sequence above,

$t_1 = 1, t_2 = 2, t_3 = 3, t_4 = 4, \ldots, t_n = n, \ldots$

General Term

A sequence is a function of the natural numbers. To describe any term of a sequence, an expression is used for t_n , where n is a member of the natural numbers. The expression is called the GENERAL TERM of the sequence.

Example 1: Find the first 4 terms of the sequence...

$t_n = 3n^2 - 2n$

$f(x) = x^2$
$f(3) = (3)^2$

$t_1 = 3(1)^2 - 2(1)$
$= 3 - 2$
$= 1$

$t_2 = 3(2)^2 - 2(2)$
$= 3(4) - 4$
$= 8$

$t_3 = 3(3)^2 - 2(3)$
$t_3 = 21$

$t_4 = 48 - 8$
$= 40$

1, 8, 21, 40, ...

Find the next 3 terms in each sequence...

1) 3, 10, 17, 24, 31 , 38 , 45 , ...

2) 1, 4, 9, 16, 25 , 36 , 49 , ...

3) 1/2, 1/4, 1/8, 1/16, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, ...

4) $3x^2$, $6x^2$, $12x^2$, $24x^2$, $48x^2$, $96x^2$, ...

(Arithmetic) Linear Sequences

Each successive term is formed by adding a value to the previous term.

Ex: a) 3, 5, 7, 9, 11, ...
b) -10, -15, -20, -25, ...

The difference between any two consecutive terms is called the *COMMON DIFFERENCE*.

Finding the equation of a linear sequence...

$$t_n = t_1 + d(n-1)$$

t_n - is the general term
t_1 - is the first term (sometimes "a")
d - is the common difference
n - is the term position

EXAMPLES...

#1. State the first five terms of the arithmetic sequence given that...
a = 9 & d = -3

$t_1 = 9$
$t_2 = 9 + (-3)(2-1)$
$= 9 + -3$
$= 6$
$t_n = 9 + (-3)(n-1)$
$t_3 = 9 + (-3)(3-1)$
$= 9 + (-3)(2)$
$= 3$ ...
→ 9, 6, 3, 0, -3

#2. Find t_{67} and t_n for the following sequence...
7, -1, -9, -17, -25, ...

$d = -8$ $t_1 = 7$
$t_n = t_1 + d(n-1)$
$t_{67} = 7 - 8(67-1)$
$= 7 - 8(66)$
$= -521$

#3. How many terms are in the following sequence?...
-5, -2, 1, 4, ..., 103 ($t_?$)

$t_n = t_1 + d(n-1)$
$103 = -5 + (3)(n-1)$
$103 = -5 + 3n - 3$
$103 + 5 + 3 = 3n$
$\frac{111}{3} = \frac{3n}{3}$
$37 = n$

#4. How many multiples of 5 would be found between 37 and 498 754?

99743

40, 45, 50, ..., 498750

$498750 = 40 + (n-1)(5)$

$498750 = 40 + 5n - 5$

$n = 99743$

ONE MORE EXAMPLE... System of equations???

#5. Given that $t_{10} = -22$ & $t_{35} = -97$, determine t_n.

$t_{10} = t_1 + (10-1)d$

$-22 = t_1 + 9d$

Attachments

Worksheet - Linear Sequences.doc