Warm Up

Review of laws of logarithms...

Solve the following equation: $\frac{3^{x-1}}{5 * 2^{3x}} = 6^{1-2x}$

$$\ln\left(\frac{3^{x-1}}{5 \cdot 2^{3x}}\right) = \ln 6^{1-2x}$$

$$\ln 3^{x-1} - \ln 5 - \ln 2^{3x} = \ln 6^{1-2x}$$

$$(x-1)\ln 3 - \ln 5 - 3x(\ln 2) = (1-2x)\ln 6$$

$$x = \frac{\ln 6 + \ln 5 + \ln 3}{\ln 3 - 3\ln 2 + 2\ln 6}$$

Curve Sketching Review...Test on Friday!!

Consider the function : $f(x) = -\dfrac{27(x-2)^2}{(x-3)^3}$

given $f'(x) = \dfrac{27x(x-2)}{(x-3)^4}$ and $f''(x) = \dfrac{54(x^2-3)}{(x-3)^5}$

Supply the information requested in the boxes at right and give a careful sketch of f on the axes below.

x-Int: (y=0)

$0 = -\dfrac{27(x-2)}{(x-3)^3}$

$x = 2$

$(2,0)$

y-Int: (x=0)

$y = -\dfrac{\cancel{27}(0-2)^2}{\cancel{(0-3)^3}}$

$= 4$

$(0,4)$

Vertical Asymptote:

* f(x) undefined

$(x-3)^3 = 0$

$x = 3$

Horizontal Asymptote:

$(x-3)^3 = (x-3)^2(x-3)$

$= (x^2-6x+9)(x-3)$

$= x^3-3x^2-6x^2+18x+9x-27$

$= x^3-9x^2+27x-27$

$$\lim_{x\to\pm\infty} \frac{-27(x^2-4x+4)}{x^3-9x^2+27x-27}$$

$$\lim_{x\to\pm\infty} \frac{-\frac{27x^2}{x^3}+\frac{108x}{x^3}-\frac{108}{x^3}}{\frac{x^3}{x^3}-\frac{9x^2}{x^3}+\frac{27x}{x^3}-\frac{27}{x^3}}$$

$$= \frac{0+0-0}{1-0+0-0}$$

$\boxed{y=0}$

$$f'(x)=\frac{27x(x-2)}{(x-3)^4}$$

Critical Values:

$x=0,2,3$

	$27x$	$x-2$	$(x-3)^4$	f'	f
$(-\infty,0)$	−	−	+	+	Inc
$(0,2)$	+	−	+	−	Dec
$(2,3)$	+	+	+	+	Inc
$(3,\infty)$	+	+	+	+	Inc

Increase

$(-\infty,0)$ & $(2,3)$ $(3,\infty)$

~~or~~ $(2,\infty)$

Decrease

$(0,2)$

Local Max.

$(0,4)$

Local Min.

$(2,0)$

$$f''(x)=\frac{-54(x^2-3)}{(x-3)^5}$$

Critical Values:

