Express the following as a single logarithm in simplest form:

$\log_2 x^{-69}$

$$6\log_2 \sqrt{x} - \frac{3}{4}\left[4\log_2 x^3 + 12\left(\log_2 x - 3\log_2 x^{-2}\right)\right]$$

$= 6\log_2 x^{1/2} - 3\log_2 x^3 - 9(\log_2 x - 3\log_2 x^{-2})$

$= 6\log_2 x^{1/2} - 3\log_2 x^3 - 9\log_2 x + 27\log_2 x^{-2}$

OR

$= 3\log_2 x - 9\log_2 x - 9\log_2 x - 54\log_2 x$

$= -69\log_2 x = \log_2 x^{-69}$

$= \log_2 x^3 - \log_2 x^9 - \log_2 x^9 + \log_2 x^{-54}$

$= \log_2\left(\dfrac{x^3 x^{-54}}{x^9 x^9}\right)$

$= \log_2\left(\dfrac{x^{-51}}{x^{18}}\right)$

$= \log_2\left(x^{-69}\right)$

Given that $\log_r x = -6$, $\log_r y = -3$, and $\log_r z = 8$, evaluate the expression $\log_r\left(\dfrac{\sqrt{x^5 z^3}}{r^{-3} y^5}\right)$.

$= \log_r x^{5/2} + \log_r z^{3/2} - \log_r r^{-3} - \log_r y^5$

$(x^5 z^3)^{1/2}$

$= \frac{5}{2}\log_r x + \frac{3}{2}\log_r z + 3\log_r r - 5\log_r y$

$\log_b b^m = m$

$= \frac{5}{2}(-6) + \frac{3}{2}(8) + 3(1) - 5(-3)$

$= -15 + 12 + 3 + 15$

$= 15$

Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate $y = e^x$.

What other function could this model?

$\log_e y = x$

$\ln y = x$

Natural logarithm: $\log_e \Rightarrow \ln$

Base "e"

Try to differentiate → $y = \ln x$.

$e^y = x$

$e^y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$\frac{dy}{dx} = \frac{1}{x}$

Differentiate: $y = \ln x^3$

$e^y = x^3$

$e^y \frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{3x^2}{e^y}$

$\frac{dy}{dx} = \frac{3x^2}{x^3}$ $\Longrightarrow d(\ln u) = \frac{du}{u}$

$\frac{dy}{dx} = \frac{3}{x}$ $= \frac{1}{u} \cdot du$

Rule: $d(\ln u) = \frac{1}{u} du$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Differentiate:

$$y = \log_6 x^3$$

$$y = \frac{\ln x^3}{\ln 6}$$

$$y = \frac{1}{\ln 6} \ln x^3$$

$$y' = \frac{1}{\ln 6}\left(\frac{3x^2}{x^3}\right)$$

$$y = \log\left(5x^4\right)$$

$$y' = \frac{1\ (20x^3)}{5x^4(\ln 10)}$$

$$\boxed{d(\log_b u) = \frac{du}{u \ln b}}$$

Rule: $d\left(\log_b u\right) = \dfrac{1}{u \ln b}\, du$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

$\ln y = \ln 3^{9x}$

$\ln y = 9x \ln 3$

$\frac{1}{y}\frac{dy}{dx} = 9 \ln 3$

$\frac{dy}{dx} = y(9)\ln 3$

$\frac{dy}{dx} = 3^{9x}(9)(\ln 3)$

$d(3^{9x}) = 3^{9x}(9)\ln 3$

Try this one... $y = \pi^{x^5}$

$\log_\pi y = \log_\pi \pi^{x^5}$

$\log_\pi y = x^5 \log_\pi \pi$

$\frac{1}{y \ln \pi}\frac{dy}{dx} = 5x^4(1)$

$\frac{dy}{dx} = y \ln \pi (5x^4)$

$\frac{dy}{dx} = \pi^{x^5} \ln \pi (5x^4)$

$y' = \pi^{x^5}(\ln \pi)(5x^4)$

$d(b^u) =$

Rule:

$$d(b^u) = b^u(\ln b)du \ , where \ \ b \in R$$

examples:

$$y = \log_7 x^8 - \ln(x^2-3)^5 + 8^{\cot x^3} - e^{\sin^{-1}(\sqrt{x})}$$

$$d(b^u) = b^u \ln b \, du \qquad d(\log_b u) = \frac{1}{u \ln b} du$$

$$d(e^u) = e^u \cdot du \qquad d(\ln u) = \frac{1}{u} du$$

$$y' = \frac{1}{x^8 \ln 7}(8x^7) - \frac{5(x^2-3)^4(2x)}{(x^2-3)^5} + 8^{\cot x^3} \ln 8 \left(-\csc^2 x^3 (3x^2)\right) - e^{\sin^{-1}\sqrt{x}} \left(\frac{1}{\sqrt{1-x}} \frac{1}{2} x^{-\frac{1}{2}}\right)$$

$$y = \frac{\log^3\left(\cot^{-1}\left(\ln x^5\right)\right)}{\left(8^{5x^4}\right)\ln\left(\sec x^5\right)}$$

$$\log^3 x = \left(\log x\right)^3$$

Practice Problems:

Page 383 - 384

#1 #2 a #3 #4

#5 #6 #7 #8

Page 367

#4, 5, 6, 8, 9, 10,

Attachments

Worksheet - Primary Trig Ratios.doc