$x^2-3=0 \rightsquigarrow (x-\sqrt{3})(x+\sqrt{3})=0$

$x^2=3$

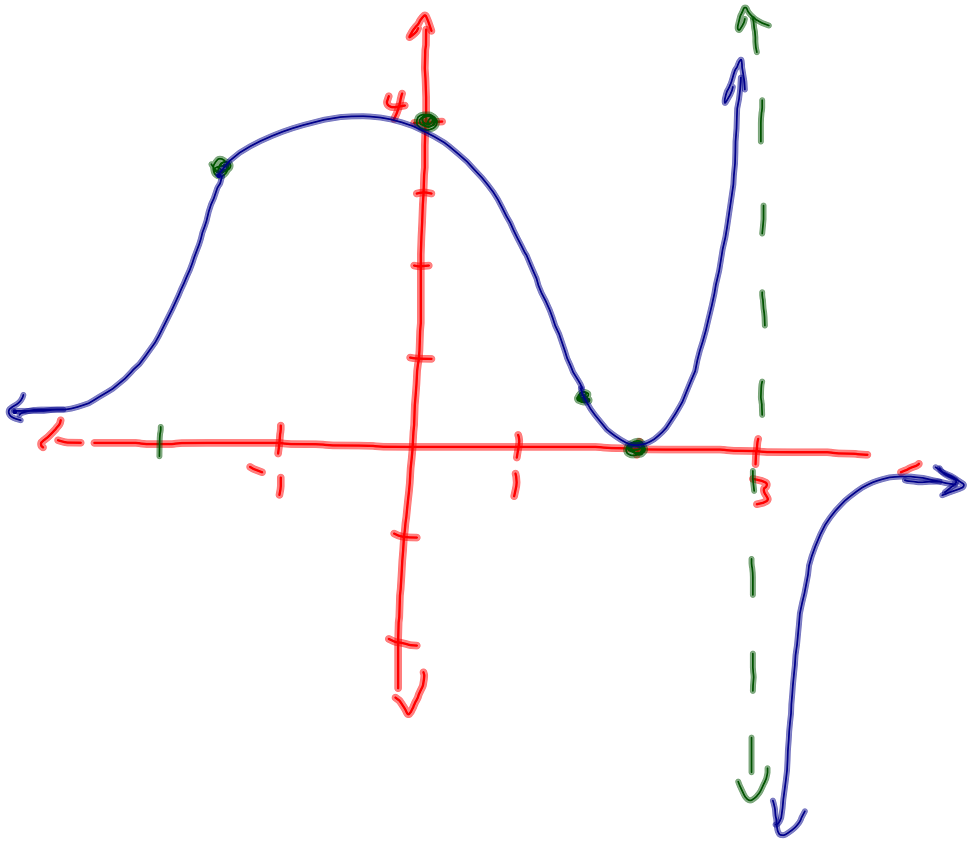
$x=\pm\sqrt{3}$

$x=\pm\sqrt{3}$ and $x=3$

	54	x^2-3	$(x-3)^5$	f''	f
$(-\infty,-\sqrt{3})$	~~+~~	+	−	−	~~Down~~ Up
$(-\sqrt{3},\sqrt{3})$	~~+~~	−	−	+	~~Up~~ Down
$(\sqrt{3},3)$	~~+~~	+	−	−	~~Down~~ Up
$(3,\infty)$	~~+~~	+	+	+	~~Up~~ Down

Inflection Points:

$(-\sqrt{3},3.6)$ $(\sqrt{3},0.95)$


4
-1
1
3

Logarithmic Differentiation

A differentiation process that requires taking the logarithm of both sides before differentiating.

This process will be used in TWO circumstances:

I. Simplifying messy products and quotients

What would it involve to differentiate the following?

$$y = \frac{(x^2-1)^5\sqrt{2x+9}\,(5x^3+2)^8}{(10x-1)\sqrt{5-x^7}}$$

- Quotient rule, multiple product rules and chain rules...

  This would be possible but it would be easier to differentiate a group of terms added and subtracted rather than multiplied and divided

Laws of logarithms will do exactly that...turn this mess into a addition and subtraction of terms.

$$y = \frac{(x^2-1)^5\sqrt{2x+9}\,(5x^3+2)^8}{(10x-1)\sqrt{5-x^7}}$$

$$\ln y = \ln\left(\; - \; - \; - \; - \; \right)$$

$$\ln y = 5\ln(x^2-1) + \frac{1}{2}\ln(2x+9) + 8\ln(5x^3+2) - \ln(10x-1) - \frac{1}{2}\ln(5-x^7)$$

$$(y)\,\frac{1}{y}\frac{dy}{dx} = \left[\frac{5(2x)}{x^2-1} + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{2x+9}\right) + 8\left(\frac{15x^2}{5x^3+2}\right) - \frac{10}{(10x-1)} - \frac{1}{2}\left(\frac{-7x^6}{5-x^7}\right)\right](y)$$

$$\frac{dy}{dx} = \left(\frac{10x}{x^2-1} + \frac{1}{2x+9} + \frac{120x^2}{5x^3+2} - \frac{10}{10x-1} + \frac{7x^6}{2(5-x^7)}\right)\left[\frac{(x^2-1)^5\sqrt{2x+9}\,(5x^3+2)^8}{(10x-1)\sqrt{5-x^7}}\right]$$